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ROBUST VIBRATION CONTROL OF FLEXIBLE STRUCTURES BY USING PIEZOELECTRIC DEVICES

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1. INTRODUCTION

Flexible structures are characterized by low damping factors, hence an active control scheme is necessary to increase the damping factor. The design of these active vibration controllers calls for a detailed dynamic model of the flexible structure. Unfortunately the motions of flexible systems are described by partial differential equations and, except for some simple cases, no close solution of these equations can be expected. To overcome this difficulty the partial differential equations are usually replaced, via spatial discretization, by a finite set of simultaneous ordinary differential ones. Implicit in this approach is a system truncation: a system of infinite order is replaced by a finite order one [1].

It has been shown [2] that a controller, designed on a finite dimensional model approximation, can destabilize the real system (*spillover*). The *spillover* phenomenon, due to the interaction between modelled and unmodelled dynamics, is emphasized in the presence of dynamic observers needed for state feedback controllers. The spillover phenomenon may be suppressed by avoiding the interaction between modelled and un-modelled dynamics. Moreover, it has been shown [3] that a controller for active vibration control is in general very sensitive also to the modelling errors inside the controller bandwidth. These modelling errors can take the form of inaccurate model shape descriptions, errors in the structural natural frequencies as well as neglected actuator and sensor dynamics. Various approaches have been proposed in literature to avoid both the spillover phenomenon [4] and the sensitivity to bandwith modelling errors [5].

In this work we compare the performance of three active vibration control schemes for the rejection of slowly varying multiple frequencies force disturbances acting on a flexible beam. The control system makes use of piezoelectric actuators and sensors. The first control technique combines a feedforward

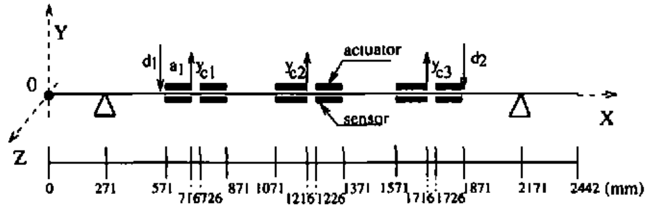

Figure 1: Constrains, force disturbances and piezo sensors/actuators

control action which tries to compensate the disturbances with a collocated feedback control action which confers robustness properties with respect to umodelled dynamics to the overall control system. The second control technique is based on most recent results in the theory of robust control: a dynamic compensator is designed by use of structured singular values design techniques, which allow a trade-off between attained level of disturbance rejection and prescribed level of robustness to unmodelled dynamics and parameter uncertainties and/or variations in the structure. The last technique is based on a suitable LQ controller and a spline based reconstructor in order to modify only the first modes of the structure. In particular the spline reconstructor operates as a spatial filter, hence it is able to screen out the high-frequency unmodelled dynamics from the measurementes then increases the robustness with respect to spillover phenomenon.

2. CONTROL PROBLEM DESCRIPTION

The vibrating mechanical structure considered in this paper is constituted by a beam 2442 mm long, with a non standard cross-section, pinned-pinned by means of two constraints placed respectively at 271 mm far from beam tips (Fig. 1). A low order model of this structure with piezoelectric actuators and sensors bonded on it has been obtained by using a finite-element+lagrangian method developed by the authors [6].

As regards the force disturbances, two point disturbance forces $d_1(t), d_2(t)$ are applied to the structure along the vertical axis as shown in Fig. 1. In particular, the disturbances forces are described by means of two slowly varying $(0 - 250Hz)$ multiple frequencies signals. As far as the piezoelectric sensors and actuators are concerned, six couples of 22×145 mm piezoelectric plates have been considered. The longitudinal position of these piezo electric plates are shown in Fig. 1. The control objective is to attenuate the vibration of the flexible structure, induced by the given disturbances, at the selected accelerometer locations, and to guarantee robustness with respect to unmodelled dynamics and parameter uncertainties.

3. FEEDBACK+FEEDFORWARD CONTROL

In this section it will be described an approach to design a control algorithm that, based on sensor signals, calculates on line the piezo actuator voltages necessary to attenuate the vibrations induced on the structure by the external disturbances. The vibration level is measured by evaluating the vertical accelerations measures in three different points along the beam.

The first problem we must take into account is that the low order simplified model of the flexible structure described in [6] is exact only in a limited frequency spectrum (0-250 Hz). Moreover in order to avoid the excitation of high frequency unmodelled dynamics (spillover) the control signals frequency magnitude must be significant only within the disturbance bandwidth.

In general it is difficult to reach this control requirements by using feedback controllers. On the other hand while the feedforward control algorithms allow avoiding the spillover phenomenon, at the same time they are very sensitive to parameter variations and/or uncertainties because they require an exact mathematical model of the structure to be controlled.

We propose a control technique which includes a feedforward control action to compensate the disturbances and a feedback control action to confer robustness properties with respect to parameter uncertainties to the overall control system, as shown in Fig. 2. Based on an estimation of disturbance forces acting on the structure, the feedforward control action generates the voltage actuator signals u_{ff} having the same shape of the disturbance signals described in the previous section. The output disturbance signal $\hat{y}_{cd}$ is estimated via least squares estimation algorithm, and the feedforward control signal results multiplying by the pseudo-inverse $W^{\dagger}$ of the transfer function of piezo actuators to vertical accelerations.

Aim of the feedback action, consisting in a decentralized [7] collocated feedback between the piezo sensors and actuators, is to reduce the feedforward control algorithm sensitivity with respect to parameter variations by actively increasing the structure damping factor.

4. MU SYNTHESIS CONTROLLER

The $H_\infty - \mu$ synthesis approach allows the designer to formulate a control problem in which the disturbance rejection requirements (*performance goal*) and the insensitivity of the feedback system to modelling uncertainties and/or variations (*robustness goal*) can be reached all together [9].

The controller is obtained via the solution of an optimization procedure in which the defined objectives are traded-off among each other.

The control designer is requested to give realistic specifications both in terms of performance and robustness by means of frequency dependent weigthing fucntions. The selection of uncertainty description plays a major role in the trade-off between robustness and performance requirements [8].

The application of this technique requires the plant to be described by the General Interconnection Structure given in Fig. 3. In this figure the plant P has three sets of inputs and three sets of outputs.

As concerns the inputs, u consists of manipulable control signals and d consists of the external signals, such as disturbances and measurement noise. The output y contains all the measured signals which are available to the feedback compensator K, while e consists of the error signals such as for example performance index signals, actuator signal levels, and so on. The inputs w and outputs z of P are introduced to take into account possible causes of model uncertainties, in the form of unknown, but norm bounded, linear time invariant systems Δ.

In the μ-synthesis control design process an iterative procedure is implemented in order to find the controller that minimizes the $\mathcal{H}^\infty$-norm of the transfer functions between external disturbances and performance signals, for all plant models, as defined by the uncertainty description.

5. LQ AND SPLINE BASED CONTROL

The main aim of an active vibration controller is to subtract elastic energy to the flexible structure. Since the disturbances are bandwidth limited we propose to use an LQ regulator which minimizes a performance index weigthing only the modes within the bandwidth and the control activity. In [10] it is shown that the associated control law depends only on the weighted modes of the structure and modifies the dynamic response of these modes only.
In order to implement this controller an estimation of at least the weighted modes is requested. This may be obtained using the continuity properties of the state variables (which arise from the spatial discretization of the beam model). In particular the measurements of the piezoelectric plates and the boundary conditions can be used with a spline function method to reconstruct the slope rate deformation of the beam. The remaining state variables can be reconstructed by the rate of change of the slope by a mix of algebraic manipulation and time integrators [10]. It is interesting to note that this reconstructor is not based on the mathematical model of the beam then it is insensible to parameter variations. An interesting feature of this reconstructor is its intrinsic filtering property. As a matter of fact, the beam deformation is approximated, at each time instant, by means of a cubic spline. Then the deformation is approximated by means of the lowest spatial frequency modes. This spatial filtering, unlike the classical time-filtering, does not introduce phase-lag, hence it does not deteriorate the stability properties. A practical consequence of this filtering action is that, even thought the disturbance inputs excite the high-frequency modes of the system, they tend to be screen out in the reconstructed state variables.

6. CONCLUSIONS

In this section the performance and robustness properties of the three control schemes previously described, are evaluated by means of numerical simulations with reference to the system described in section 2. All the controllers are designed on a reduced order model of the beam and evaluated on a more complete model.

Let's define the beam vibration level as the sum of the effective values of the acceleration signals y_{c1}, y_{c2}, y_{c3} shown in Fig. 1. In Fig. 4 the reduction of the vibration level obtained by applying respectively the feedforward action only, the feedback action only and the feedback+feedforward action, described in the section 3 is shown. In order to evaluate the robustness of this control actions we performed closed-loop numerical simulations in the presence of a significant parameter uncertainty (10% variation of the beam density mass). It must be noted that it is necessary to use a feedback control in order to confer to the feedforward control action adeguate robustness properties with respect to parameter uncertainties. Unfortunately there are some difficulties in the practical implementation of the collocated feedback. It has been shown [7] that the phase lag introduced by the piezo actuators can cause the spillover phenomena.

By using the $H_\infty - \mu$ approach described in the section 4, we designed a MIMO (MultiInput-MultiOutput) dynamic feedback controller between accelerations signals and piezo actuator voltages. Instead of resisting the motion at all frequencies, as collocated feedback does, the $H_\infty - \mu$ controller behaves like counter resonances which eliminate the effects of the mechanical resonances at system output. Moreover by specifying adeguate weigthing functions it is possible to select the resonance frequencies to be damped, so as to avoid spillover phenomena. Fig. 5 shows the attenuation in the maximum singular value plots of the transfer functions between the disturbance d_1 and accelerations. From the plot it is evident that the required attenuation has been achieved in the frequency range of interest. The major weakness of this method is that, in general, it leads to high order controllers, so implying possible difficulties in implementing it into a digital computer. Moreover it requires a good knowledge of the plant dynamics in the disturbance bandwidth.

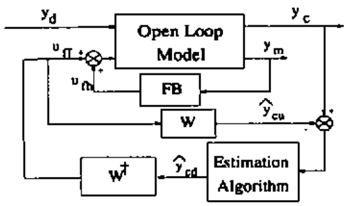

Figure 2: Feedback+Feedforward control scheme

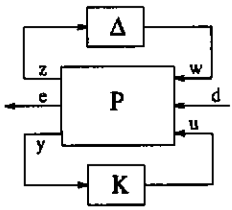

Figure 3: General Interconnection Structure

When applying the LQ with spline reconstructor approach only the first two modes were weighted in the performance index. On the other side the availability of three piezoelectric sensors allows one to correctly reconstruct these modes. In Fig. 4 it is also presented the reduction of vibration level for this controller. The major drawback of this method is the sensitivity of the reconstructor to the number of modes in the disturbance bandwidth. Although the closed loop stability cannot be analitically proved, the stability analysis showed that the presence of the spline reconstructor gives better stability margins than simple LQ at the output of the systems.

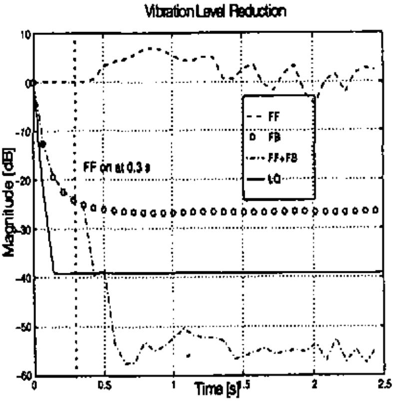

Figure 4: Feedback + Feedforward and LQ + spline controllers performances

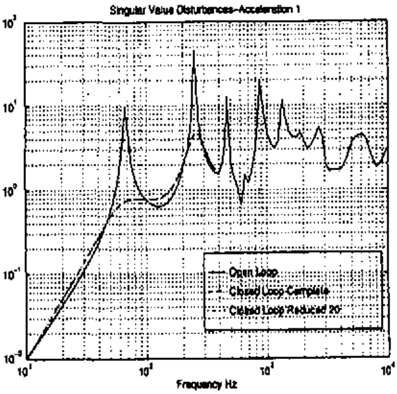

Figure 5: $H_\infty - \mu$ controller performances

REFERENCES

[1] L. Meirovitch, Dynamics and Control of Structures (Wiley, New York, 1990).

[2] L. Meirovitch, and A. Norris, J. of Sound and Vibr., 144, 365 (1991).

[3] G.J. Balas, P.M. Young, J. of Guid. Cont. and Dyn., 18, March-April (1995).

[4] Y. Sharony and L. Meirovitch, J. of Guid. Cont. and Dyn., 14, 268 (1991).

[5] E.G. Collins, J.A. King,D.J. Philips, D.C. Hyland, J. of Guid. Cont. and Dyn., 15, no.4, (1992).

[6] G. Ambrosino, G. Celentano, and R. Setola, 'Integrated Model of Flexible Beam with Piezoelectric Plates for Active Vibration Control', 13th IFAC World Congress, (1996).

[7] E.T. Falangas, J.A. Dworak, S. Koshigoe, IEEE Trans.CST, 34, (August 1994)

[8] G.J.Balas, J.C.Doyle, IEEE Trans.CST, 2, 352 (1994).

[9] G.J.Balas,J.C.,Doyle,K.Glover,A.Packard,R.Smith, $\mu-$Analysis and Synthesis MATLAB Toolbox (The Mathworks, Natick, 1994).

[10] G. Ambrosino, G. Celentano, and R. Setola, 'A Spline Approach to State Reconstruction for Optimal Active Vibration Control of Flexible Systems', 4th IEEE CCA, 896 (1995).