

MODELLING OF PLATE VIBRATION USING MIRROR-SOURCE APPROACH

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1. INTRODUCTION

Computation of structural vibration is done nowadays using sophisticated numerical methods. These in turn often need considerable computational power. Analytical methods maintain however their important place in vibration analysis. This applies in particular to cases where physical understanding of the phenomena involved is required.

A flexurally vibrating thin plate is one of the most frequently used models for representing typical behaviour of complex multi-modal structures. The characteristic equation for the plate allows exact solutions only for the simplest geometrical and boundary conditions obtained by eigenfunction expansion. Apart from tolerating only simple conservative plate boundary, such a solution is restrictive in yet another way: its modal character imposes specific type of the plate dissipation, i.e. modal damping, which is notorious for being physically unrealistic.

This paper investigates possibilities for an alternative treatment of plate vibration which conserves the wave character of vibration and allows for release of boundary in one or more directions.

2. ILLUSTRATION OF MIRROR-SOURCE PRINCIPLE

Mirror-source approach is well known technique in acoustics. The solution for the acoustical pressure within a bounded acoustical medium is here obtained by replacing the boundaries with conveniently distributed sources outside the bounded zone, such that the boundary conditions remain unchanged. Two particular boundary types are often simulated this way: free or pressure-release boundary and rigid or pressure-blocking boundary.

An analogous approach can be adopted where vibration of flat plates are concerned. Such an approach can be illustrated using the example of a simply supported beam. The mirror-source solution for the displacement of the finite beam excited at "a" from the left support can be constructed from the solution for an infinite beam, [1]:

$$w(x) = \omega F / (4jBk^3) \cdot [\exp(-jkx) - j \cdot \exp(-kx)] \quad (1)$$

by adding excitation forces in the way indicated on Fig. 1. The symbols are: ω - frequency, k - wavenumber, F - excitation force, B - flexural stiffness and j - imaginary unit.

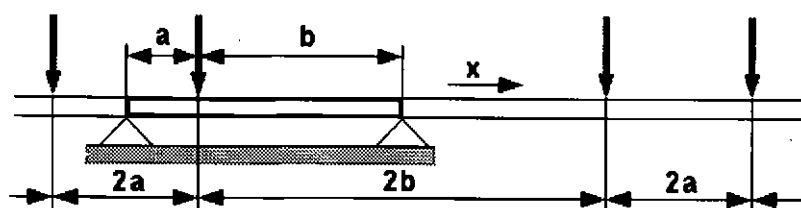


Figure 1: Construction of mirror sources for a beam

The objective is to achieve the desired boundary conditions at the two hypothetical supports. In the case of simple supports this mean vanishing of both w and $\partial^2 w / \partial x^2$. An infinite number of mirror sources of alternating sign become necessary to fully satisfy these requirements. For making the numerical computation possible truncation of far sources is needed. This will converge only for complex k , i.e. for damped beams.

For a straight beam vibrating in flexure, the exact analytical solution can be obtained either in a closed form or by eigenvalue expansion. The two approaches will not give an identical solution in the presence of non-zero damping. Fig. 2, top, shows the RMS level distribution along the beam excited at a frequency halfway between the 3rd and 4th resonance. The bottom diagram shows the error for both the modal (thin line) and mirror (thick line) approach, caused by 60 dB truncation applied to both of the series solutions. The mirror approach is seen to give much better result. Both approaches produce a significant rise in error at the positions where the vibration level is very low.

Fig. 3 shows the overall RMS level error of the mirror-source approach as a function of the truncation threshold and loss factor. This threshold is the ratio of the contribution of the main and a mirror source at the position within the beam span. The error is negligible even for a fairly low truncation threshold of 20 dB, and it drops further with increase of the threshold.

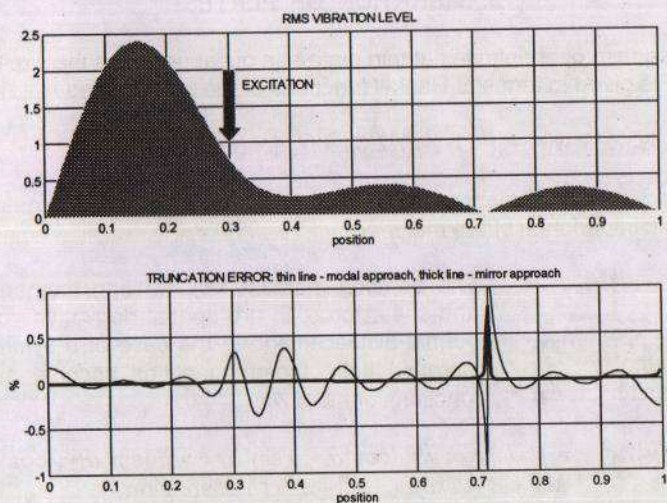


Figure 2: Level and error distribution along the beam

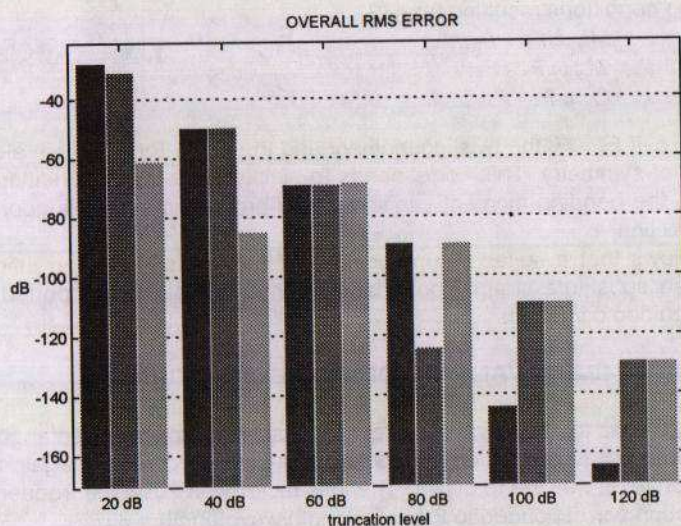


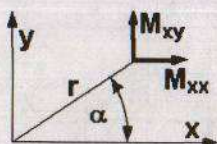
Figure 3: Truncation error. Left - $\eta=0,1\%$, middle - $\eta=0,5\%$, right - $\eta=2\%$

3. VIBRATION OF PLATES

The response of an infinite flat thin plate at a distance r from the excitation force F is given in terms of Hankel functions of the second kind H , [1]:

$$w(r) = jF \cdot (1-\nu^2)^{1/2} / (2.3\omega\rho ch^2) \cdot [H_0(kr) - H_0(-jkr)] \quad (2)$$

where w - normal displacement, ν - Poisson's coefficient, h - thickness, ρc - specific impedance of the material.

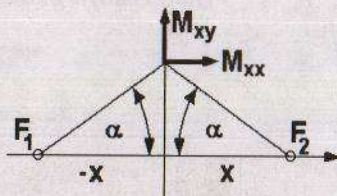


The bending moment M_{xy} , twisting moment M_{xx} and shear force Q_x are spatial derivatives of the normal displacement. In the case of a symmetric vibration field, these quantities become simple functions of r and α :

$$\begin{aligned} M_{xy} &\propto \partial^2 w / \partial x^2 + \nu \cdot \partial^2 w / \partial y^2 \propto w^{(2)} \cdot (\cos^2 \alpha + \nu \sin^2 \alpha) + w^{(1)} \cdot (\sin^2 \alpha + \nu \cos^2 \alpha) / r \\ M_{xx} &\propto \partial^2 w / \partial x \partial y \propto (w^{(2)} - w^{(1)} / r) \cdot \cos \alpha \cdot \sin \alpha \\ Q_x &\propto \partial(\Delta w) / \partial x \propto (w^{(3)} + w^{(2)} / r - w^{(1)} / r^2) \cdot \cos \alpha \end{aligned} \quad (3)$$

Two forces F_1 and F_2 , simultaneously acting on the plate as shown, will yield along the line halfway between them the following conditions resulting from (3):

$$\begin{aligned} M_{xy} &\propto F_1 + F_2 \\ M_{xx} &\propto F_1 - F_2 \\ Q_x &\propto F_1 - F_2 \end{aligned} \quad (4)$$



Therefore, if $F_1 = F_2$ the twisting moment and the shear force vanish along the line of symmetry. This corresponds to simply guided end condition. If $F_1 = -F_2$ the bending moment vanishes, thus producing, simply supported end condition.

It follows that a suitable superposition of forces acting on an infinite plate can substitute straight boundaries with either simply-supported or simply-guided conditions.

4. EQUIVALENCE OF PLATE SOLUTIONS

Figure 4 shows the RMS vibration level of a simply-supported rectangular plate. The excitation position is denoted by a small circle. Here again the truncation level was set to 60 dB. The loss factor was 20%, the frequency of excitation corresponded to the ratio length/wavelength = 3,5.

Fig. 5 shows the difference in the RMS levels obtained using modal and mirror approach. A very good matching can be seen between the two results, the maximum difference of 15% occurring at the excitation point.

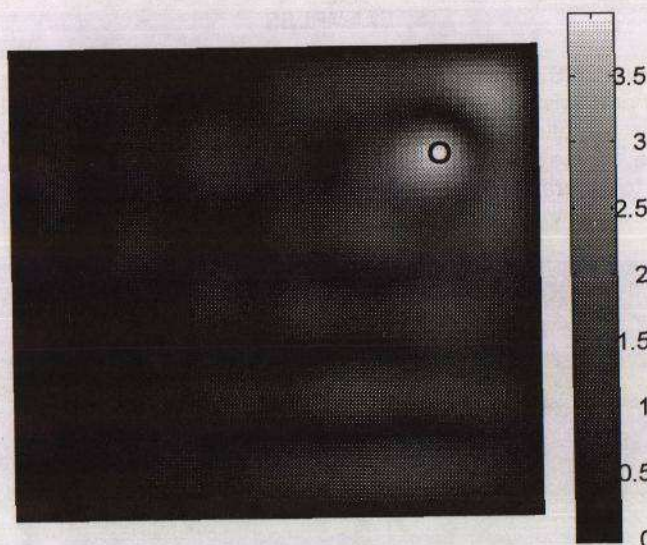


Figure 4: RMS level across the plate - units in mean RMS value

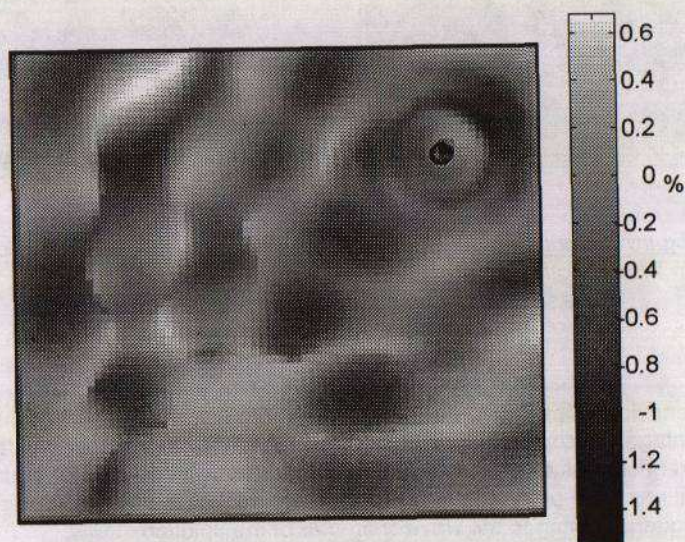


Figure 5: Difference of RMS levels using modal and mirror approach (%)

5. EXAMPLES

Fig. 6 shows RMS vibration levels obtained by the mirror-source approach for few characteristic cases: one fully and three partially bounded plates. Sinusoidal excitation is assumed. The number of mirror sources ranges from three, for the plate bounded by two perpendicular boundaries (upper right), up to several hundred for the fully bounded plate (upper left).

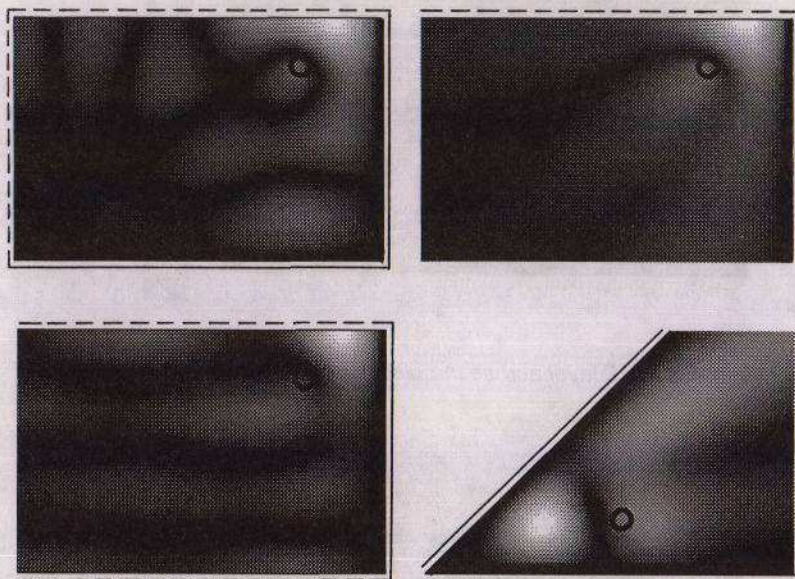


Figure 6: Boundary: full - simply supported, dashed - simply guided
Wavelength = 40% of the displayed length. Loss factor = 5%.
Dynamics: top figures: 30 dB, bottom figures: 20 dB.

6. CONCLUSIONS

The principle of mirror-imaging applied to plate vibrations offers numerous advantages in comparison with the modal method of response synthesis. Different shapes can be analysed in conjunction with 2 types of boundary conditions. The plate damping can be specified as a continuous function of frequency owing to the wave character of the solution.

REFERENCE: L. Cremer, M. Heckl *Structure Borne Sound*, Springer 1988