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A SUBDOMAIN DECOMPOSITION METHOD FOR LARGE ACOUSTIC CAVITIES

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1.INTRODUCTION

Ground, sea and aerospace transportation industries can provide vehicles with large dimensions as busses, trains, ships and planes.
Consequently, the size of the eigenvalue problem providing modes of acoustic cavities requires a very large computational effort. An efficient subdomain decomposition technique is proposed in order to compute the eigenproperties and the frequency response of large acoustic cavities.

2.THE SUBDOMAIN DECOMPOSITION METHOD

Let us consider an acoustic cavity Ω surrounded by a rigid wall S. The acoustic pressure P satisfies the Helmholtz equation (1) in the cavity with associated boundary condition .

$$\left.\begin{array}{c} \Delta P(M) + k^2 P(M) = 0 \\ \left.\dfrac{\partial P}{\partial n}\right]_S = 0 \end{array}\right\} \qquad (1)$$

where $k = \frac{\omega}{c}$ is the wave number given in terms of the circular frequency ω and the acoustic velocity c within the acoustic domain.
To solve the eigenvalue problem (1), we decompose Ω into several subdomains Ω_i separated by fictitious interfaces Γ_{ij} [1].

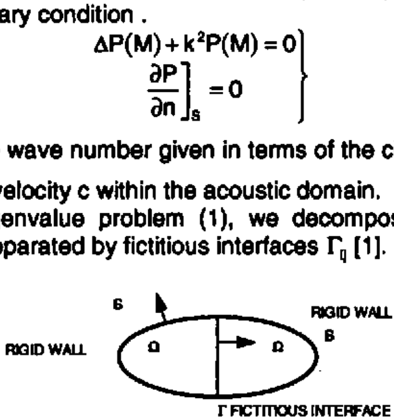

Fig.1 Subdomain decomposition of a cavity

$$\Delta P_1(M)+k^2P_1(M)=0 \qquad (2)$$
$$\left.\frac{\partial P}{\partial n}\right]_{S_1}=0 \qquad (4)$$
$$\Delta P_2(M)+k^2P_2(M)=0 \qquad (3)$$
$$\left.\frac{\partial P}{\partial n}\right]_{S_2}=0 \qquad (5)$$
$$\left.(P_1-P_2)\right]_\Gamma=0 \qquad (6)$$
$$\left.\frac{\partial P_1}{\partial n}\right|_\Gamma=\left.\frac{\partial P_2}{\partial n}\right|_\Gamma \qquad (7)$$

Here, we consider only two subdomains (fig.1).
This leads to calculate the acoustic pressures P_1 and P_2 in each subdomain Ω_1 and Ω_2 which verify the Helmholtz equations (2) and (3), the rigid wall condition (4) and (5) and compatibility conditions which are the continuity of the acoustic pressure (6) and the continuity of the acoustic velocity (7) on the fictitious interface Γ.

Therefore, the solution of the problem (1) renders the following functional stationnary [2]:

$$\begin{aligned}\Pi(P_1,P_2,\lambda)=&\frac{1}{2}\int_{\Omega_1}\left\langle\vec{\nabla}P_1(M),\vec{\nabla}P_1(M)\right\rangle-k^2.P_1(M).P_1(M)\,dV\\&+\frac{1}{2}\int_{\Omega_2}\left\langle\vec{\nabla}P_2(M),\vec{\nabla}P_2(M)\right\rangle-k^2.P_2(M).P_2(M)\,dV\\&-\int_\Gamma\lambda(P_1-P_2).d\Gamma\end{aligned} \qquad (8)$$

where λ is the Lagrange multiplier which represents the normal acoustic pressure derivative on the interface

The discretization of the functional Π leads to the following symmetric eigenvalue problem (9) :

$$\left(\begin{bmatrix}H_1 & 0 & -C_1^T\\ 0 & H_2 & C_2^T\\ -C_1 & C_2 & 0\end{bmatrix}-k^2\begin{bmatrix}Q_1 & 0 & 0\\ 0 & Q_2 & 0\\ 0 & 0 & 0\end{bmatrix}\right)\begin{Bmatrix}P_1\\P_2\\\lambda\end{Bmatrix}=\begin{Bmatrix}0\\0\\0\end{Bmatrix} \qquad (9)$$

where,
$[H_1]$ and $[H_2]$ are the "stiffness" matrices of the subdomain Ω_1 and the subdomain Ω_2.
$[Q_1]$ and $[Q_2]$ are the mass matrices of the subdomain Ω_1 and the subdomain Ω_2.
$[C_1]$ and $[C_2]$ represents the coupling between the subdomains Ω_1 and Ω_2 through the interface Γ and the Lagrange multiplier λ.
This system is larger than the global one, obtained without using the subdomain decomposition method.

To reduce the size of the algebric system (9), a modal synthesis approach is used [1]. The acoustic pressure in each subdomain is expended on an appropriate basis $\left[\Phi_k\right]$:

$$\left\{P_k\right\} = \left[\Phi_k\right]\left\{\alpha_k\right\} \tag{10}$$

where $\left\{\alpha_k\right\}$ are the modal components.

This leads to the following reduced system (11) :

$$\left(\left[\begin{array}{c|c|c} \Phi_1^T H_1 \Phi_1 & 0 & -\Phi_1^T C_1^T \\ \hline 0 & \Phi_2^T H_2 \Phi_2 & \Phi_2^T C_2^T \\ \hline -C_1\Phi_1 & C_2\Phi_2 & 0 \end{array}\right] - k^2 \left[\begin{array}{c|c|c} \Phi_1^T Q_1 \Phi_1 & 0 & 0 \\ \hline 0 & \Phi_2^T Q_2 \Phi_2 & 0 \\ \hline 0 & 0 & 0 \end{array}\right]\right)\left\{\begin{matrix} \alpha_1 \\ \alpha_2 \\ \lambda \end{matrix}\right\} = \left\{\begin{matrix} 0 \\ 0 \\ 0 \end{matrix}\right\} \tag{11}$$

3.EIGENMODE EXTRACTION OF THE SUBDOMAIN COMPONENT

The local modes Φ_k verify the Helmholtz equation (12), the rigid wall condition (13) combined with an *ad hoc* boundary condition on the fictitious interface (14), (15) or (16).

$$\Delta\Phi + k^2\Phi = 0 \tag{12}$$

$$\left.\frac{\partial\Phi}{\partial n}\right]_{S_k} = 0 \tag{13}$$

Three kinds of boundary conditions on the fictituous interface are used :

a) the homogenous Neumann condition : $\left.\frac{\partial\Phi}{\partial n}\right]_\Gamma = 0$ (14)

b) the homogenous Dirichlet condition : $\Phi\big]_\Gamma = 0$ (15)

c) the mixed Robin condition [3]: $\left.\frac{\partial\Phi}{\partial n} - \frac{\omega^2}{c^2}\kappa\Phi\right|_\Gamma = 0$ (16)

The Robin condition links normal pressure derivative and pressure on the interface. This condition generates local modes on the interface.

The κ parameter determines the number of local modes in a given frequency range.

4.NUMERICAL IMPLEMENTATION

The subdomain decomposition method envolves the implementation of the following steps :

a) decomposttion of the global cavity
b) computation of the local modes for each subdomain (12),(13).
   Three kind of boundary condition on the interface connecting the subdomain are proposed (14),(15),(16)
c) solving the eigenvalue problem (11)
d) synthesis of the modeshapes of the acoustic cavity (10)

5.NUMERICAL RESULTS

A rectangular acoustic cavity with rigid walls (2 m×0.8 m) is used to validated the proposed subdomain decomposition method. The cavity is split in two subdomains (1. m×0.8 m and 1 m×0.8 m). Figure 2 shows a comparison between analytical and numerical results obtained with the three kinds of boundary conditions on the interface.

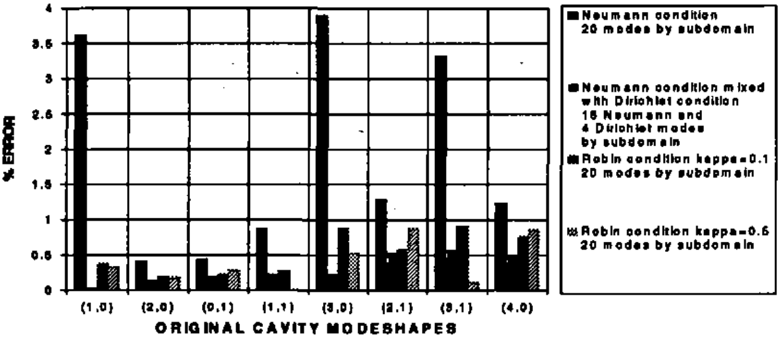

Fig.2 : Comparison between analytical and numerical eigenvalues

The fictitious interface is located on a nodal line of the global modes (1,0), (1,1), (3,0) and (3,1). Thus, the component modes computed with the Neumann condition are not well adapted to synthesize these global modes. Results are improved if these component modes are combined with those computed with the Dirichlet condition. The error between analytical and numerical eigenfrequencies is less than 1% when component modes are computed with the Robin condition on the interface. This condition allows to take less number of component modes to obtain global eigenproperties.

6.CONCLUSION

A modal synthesis method is presented to calculate eigenproperties of large acoustic cavities. The method is based on a modular decomposition and authorizes small memory space. Numerical results are in good agreement with analytical results. This method will be adapted for solving vibro-acoustic coupling problems.

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