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USE OF THE PE METHOD FOR PREDICTING NOISE OUTDOOR

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INTRODUCTION

The atmosphere is neither static nor uniform and the inhomogeneities of this medium have a very significant effect on acoustical propagation outdoors. The nature of the ground also has an effect on the propagation. The PE method is capable of taking into account the combined effects of a finite ground impedance, refraction due to sound speed profiles and scattering by atmospheric turbulence. The use of the PE method therefore requires several environmental input parameters. The complex impedance of the ground must be measured or estimated. The sound speed profiles must be inferred from point measurements of wind speed, wind direction and temperature. Similarity scaling expressions may be employed. Finally, the strength and scale of wind and thermal turbulence is required. This paper reviews the basic steps of the PE method, and outlines the measurement procedures used at the National Research Council to obtain the environmental parameters required as input to the PE.

THE PARABOLIC EQUATION METHOD

The PE method assumes simple harmonic time dependence exp($-i\omega t$) and begins with the Helmholtz equation

$$[\nabla^2 + k^2]p(r,z) = -4\pi\delta(r, z-z_s) \tag{1}$$

where, for a realistic atmosphere, the wavenumber $k(r,z) = \omega/c(r,z)$, r is the horizontal range and z is the height above the ground. The speed of sound therefore generally depends on both height and range. Reflection from a porous ground is described by the boundary condition

$$[\partial p/\partial z + ik\beta p]_{z=0} = 0 \tag{2}$$

at the ground surface where β is the normalized complex surface admittance.

The PE employs an assumption that wave motion for a particular problem is always directed away from the source or that there is very little backscattering. Writing the operator ∇^2 in cylindrical coordinates and setting $U = pr^{1/2}$, the Helmholtz equation is factored into propagation of incoming and outgoing waves,

$$(\partial/\partial r + i\sqrt{Q})(\partial/\partial r - i\sqrt{Q})U = 0 \tag{3}$$

Considering only the outgoing wave leads to the *one-way wave equation*

$$\partial U/\partial r = i\sqrt{Q}U \tag{4}$$

where $Q = \partial^2/\partial z^2 + k^2$. Most implementation of the PE can be traced back to Eq. (4). The approach for advancing the field in range is the point of departure for the PE methods.

The Green's function formulation of Gilbert and Di [1] has been implemented [2] at the National Research Council. Writing $u = U\,exp(ik_0\, r)$, the PE allows a "marching" solution whereby the known solution at r is used to calculate the solution at $r + \Delta r$ using an exponential propagator;

$$u(r+\Delta r, z) = \exp[i\Delta r(\sqrt{Q} - k_0)]u(r) \tag{5}$$

The implementation [2] uses a split step approximation and further separates the wavenumber into two parts that includes a deterministic component $c_d\,(z)$ and a stochastic component $\mu(r,z)$,

$$k(r,z) = k_0[c_0/c_d(z) + \mu(r,z)] \tag{6}$$

With these approximations, the propagator has the form;

$$\exp[i\Delta r(\sqrt{Q} - k_0)] \approx \exp[i\Delta r(\sqrt{Q_d} - k_0)]\exp[i\Delta r S_\mu k_0] \tag{7}$$

The functions Q_d and S_μ in Eq. (7) are defined in Ref. 2 and represent the deterministic and stochastic parts of the operator, respectively.

Thus the PE requires the values of the complex ground impedance $Z = 1/\beta$ to satisfy Eq. (2), the speed of sound as a function of height z

above the ground required in Q_d , and a description of the random function $\mu(r,z)$ required in S_μ.

IMPEDANCE OF THE GROUND

In recent years the measurement of level differences has become a popular method to obtain the impedance of the ground. The sound from a point source is measured at two heights (see Fig. 1) and the difference in levels at the two microphones is plotted as a function of frequency. This method is greatly simplified if the measured level differences are compared to level differences calculated from a model for ground impedance. Although there are several different models, two popular models have been a one parameter model by Delany and Bazley [3] and a two parameter model by Attenborough [4]. The impedance according to the model of Attenborough is given by;

$$Z=0.484(1+i)[\sigma_e/f]^{1/2}+30i(\alpha_e/f) \tag{8}$$

where σ_e and α_e are effective parameters used as constants adjusted for best fit with measured results. The technique summarized here is currently under consideration as an American National Standard procedure to obtain the impedance of natural ground surfaces.

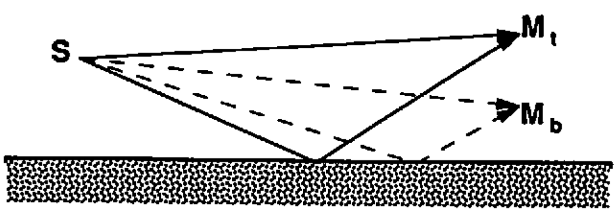

Fig. 1 Level difference measurement to obtain the ground impedance.

METEOROLOGICAL MEASUREMENTS

The deterministic component of the PE propagator requires a knowledge of the speed of sound as a function of height above the ground. The sound speed is obtained from measurements of temperature, wind velocity and wind direction. Figure 2 shows a sketch of a meteorological tower used for these measurements. Temperature is measured at two heights using platinum resistance thermometers. These probes are mounted in fan-aspirated radiation shields to minimize the effects of direct solar radiation. The wind speed and direction are determined at a height of 10 m using cup anemometer and windvane, respectively.

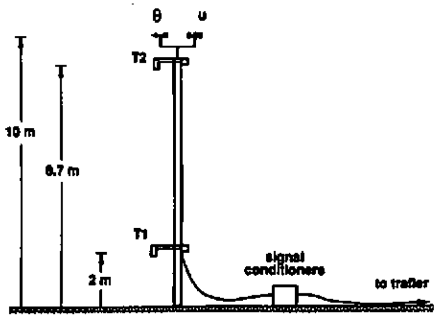

Fig. 2 Meteorological tower

Additional sensors at other heights are also sometimes used

The temperature difference between the two temperature probes and the average wind direction lead to an estimate of $c_d(z)$ by applying similarity theory (e.g., Panofsky and Dutton, 1984).

<u>Wind</u>:
$$u(z) = \frac{u*}{\kappa_a}\left[\ln(z/z_o) - \psi_m(z/L) + \psi_m(z_o/L)\right] \qquad (9)$$

$$\psi_m = -5z/L \qquad , L>0$$
$$\psi_m = \ln[(1+x^2)(1+x)^2/8] - 2\arctan x + \pi/2 \ , L<0 \qquad (10)$$
$$\text{with} \qquad x = (1-16z/L)^{1/4}$$

<u>Temperature</u>:
$$T(z) = T_o + \frac{T*}{\kappa_a}\left[\ln(z/z_o) - \psi_h(z/L) + \psi_h(z_o/L)\right] \qquad (11)$$

$$\psi_h = -5z/L \qquad , \quad L>0$$
$$\psi_h = 2\ln\left[\tfrac{1}{2}\left(1+\sqrt{1-16z/L}\right)\right] \ , \quad L<0 \qquad (12)$$

<u>Similarity relation:</u> $\kappa_a g L T* = T_s u*^2$ (13)

where L is the Monin-Obukhov length and z_o = 0.005 m; κ_a = 0.4; T_s

= 300 K; g = 9.81 ms^{-2}. These empirical expressions describe both the temperature profile $T(z)$ and the wind speed profile $u(z)$; these profiles are not independent, though, but are linked through the Monin-Obukhov length. Following the procedure described by L'Espérance *et al.* (1993), the measurements of temperature and wind provided by our tower can be used to determine the profiles. Once obtained, the effective sound speed profile is

$$c(z)=c_0\sqrt{1+T(z)/273}+u(z)\cos\psi \tag{14}$$

where c_o is the sound speed at 0°C and ψ is the angle between wind and propagation directions.

The stochastic component of the PE propagator is specified through the spatially-varying component $\mu(r,z)$ of the acoustical index of refraction. A Fourier approach [2] is followed to generate individual realizations of a turbulent atmosphere. The PE propagation code is run for each realization and the results of 200 realizations averaged (in power) to give predicted sound pressure levels. These may be compared to the measured levels.

There are different functional forms to characterize the turbulence. A conceptually simple form that has been popular is a Gaussian spectrum specified through two measurable parameters, a variance $<\mu^2>$ and a correlation length ℓ. The wind and temperature are measured using fast response anemometer and temperature probe, respectively, and the standard deviation in wind speed (σ_v) and temperature (σ_T) is obtained from the fluctuating time signal. Then a good approximation is

$$<\mu^2>=(\sigma_v/c_0)^2+(\sigma_T/2T_0)^2 \tag{15}$$

The correlation length ℓ is obtained by correlating the time signals.

COMPARISON OF THEORY AND EXPERIMENT

The calculation of the sound field for a single realization of a turbulent atmosphere is shown in Figure 3. For this calculation, the sound speed profile showed relatively strong upward refraction. A Gaussian spectrum of turbulence was used with ℓ = 1.1 m and $<\mu^2>$ =2x10^{-6}. The limiting caustic, separating the insonified region from the acoustic shadow, appears as a region of higher sound pressure levels. Within the shadow, there is considerable acoustic energy due to scattering from the turbulent structures in the atmosphere. The dominant region scattering sound energy toward a receiver on the ground is approximately halfway between source and receiver and above the limiting caustic in elevation

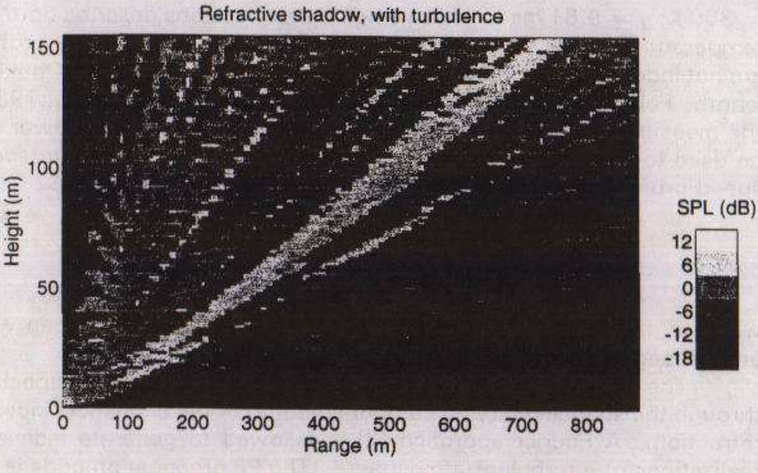

Fig. 3 Refractive shadow with turbulence

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