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ON THE FUZZ IN A STRUCTURAL FUZZY

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1. INTRODUCTION

A master structure may be adjoined by another in order to subdue its response. In the steady state, this control of the response is usually accomplished by significantly siphoning energy off the master structure into the adjunct structure and providing the adjunct structure with efficient means to dissipate its stored energy. When the adjunct structure is statistically defined the resulting combined structural complex is commonly designated structural fuzzy. In this structural fuzzy, the adjunct structure constitutes the fuzz. In this paper the master structure is also statisticalized so that the statistical energy analysis (SEA) may be used to decipher the response behavior of the structural fuzzy.

The statistical energy analysis (SEA) is an analytical tool of some experience; it has been developing since 1962 and is being used extensively nowadays in Noise Control Engineering. Several computer programs have been developed and skilled personnel trained to handle its employment. In this paper SEA is deployed in its rudimentary form in which many possible refinements are not included. However, such refinements can be made when the structures are defined in greater detail. In this paper, the structures are of merely generic design. Nonetheless, quantities and parameters are considered to be frequency dependent so that local dependencies on frequency can be accounted for. Thus, variations with frequency in the ratio of modal stored energies and of the ratio of modal densities in the two structures are permitted in the analysis. SEA quantities and parameters are stripped of spatial dependencies so that local spatial dependencies are suppressed within each of the structures. For example, inquiry of the spatial variations in the response (stored energy) within a single structure is illegitimate in SEA. Yet, variations in the response (stored energy) from one structure to another is a valid inquiry. As already explained, however, the stored energies may be functions of frequency, which SEA readily accommodates. It is to be assumed that the reader is familiar with SEA.

2. STATISTICAL ENERGY ANALYSIS MODELING OF A STRUCTURAL FUZZY

Quantities and parameters are averaged a'la SEA; those in reference to the master structure are designated by the subscript (s) and those in reference to the adjunct structure by (b):

$\eta_{ss}(\omega)$, $\varepsilon_s(\omega)$, $n_s(\omega)$ and $E_s(\omega)$ are the loss factor, the modal stored energy, the modal density and the stored energy in the master structure, respectively.

$\eta_{bb}(\omega)$, $\varepsilon_b(\omega)$, $n_b(\omega)$ and $E_b(\omega)$ are the loss factor, the modal stored energy, the modal density and the stored energy in the adjunct structure, respectively.

The averages are carried out over a frequency band of bandwidth $\Delta\omega$ centered at the frequency ω and thus

$$E_\alpha(\omega) = \Delta\omega\, n_\alpha(\omega)\, \varepsilon_\alpha(\omega) \quad ; \quad \alpha = s \text{ or } b \tag{1}$$

The coupling between the master structure (the s-structure) and the adjunct structure (the b-structure) is defined by the coupling loss factor $\eta_{bs}(\omega)$ and in the reverse direction by $\eta_{sb}(\omega)$. The modeling of the coupled structures in SEA is completed by this definition. A SEA model of the coupled master structure to the adjunct structure is depicted in Fig. 1.

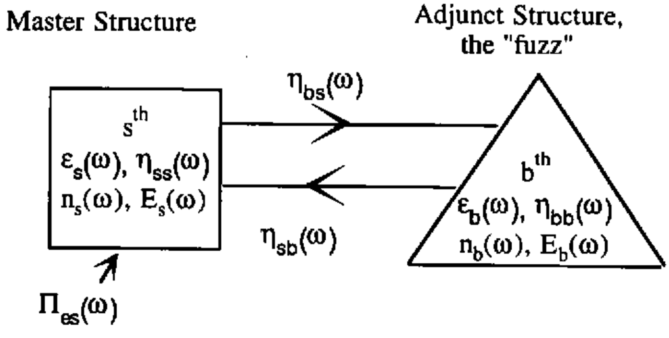

Figure 1. A SEA model of two coupled structures.

3. FORMULATION OF SEA

If only the master structure is externally driven, the ratio $\alpha_s^b(\omega)$ of the modal stored energies $\varepsilon_b(\omega)$ and $\varepsilon_s(\omega)$ is derived to be

$$\alpha_s^b(\omega) = [\varepsilon_b(\omega)/\varepsilon_s(\omega)] \simeq [1 + v_{sb}^b(\omega)]^{-1} ; \qquad (2)$$
$$v_{sb}^b(\omega) = [\eta_{bb}(\omega)/\eta_{sb}(\omega)] \quad ,$$

where $v_{sb}^b(\omega)$ is the coupling quotient for the union; v_{sb}^b defines the strength of the coupling between the structures. For a weak coupling $v_{sb}^b \gg 1$ and for a strong coupling $v_{sb}^b \ll 1$. A consistency relationship in SEA requires the ratio $\lambda_s^b(\omega)$ of the coupling loss factors to be also equal to the ratio of the modal densities; namely

$$[\eta_{bs}(\omega)/\eta_{sb}(\omega)] = \lambda_s^b(\omega) \quad ; \quad \lambda_s^b(\omega) = [n_b(\omega)/n_s(\omega)] \quad , \qquad (3)$$

From Eqs. (1) - (3) one may derive the expression for the ratio $\zeta_s^b(\omega)$ of the stored energies in the form

$$\zeta_s^b(\omega) = [E_b(\omega)/E_s(\omega)] = \alpha_s^b(\omega)\, \lambda_s^b(\omega) \qquad (4)$$
$$\zeta_s^b(\omega) = [\eta_{bb}(\omega) + \eta_{sb}(\omega)]^{-1}\, \eta_{bs}(\omega) \quad ,$$

The "effective loss factor" $\eta_e(\omega)$ for the combined structure (structural fuzzy) can be defined

$$\omega\eta_e(\omega)\, E(\omega) = \omega\eta_{ss}(\omega)\, E_s(\omega) + \omega\eta_{bb}(\omega)\, E_b(\omega) \quad ; \qquad (5)$$
$$E(\omega) = E_s(\omega) + E_b(\omega) \quad ,$$

where $E(\omega)$ is the total energy stored in the coupled master and adjunct structures. Equation (5) equates the dissipation in the combined structure to that in the individual structures. Substituting the ratio $\zeta_s^b(\omega)$ of the stored energies, stated in Eq (4), in Eq. (5) yields

$$\eta_e(\omega) = [\eta_{ss}(\omega) + \eta_{bb}(\omega)\, \zeta_s^b(\omega)]\; [1 + \zeta_s^b(\omega)]^{-1} \qquad (6)$$

In the absence of the adjunct structure $\eta_e(\omega) \rightarrow \eta_{ss}(\omega)$. Equation (6) suggests two design criteria for achieving a high $\eta_e(\omega)$: (A) the loss factor $\eta_{bb}(\omega)$ of the adjunct structure needs to exceed the loss factor $\eta_{ss}(\omega)$ of the master structure and (B) a significant portion of the stored energy must be placed in the adjunct structure so that $\zeta_s^b(\omega)$ approaches or exceeds unity. If these criteria are satisfied then the effective loss factor $\eta_e(\omega)$ approaches the higher value of $\eta_{bb}(\omega)$ A failure in one and/or the other criteria; (A) and/or (B), may scuttle the intended design to subdue the response of the master structure by adjoining to it an adjunct structure. The statements just made are illustrated by artificial but typical examples.

4. ILLUSTRATIVE EXAMPLES

In Figs. 2a, b and c the ratio $\zeta_s^b(\omega)$ of the stored energies is specified in terms of three values; $\zeta_s^b(\omega) = 10$, 1 and 10^{-1}, respectively. In each figure a set of values is assigned to $\eta_{ss}(\omega)$ and $\eta_{bb}(\omega)$ as indicated in the figures. Also, in these figures the satisfaction of (A) and (B) are indicated; the stronger the satisfaction the bolder the designating letter. Although intuitively obvious, Figs. 2a, b and c clearly demonstrate the design criteria that one needs to seek to render the structural fuzzy an effective noise control procedure.

In Figs. 3a and b a situation is depicted in which the ratio $\zeta_s^b(\omega)$ of the stored energies may vary considerably with frequency. Variations of this kind may arise from modal clumping (bunching) in the master structure, in the adjunct structure or in both. [cf. Eq. (3).] Modal clumping often occurs in structures in which modal degeneracy prevails; e.g., this kind of degeneracy may result from undue structural symmetries. In Fig. 3a the ratio $\zeta_s^b(\omega)$ of the stored energies is artificially assumed to be

$$\zeta_s^b(\omega) = \left\{ 10^{-1} + 10^2 \cos^2[\{ I(\omega/20\pi) - 1I(2\pi \times 10^{-2}) \}^{1/2}] \right\}^{-1} \qquad (7)$$

In those frequency ranges where $\zeta_s^b(\omega) << 1$, the ratio of the stored energies is in violation of (B). Assuming that (A) is amply satisfied; namely, $\eta_{bb}(\omega) >> \eta_{ss}(\omega)$, the effective loss factor $\eta_e(\omega)$ for this case is depicted in Fig. 3b. It is apparent that whenever (B) is severely violated the adjunct structure fails to properly perform its intended role.

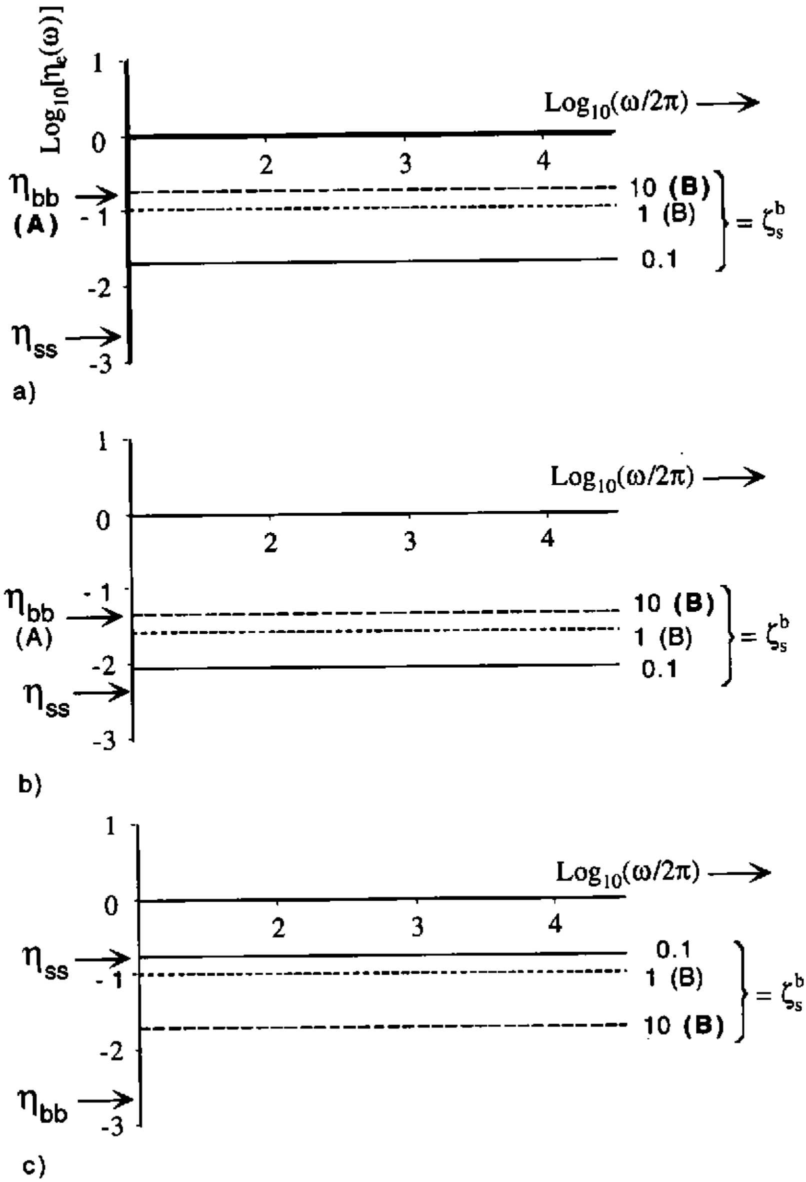

Figure 2. The effective loss factor $\eta_e(\omega)$ as a function of frequency $(\omega/2\pi)$ for three values of the ratio $\zeta_s^b(\omega)$ of stored energies for three values of the ratio $[\eta_{bb}(\omega)/\eta_{ss}(\omega)]$ of loss factors: a) 10^2, b) 10 and c) 10^{-2}.

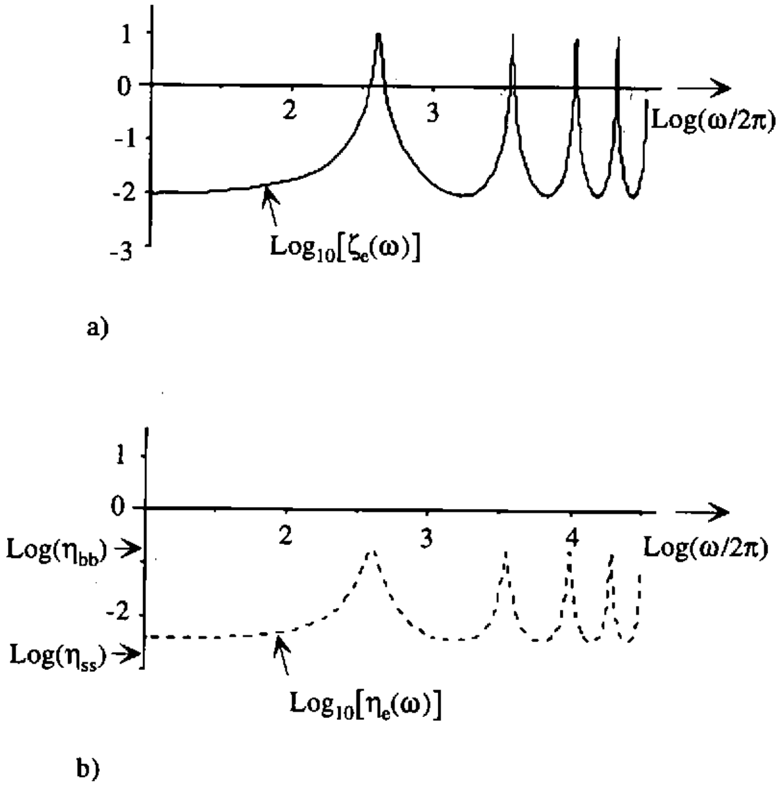

Figure 3. Example in which (A) is amply satisfied; $\eta_{bb}/\eta_{ss}) = 10^2$, but (B) is violated in certain frequency ranges by an undulating ratio $\zeta_s^b(\omega)$ of stored energies, as depicted in a). The effective loss factor $\eta_e(\omega)$ under the influence of the undulations in $\zeta_s^b(\omega)$ is depicted in b).