TIME DOMAIN BOUNDARY ELEMENT METHODS FOR ACOUSTIC SCATTERING FROM CURVED SURFACES

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1 INTRODUCTION

This paper discusses the time domain boundary element method (TDBEM) for the wave equation. Applications in room acoustics include scattering from reflectors and other objects. Progress in TDBEM has been slow, mainly due to numerical issues related to stability and numerical integration of singular integrals. To address this, a few unconventional approaches have been investigated. They include expressing strongly and hypersingular integrals by vector potentials, and utilising Hermite field interpolation both in space and time. The geometry is moreover discretised using curved surfaces that are defined by nodal points and principal curvatures. A brief discussion on numerical limitations of the method is included.

2 PREVIOUS RESEARCH

TDBEM is a technique that is especially well suited for studying acoustic scattering of transient signals from isolated objects in unbounded homogeneous media, especially since only the bounding surface of the domain has to be discretised. Typical approximate situations include statues or reflectors in a large auditorium for example. Pioneering work started about 40 years ago[1,2,3], but despite the promising features of TDBEM, progress has been relatively slow due to the complexity and subtleties in the theoretical formulations of boundary element methods in general.

2.1 BEM in the frequency domain

When studying the traditional BEM formulation of Helmholtz' equation for unbounded volumes, the solutions in the frequency domain is non-unique at specific frequencies that are caused by resonant solutions of a related solution on the complement of the original domain[4]. The non-uniqueness is, however, a consequence of the BEM formulation, and holds no physical significance to the original problem of solving Helmholtz' equation with prescribed boundary conditions.

The solution to this problem is in essence to enforce the prescribed boundary conditions[5] by applying them to the observational variables and not to the integration variables only. One way to achieve this enforcement is to differentiate the original integral equation resulting from the BEM formulation, which makes explicit expressions for boundary condition terms appear in terms of observational variables. However, the process of doing this introduces hypersingular integrals in addition to already existing weakly and strongly singular integrands, and the numerical treatment of these needs to be handled with great care.

Using Taylor expansions of singular integrands, one may subtract a suitable number of terms so that the resulting integrand becomes at most weakly singular, which in turn makes it amenable to standard quadrature techniques. The same terms however need to be added back to restore the validity of the formulation, and then integrated analytically or semi-analytically. Changing to polar coordinates in parameter space and a subsequent Laurent expansion in the radial coordinate may solve this last step[6,7,8]. Another approach is to identify divergence-free parts of these singular parts of the integrands, find the corresponding vector potential and apply Stokes' theorem so that the surface integral becomes a line integral[9,10,11,12,13].

The afore-mentioned Taylor expansions impose regularity demands on the field variables; demands that appear unnecessarily strong. Because of this, considerable attention has been devoted to minimum requirements on regularity and whether they may be weakened, unfortunately without clear consensus, since violating these requirements tend to give reasonable results anyway[14,15]. One reason for the need to have access to weak regularity conditions on the field variables is to theoretically motivate constant field approximations on each discretised surface element patch. This in turn makes implementations of BEM relatively simple since collocation points are placed at the centre of each surface element, thereby avoiding issues related to discontinuities on the boundary. In addition to this, advantages appear in terms of decreased computational time since integrands take less time to evaluate.

Another problem that has occupied researchers in the field of BEM is how to handle non-smooth boundaries and limiting procedures when the observational variable approaches the collocation point on the boundary[16]. This involves differential geometry and delicate limiting procedures in the evaluation of hypersingular integrands in parameter space[17].

2.2 TDBEM

It was assumed that by studying the problem directly in the time domain instead of in the frequency domain, the problems related to non-uniqueness would be resolved, since strictly harmonic solutions become forbidden by assuming a quiescent past. It however turns out that the problem reappears as a stability problem, sensitive to the exact choice of discretisation in space and time, when the incident field's spectrum contains frequency components that correspond to the resonant frequencies causing non-uniqueness in the frequency domain formulation.

The stability of approximate numerical marching-on-in-time (MOT) solutions of the resulting integral equations in TDBEM is therefore of central importance[18,19]. The problem has been remedied by methods similar to those for the frequency domain, i.e. by enforcing the boundary conditions, but the method is still somewhat heuristic since it introduces an arbitrary coupling constant[20,21]. A thorough systematic and theoretical study of stability criteria of MOT schemes, preferably using z-transform techniques, might shed some light on the situation.

3 ALTERNATIVE APPROACHES

Instead of assuming the relatively simple implementation technique that results when a constant field approximation is assumed on flat elements (which however seems to imply a systematic error and a violation of prerequisites), the field may be modelled to be smoothly varying throughout the entire bounding surface, including across elements and with respect to time. For the geometry discretisation, smoothness may also be achieved using suitably curved elements. This however results in a natural choice to collocate on nodes where surface elements meet, instead of on element centres, with corresponding complications in implementation and theoretical limiting procedures.

For the case of a smooth field situation, Hermite interpolation[22] is used, which may lead to problem formulations where not only the field values on nodes are unknowns to be solved for but also their tangential and temporal derivatives. This requires a coupled system of hypersingular integral equations to be solved by a marching-on-in-time (MOT) scheme, and then the coupling constant remedy above may be insufficient. An alternative is to model temporal derivatives using finite differences of current and previous values of the field variables using constant sampling intervals in time, but an analogous procedure for the tangential derivatives seems problematic because of the general form of the bounding surface.

With this approach however, a more theoretically sound basis of investigation is initiated, where consequences of weakening various assumptions such as a non-smooth boundary or a non-smooth field can be analysed systematically, since comparison with results for stronger restrictions are readily available. This has previously been explored for the frequency domain[23], while similar treatments in the time domain have been hard to find.

It has recently been shown that all singular parts of the integrands in the time domain formulation of BEM are divergence-free[24]. Moreover, the corresponding vector potentials are known in closed form. This might lead to many interesting alternative formulations and implementations, since all singular surface integrals can be converted into line integrals using Stokes' theorem. Moreover, since the boundary element method primarily deals with closed surfaces, these contributions taken together add to zero, since a closed surface has no boundary. The practical and theoretical consequences of this approach need further investigation.

4 PRELIMINARY RESULTS

Pilot studies of the implementation for a sphere with Neumann conditions, having set both temporal and tangential derivatives at nodes equal to zero in the Hermite interpolations, show stable although noisy results using a Burton-Miller approach for enforcing the boundary conditions. See Figure 1 and 2. As mentioned however, this does not seem to be easily translated to cases when nodal derivatives are not forced to zero. Figure 3 schematically indicates how a hypersingular integral encountered in TDBEM is regularised using Stokes' theorem. Figure 4 finally demonstrates the potential usefulness of discretisation by curved triangles. See Ref. 24 for more specific information and details.

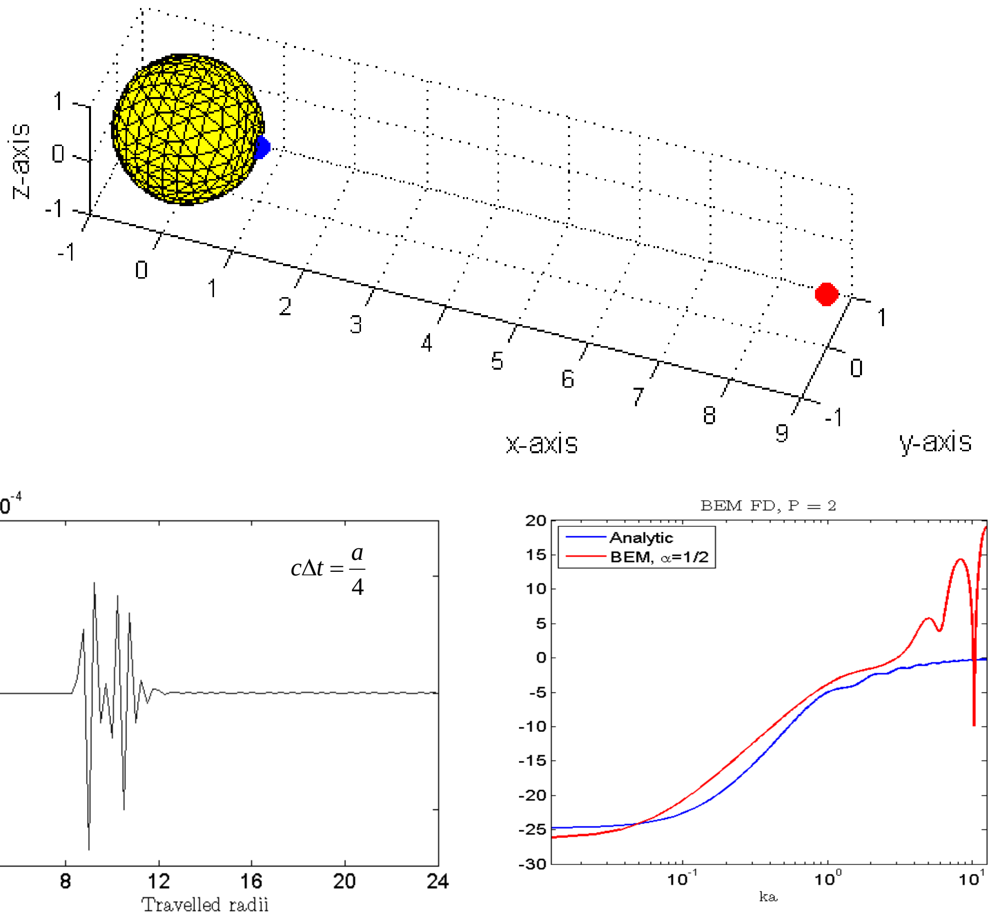


Figure 1: Top: A sphere centered at (0,0,0) that has been discretised by 1024 curved triangles. Collocation point at (1,0,0) and point source at (9,0,0). Bottom left: The impulse response for the scattered part using TDBEM. The result is stable but noisy. Bottom right: Comparison with analytic solution in the frequency domain. The virtual sound sources on the boundary have cardioid-directivity.

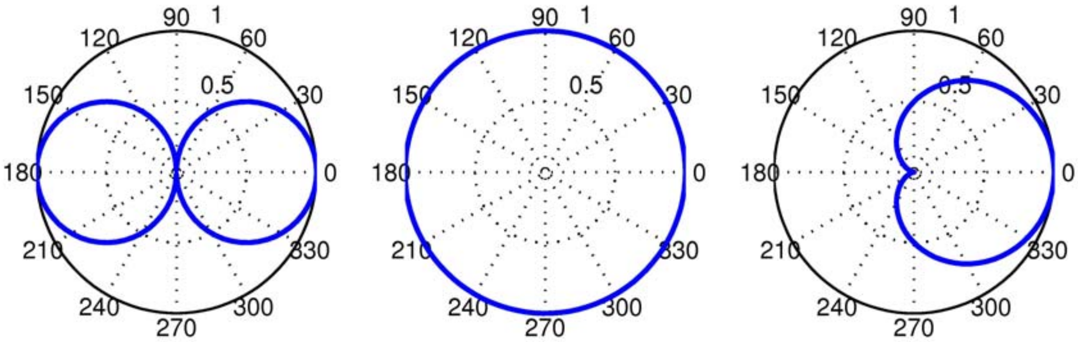


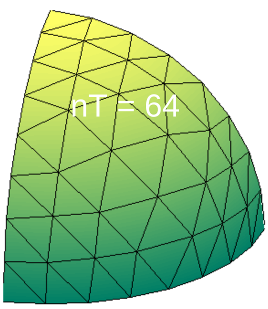
Figure 2: Left: Dipole directivity. Middle: Monopole directivity. Right: Cardioid-directivity resulting from combining equal amounts of dipole- and monopole directivity. This is the directivity of the virtual sound sources in TDBEM that results from the Burton-Miller approach with a coupling constant equal to ½.

$$\mathbf{M}_s = \frac{3\hat{n}_0 \cdot \hat{R}\,\hat{R} - \hat{n}_0}{R^3}\left(1 + R\frac{\partial}{c\partial t}\right) + \frac{\hat{n}_0 \cdot \hat{R}}{R^2}\nabla$$

$$\int \mathbf{M}_s \left\{\phi - \phi_0 - \nabla\phi\big|_0 \cdot \mathbf{R}\right\} \cdot d\mathbf{A} = \int (\nabla \times \mathbf{m}) \cdot d\mathbf{A} = -\oint \mathbf{m} \cdot d\mathbf{r}$$

$$\mathbf{m} = \phi_0 \frac{\hat{n}_0 \times \hat{R}}{R^2} + \nabla\phi\big|_0 \cdot \hat{n}_0 \mathbf{l} + \frac{\nabla\phi\big|_0}{R} \times \left(\hat{n}_0 + \hat{n}_0 \cdot \hat{R}\,\hat{R}\right)$$

Figure 3: A hypersingular surface integral appears in TDBEM formulations. The figure shows typical expressions, see Ref. 24 for details. It turns out that the integrand is divergence-free. The corresponding vector potential has been found. The surface integral may thus be rewritten as a line integral using Stokes' theorem. The numerical evaluation simplifies since the line integral is taken around the singularity, and thus avoids it.



nT	**Flat** triangles	**Curved** triangles
1	0.866025403784439	1.607347918859648
4	1.302218940169728	1.571824991316277
16	1.494361453845343	1.570813664341333
64	1.551022973447906	1.570796600319646
256	1.565809983587370	1.570796331076317
1024	1.569547029650318	1.570796326861769
4096	1.570483832679483	1.570796326795939
16384	1.570718192645907	1.570796326794909
Exact	1.570796326794897	1.570796326794897

Figure 4: Calculation of the area of 1/8 of a unit sphere - comparison between flat and curved triangles. Note the convergence rate when discretising the geometry by curved triangles.

5 DISCUSSION

The results are promising when looked at in the frequency domain, but in the time domain they are noisy due to less accuracy at higher frequencies. Higher sampling frequency and finer discretisation may improve the situation, but with this comes increasing problems associated with numerical evaluation of singular and near-singular integrals.

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