ATTENUATION OF STONELEY WAVE IN FLUID-FILLED BOREHOLE DUE TO ITS SCATTERING ON TWO-DIMENSIONAL STATISTICALLY ROUGH WELL SURFACE.

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1 INTRODUCTION

Attenuation of Stoneley waves at their propagation along a borehole is currently considered as an important information source about porosity and permeability of surrounding layers, which are crossed by a borehole. Usually the attenuation mechanism is associated with a fluid flow between borehole and porous permeable surrounding medium. But there are some other mechanisms which lead to attenuation of wave field in a borehole. In particular, the attenuation of Stoneley wave can be occurred due to it's scattering on a rough surface of borehole. Hence, there is the necessity to estimate the contribution of scattering at data interpretation of Stoneley waves attenuation in a borehole.

The method used in the report for evaluation of attenuation coefficients is analogous to the one of the work[1], where the attenuation factor due to scattering on the rough surface of empty borehole was estimated. The main difference of the report is existence not the only Rayleigh mode but also a set of additional modes describing waves propagated mainly inside a borehole.

Applicability of the method corresponds to the small ratio of roughness amplitude to wave length.

2 PROBLEM STATEMENT

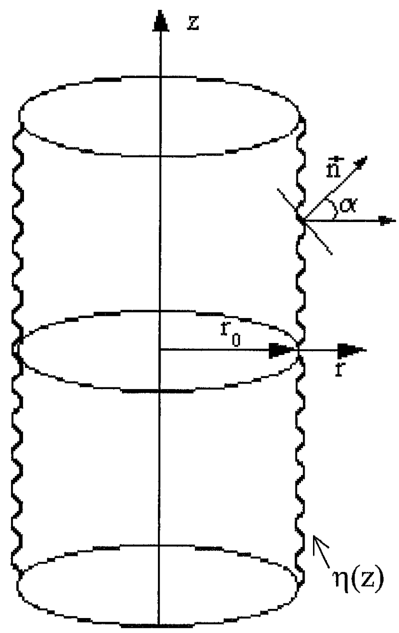

Figure 1. The scheme of borehole

The scheme of a borehole is shown on Fig.1. The well roughness is described as random function $r=\eta(z)$. The point monochromatic source radiating longitudinal waves is placed on the borehole axis.

Wave field outside the borehole is completely described by scalar and vector potentials. Wave field inside a borehole is completely described by scalar potential only. These potentials are governed by the wave equations. The problem is solved in cylindrical coordinates. The boundary conditions on the surface $r = r_0 + \eta(z)$ with arbitrary function $\eta(z)$ consist of: 1) equality of forces applied to the both sides of a borehole surface; 2) equality of normal to the borehole surface components of displacements of fluid and solid; 3) radiation conditions on the infinity; 4) the source singularity on the borehole axis $r = 0$.

3 SOLUTION

Since roughness is considered as a small in comparison with the wavelength, the boundary conditions are expanded in the Taylor series in the vicinity of the mean well radius $r = r_0$, taking into account terms of zero and first orders only. Thus, the initial problem is reduced to the one for the wave field propagation in the perfect fluid-filled borehole but with complicated arbitrary boundary conditions. After Fourier transformation by variables z (z→k) and t (t→ω), the solution of system can be written in the following form:

$$\begin{aligned}\varphi_f(r,k,\omega) &= -\frac{A}{2\pi r}K_0(\nu_f r) + C_1 I_0(\nu_f r)\\ \varphi(r,k,\omega) &= C_2 K_0(\nu_l r)\\ \psi(r,k,\omega) &= C_3 K_1(\nu_s r)\end{aligned},$$

where φ_f is scalar potential, describing wave field in fluid, φ, ψ are scalar and vector potentials, describing wave field in elastic medium; $\nu_i = \sqrt{k^2 - k_i^2}$ at $|k| \geq \frac{\omega}{c_i}$ or $\nu_i = i\sqrt{k_i^2 - k^2}$ at $|k| \leq \frac{\omega}{c_i}$, c_i – waves velocities.

Substitution of the solutions to the boundary condition leads to the matrix integral equation for coefficients C_1, C_2, C_3 in the following form:

$$\begin{aligned}L_0(k)\vec{\varphi}(k) &= L_1^*(k,\tilde{k})\vec{\varphi}(\tilde{k}) + \vec{Q}(k)\\ L_1^*(k,\tilde{k}) &= \frac{1}{2\pi}\int_{-\infty}^{+\infty} dk'\eta(k')L_1(k,\tilde{k})\end{aligned}, \qquad (1)$$

where $\vec{\varphi}(k) = (C_1, C_2, C_3)$- vector consists from coefficients of eigen functions of the solution, L_0, L_1 are matrixes of dimension 3x3, $\vec{Q}(k)$ - vector describing source, $\eta(k)$ - space Fourier spectrum of rough surface; $\tilde{k} = k - k'$ with wave number *k*.

To solve the system (1), the mean field approach is used. In the framework of this method the field is presented as a sum of mean and random fields $\vec{\varphi} = <\vec{\varphi}> + \vec{\varphi}'$. By averaging procedure of initial random operator equation (1) in the framework of mean field approach the following averaged matrix equation for the mean field can be derived:

$$\begin{aligned}&\left[L_0(k) - \sigma^2 L_2\right] < \vec{\varphi} > = \sigma^2 \vec{Q}^*\\ &L_2 = \frac{1}{2\pi}\int_{-\infty}^{+\infty} dk' W(k') L_1^*(k,\tilde{k}) L_0^{-1}(\tilde{k}) L_1^*(\tilde{k},k)\\ &\vec{Q}^* = \frac{1}{4\pi^2}\int_{-\infty}^{+\infty} dk' W(k') L_1^*(k,\tilde{k}) L_0^{-1}(\tilde{k}) \vec{Q}'(\tilde{k}) + <\vec{Q}(k)>\end{aligned},$$

where σ - roughness amplitude, *W(k)* – Fourier spectrum of correlation function: $\sigma^2 W(z-z') = <\eta(z)\eta(z')>$.

In difference with the case of the empty borehole [1], the dispersion equation

$$det\left[L_0(k) - \sigma^2 L_2\right] = 0 \qquad (2)$$

has finite number of roots at the given frequency.

4 ATTENUATION FACTOR

Since, in the considered case of weak scattering the condition $\|L_0\| >> \sigma^2 \|L_2\|$ is required, to solve the dispersion equation method of small perturbation was applied.

Thus, the solution of dispersion equation can be written as following $k_i = k_{mi} + \delta k_i$, where k_{mi} are solutions of unperturbed dispersion equation for perfect borehole. For example, $k_{m0} \equiv k_{St}$ is wave number corresponding to the Stoneley wave, k_{mi}, *i*=1,... are the wave numbers corresponding to the higher quasi-Rayleigh modes. Expression for the correction δk_i can be written as following

$$\delta k_i = \sigma^2 H(k_{mi}) \Big/ \left.\frac{\partial \det L_0(k)}{\partial k}\right|_{k_{mi}},$$

where

$$H(k) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} dk' W(k') \frac{M(k,\tilde{k})}{\det L_0(\tilde{k})}, \quad M(k,\tilde{k}) = \sum_{i,j=1}^{3} L_{2ij}(k,\tilde{k}) \text{minor}(L_0(k)_{ij}),$$

and notation $\text{minor}(L_0(k)_{ij})$ is used for minor of matrix L_0 corresponding to its *ij* component.

Attenuation coefficient is an imaginary part of the dispersion equation solution k_i. Since k_{mi} are pure real wave numbers, therefore the attenuation coefficient is an imaginary part of δk_i.

The imaginary part of the correction δk_i to the wave number is appeared due to the following features:

1) The pole of Stoneley wave $\tilde{k}_i = \pm k_{Sti}$

$$\alpha_{Sti}(\omega) = \frac{\sigma^2 k_s^{\,4}}{\left(\left.\frac{\partial \det L_0(x)}{\partial x}\right|_{x_{mi}}\right)} \left\{ \frac{W(k_{mi} - k_{St}) M(x_{mi}, x_{St}) + W(k_{mi} + k_{St}) M(x_{mi}, -x_{St})}{\left.\frac{\partial \det L_0(x)}{\partial x}\right|_{x_{St}}} \right\},$$

where *x* –dimensionless wave number defined as $k = k_s x$, with k_s - shear wave number wave, *i*=0,....

2) The poles of quasi-Rayleigh waves $\tilde{k} = \pm k_{mi}$

$$\alpha_{Ri}(\omega) = \frac{\sigma^2 k_s^{\,4}}{\left(\left.\frac{\partial \det L_0(x)}{\partial x}\right|_{x_{mi}}\right)} \sum_j \left\{ \frac{W(k_{mi}-k_{mj})M(x_{mi},x_{mj}) + W(k_{mi}+k_{mj})M(x_{mi},-x_{mj})}{\left.\frac{\partial \det L_0(x)}{\partial x}\right|_{x_{mj}}} \right\},$$

where i=0,..., j=1,....

3) The cuts $|k| \le \frac{\omega}{c_l}$, $|k| \le \frac{\omega}{c_s}$ corresponding to longitudinal and shear body waves

$$\alpha_{vi}(\omega) = \frac{\sigma^2}{2\pi} \frac{k_s^4}{\left.\frac{\partial detL_0(k)}{\partial k}\right|_{k=k_{mi}}} \operatorname{Im} \int_{c_s/c_{mi}-1}^{c_s/c_{mi}+1} dx' W(k') \frac{M(x,\tilde{x})}{detL_0(\tilde{x})},$$

where i=0,....

Only the shear waves give contribution to attenuation in the domain $x' \in [c_s/c_{mi} - 1; c_s/c_{mi} - c_s/c_l] \cup [c_s/c_{mi} + c_s/c_l\,; c_s/c_{mi} + 1]$ and the main contribution to attenuation due to scattering into longitudinal waves comes from domain $x' \in [c_s/c_{mi} - c_s/c_l\,; c_s/c_{mi} + c_s/c_l]$.

In connection with these features, the attenuation coefficient can be written as a sum of partial attenuation coefficients $\alpha_i = \alpha_{Sti} + \alpha_{Ri} + \alpha_{vi}$, due to Stoneley to Stoneley (St→St) and Stoneley to quasi-Rayleigh (St→R_i) waves scattering processes as well as due to Stoneley to the body (St→P, St→S) waves scattering processes.

The frequency dependencies of partial attenuation coefficients due to (St→St), (St→R_i), (St→P, St→S) waves scattering processes are shown on the fig.2 for different values of correlation lengths: 0,02, 0,1, 1,0 m.

The left plots show partial contributions to attenuation due to scattering into other eigen modes (Stoneley and first quasi-Rayleigh), the right plots represent contributions of body shear and longitudinal waves. The Gaussian correlation function was used for calculations:

$$W(x) = e^{-x^2/a^2}, W(k) = a\sqrt{\pi} e^{-(ka)^2/4},$$

where a is correlation length.

It is possible to see on the fig.2, what with increasing of correlation length the main contribution to attenuation comes from scattering of the eigen mode into the same eigen mode.

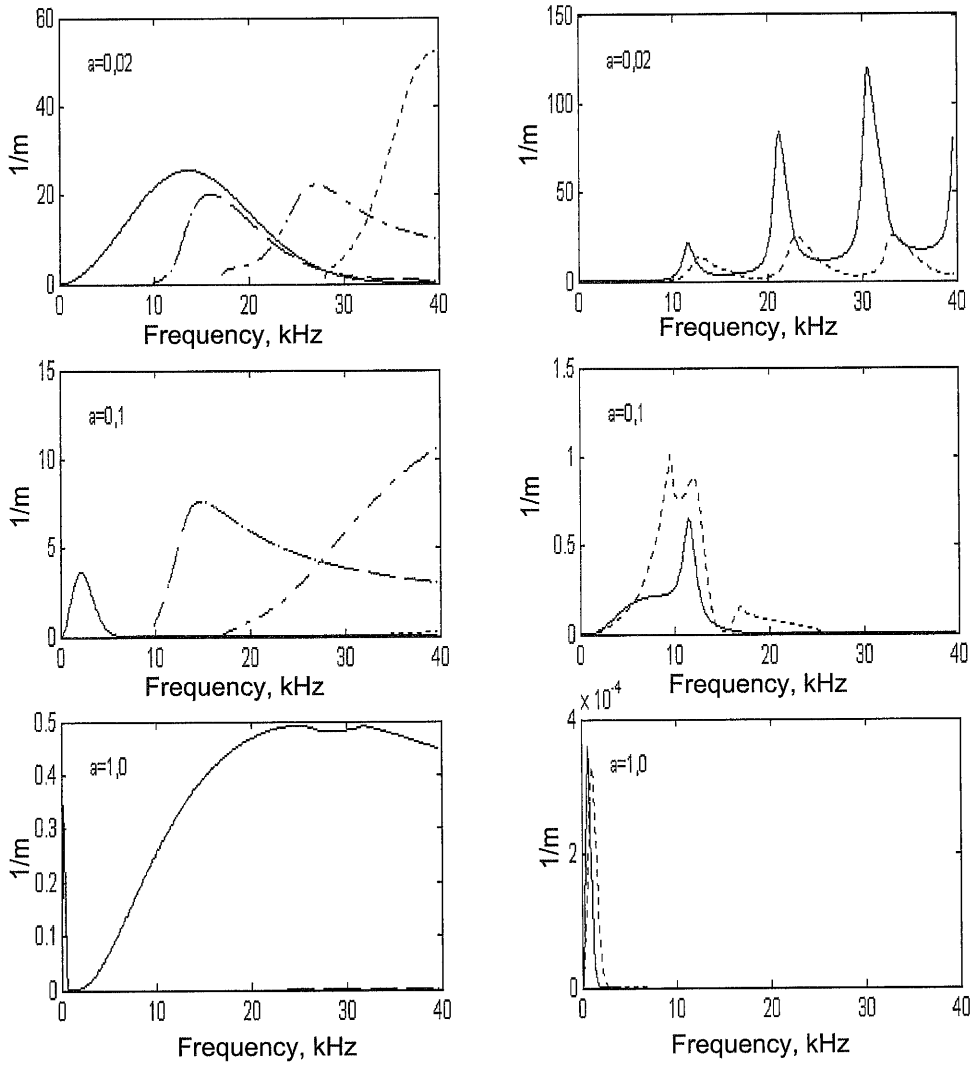

Figure 2. Frequency dependencies of partial attenuation coefficients of Stoneley wave due to different scattering processes. The left plots show attenuation due to scattering into other eigen modes: to Stoneley wave (solid line), to the first (dotted line), to the second (dashdot line), to the third (dashed line) quasi-Rayleigh waves. The right plots show attenuation due to scattering into the body waves: to longitudinal wave (solid line), to shear wave (dotted line).

5 SUMMARY

The problem about Stoneley and quasi-Rayleigh waves attenuation due to their scattering on the well roughness is considered in this report. The solution of problem is obtained in framework of small perturbation approximation and with use of the mean field approach technique.

The dispersion equation describing Stoneley and quasi-Rayleigh waves propagation along borehole with rough surface is derived. Its explicit solution is obtained in the form of correction to wave numbers of the undisturbed problem. The real and imaginary parts of these corrections describe the dispersion and attenuation factor of waves correspondingly.

The total attenuation coefficient can be represented as a sum of partial attenuation coefficients due to Stoneley to Stoneley (St→S) and Stoneley to quasi-Raileigh (St→R_i) waves scattering processes as well as due to Stoneley to body (St→P, St→S) waves scattering processes.

The numerical results showing the frequency dependencies of the attenuation coefficient for different correlation functions and different correlation lengths are presented. It is shown that for large correlation length the main contribution to attenuation of some eigen mode comes from the scattering into the same eigen mode.

6 REFERENCES

1. M.R. Chillemi and G.A. Maximov, Attenuation of Stoneley waves in empty borehole due to its scattering on statistically rough well surface, Proc. of Sci. Session of MEPhI-2002, Jan. 21.-25, 2002. V.5, Moscow, MEPhI, 2002, p.88-89.