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CHARACTERISATION OF PARTIALLY COHERENT RANDOM FORCES BY MEANS OF PCA

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Introduction

The forces acting on the mechanical structures, which in most cases are random, partially coherent, often can not be directly determined because of the impossibility for placing force transducers. Some indirect ways have been developed recently to characterize these forces from the measurement of dynamic responses on the structure, such as what is proposed by Roggentamp and Bernhard[1]. As the simultaneous response cross-spectra measurement between each response is required in this method, the acquisition of these response cross-spectra demands the experimental instruments possessing numerous channels when a great number of responses are required. In the present paper, we will address another indirect method of force determination, which adopts some reference signals and decomposes the vibration field using the technique Principal Components Decomposition (PCA), a technique usually employed to deal with the mutually uncorrelated or partially correlated source problems. This method allows it possible to measure the vibration field by the scanning acquisition technique using one laser vibrometer.

Force determination with use of PCA

Considering a MIMO system with M input forces and N output dynamic responses. If the M input forces are totally coherent, the relationship between the input forces and the dynamic responses can be expressed as follows:

$$[\mathbf{a}]_{N\times 1} = [\mathbf{H}]_{N\times M}[\mathbf{F}]_{M\times 1}, \tag{1}$$

and the unknown forces can be derived by:

$$[\mathbf{F}]_{M\times 1} = [\mathbf{H}]^{+}_{M\times N}[\mathbf{a}]_{N\times 1}. \tag{2}$$

where, $[\mathbf{a}]$ is the dynamic responses matrix whose elements represent the complex frequency spectra of vibration (acceleration, vibrating velocity or displacement) on the measured structure, $[\mathbf{H}]$ the Frequency Response Functions matrix and $[\,]^+$ denotes the pseudo-inverse operation of matrix.

We note that equations(1),(2) are applicable only in the case where the input forces are totally coherent. For incoherent or partially coherent forces, which are often the circumstances encountered in the industrial applications, it is impossible to use it directly. One possible way of reach it is to characterize the vibration field in terms of a set of reference signals [2], as realized in the technique NAH (Nearfield Acoustic Holography) on uncorrelated sound sources. If there are some linearly independent reference signals which can represent the vibration field produced by the actual force sources, these reference signals can be utilized to decompose the dynamic vibration field produced by those incoherent force sources into a set of partial fields. Each partial field is fully coherent so equation (2) is adaptable to calculate individually its corresponding force component. Since the partial vibrating fields are mutually independent, they can be summed on an energy basis in order to reconstruct the forces. The technique PCA is capable of doing it.

To perform PCA, we chose L reference signals representing the vibration field produced by the actual force sources. The spectral matrix $[\mathbf{G}_{RR}]_{L\times L}$ containing the auto-spectra of each reference signal and cross-spectra between each of them, the cross-spectral matrix $[\mathbf{G}_{Ra}]_{N\times L}$ between N vibrating responses and L reference signals should be measured. The Singular Values Decomposition (SVD) technique is employed to factorize the spectral matrix of references into the form of eigenvalues together with eigenvectors:

$$[\mathbf{G}_{RR}]_{L\times L} = [\mathbf{U}_R]_{L\times L}\lceil \mathbf{G}_{\Sigma\Sigma}\rfloor_{L\times L}[\mathbf{U}_R]^H_{L\times L}\,. \tag{3}$$

$[\mathbf{U}_R]$ is an unitary matrix containing in its columns the eigenvectors, $\lceil \mathbf{G}_{\Sigma\Sigma}\rfloor$ is a diagonal matrix consisting of r non-zero elements arranged in decreasing order. According to the virtual sources theory, $\lceil \mathbf{G}_{\Sigma\Sigma}\rfloor$ can be considered as the autopower spectra of a set of uncorrelated principal (or virtual) references related to the physical references by means of the transformation $[\mathbf{U}_R]$, if $\lceil \mathbf{G}_{\Sigma\Sigma}\rfloor$ is rank deficient ($r < L$),$\lceil \mathbf{G}_{\Sigma\Sigma}\rfloor$ can be approximated by using the r significant eigenvalues and the corresponding eigenvectors, such as

$$[\mathbf{G}_{RR}]_{L\times L} = [\mathbf{U}'_R]_{L\times r}\lceil \mathbf{G}_{\Sigma\Sigma}\rfloor_{r\times r}[\mathbf{U}'_R]^H{}_{r\times L}\,. \tag{4}$$

The reduced cross-spectral matrix between the vibration responses and virtual references can be transformed into:

$$[\mathbf{G}_{\Sigma a}]_{N\times r} = [\mathbf{G}_{Ra}]_{N\times L}[\mathbf{U}'_R]_{L\times r}\,. \tag{5}$$

Each column of this matrix $[\mathbf{G}_{\Sigma a}]$ represents the cross-spectra associating the responses to one principal reference. They can be considered as the r columns of the matrix of the complex vibrating responses $[\mathbf{a}]_{N\times r}$, each column represents an independent and spatially coherent complex vibrating response field produced by one principal reference and can be used to obtain individually its original force source component. A number of r statistical independent force components calculated by the equation (2) can be summed on a energy basis to reconstruct the original force field.

We note in this method, the forces are characterized relatively to multiple references. For a stationary field, this method enables us to measure the vibration responses on the structure by the scanning with one vibrometer instead of the parallel measurement with numerous vibrometers, which makes it very adequate for determining indirectly the forces acting on the industrial structures.

Experimental Investigation

To verify experimentally this inverse method, an experimental configuration shown in Fig.1 is utilized. Two mechanical shakers excite a rectangular plate at points F_1, F_2 with random white noise signal over the frequency range from 0 to 2000 Hz. A force transducer is inserted between each shaker and the plate. The accelerations at prescribed points $a_1, a_2 \ldots a_6$ on the plate are surveyed successively by moving one accelerometer. We take the output signals from two force transducers as the references, the cross-spectral matrix between two references $[\mathbf{G}_{RR}]_{2\times 2}$ and the cross-spectral matrix between two references and six acceleration responses $[\mathbf{G}_{Ra}]_{6\times 2}$ are measured. The FRFs matrix $[\mathbf{H}]_{6\times 2}$ is obtained separately by the single shaker excitation path. The forces F_1, F_2 obtained by this inverse method by means of PCA are compared with that measured directly by the force transducers (Fig.2). It can be seen that they are in a good agreement.

Conclusions

The utilisation of Principal Component Analysis provides another possibility to determine indirectly the random, partially coherent forces. The adoption of some references can reduce the requirement on experimental equipments and enables us to measure the vibration responses by the scanning with one laser vibrometer, which makes it very suitable for characterizing indirectly the forces applied to the industrial mechanical structures.

References

[1] T.J. Roggentamp, R.J. Bernhard, "Indirect Measurement of Multiple Random Force Spectra", Inter-noise 93, Leuven, Belgium, August 24-26 1993, 881-884.

[2] J.F. Li, J.C. Pascal, C. Carles, "Reconstruction of partially coherent sources by use of Principle Component Analysis", Inter-noise 95, Newport Beach,USA, July 10-12 1995, 1355-1358.

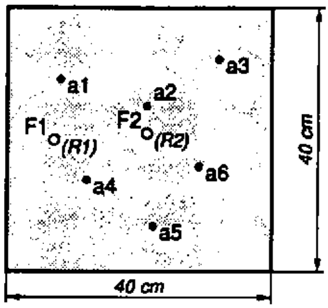

Fig.1: Sources, references and responses locations on the plate.

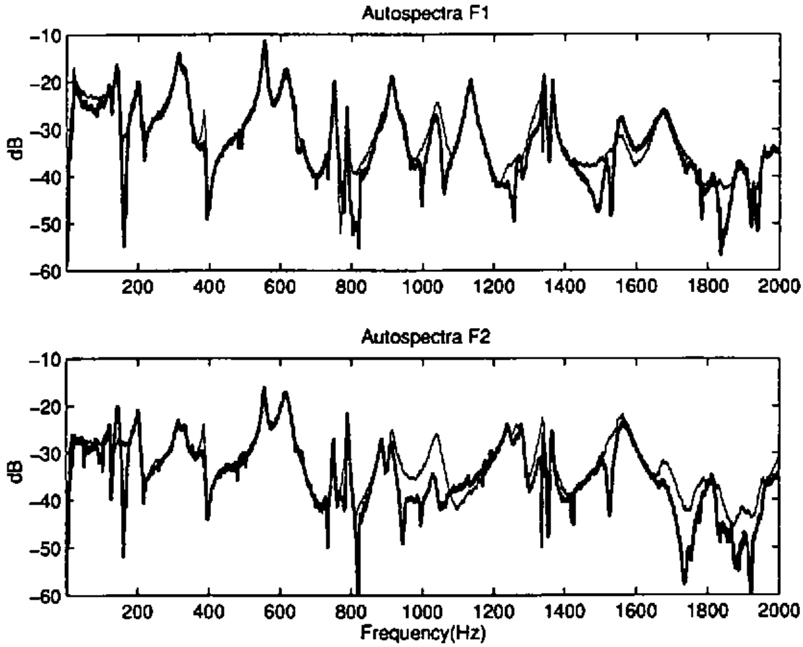

Fig.2: Comparison of the forces F_1, F_2: measured by force transducer (deep line); obtained via the inverse method using PCA (light line).