AN INVESTIGATION OF THE EFFECT OF ENVIRONMENTAL FLUCTUATIONS ON SYNTHETIC APERTURE AND LONG ARRAYS

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1 INTRODUCTION

Synthetic Aperture Sonar (SAS) is a sonar imaging technique that artificially creates a large aperture array by moving a single small aperture along a (usually linear) path. The angular resolution of a simple line array is directly proportional to the wavelength divided by the size of the aperture so SAS is an attractive way of resolving "difficult" targets.

Most current work on SAS has focused on the errors associated with deviations in the position of the elemental array and the development of auto-focusing algorithms to improve the imaging. Large-scale trials are expensive, hence much of this work has relied on simulations. Almost entirely, these simulations and image processing algorithms assume that the water is homogeneous with a single wave speed at all points. In reality this is not the case: the refractive index of the ocean varies with position, primarily due to small fluctuations in temperature caused by water motion. These fluctuations are small – a typically used rule of thumb has a mean square refractive index fluctuation of 5×10^{-9} at a mean scale of 0.6m – nevertheless they can have a significant effect on the local propagation[1].

The aim of this work is to try to assess the effect of the environmental fluctuations on the formation of a Synthetic Aperture Sonar array. As a first step, this paper looks at how the resolution of a fixed, large aperture, side-scan sonar is affected by refractive index fluctuations in the water. Previous work by Dobbins[1] focused on passive arrays whereas here we will mainly be concerned with an active configuration

2 MODELLING APPROACH

2.1 Methodology

A schematic of the geometry and symbols that have been used throughout the development is given in Figure 1. We start by considering a pressure wave, $\mathbf{P}_{in}$, incident on two point scatterers located at position $\mathbf{r}_0$ and $\mathbf{r}_1$ with respect to the origin point on the array

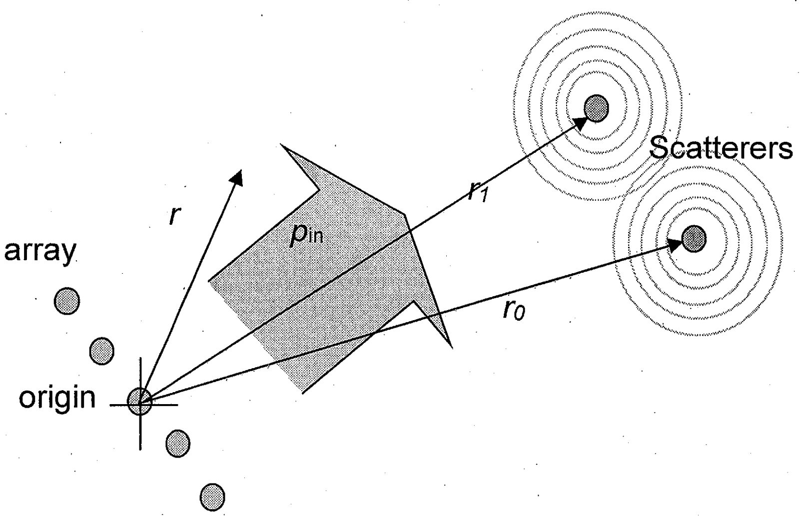

Figure 1: Point scatterers subjected to an incident pressure wave.

Physically, we expect the scattered wave at some position, $\mathbf{r}$, to have the form

$$p_{\text{scat}}(\mathbf{r}) \approx CG(\mathbf{r} \mid \mathbf{r}_0)p_{\text{in}}(\mathbf{r}_0) + CG(\mathbf{r} \mid \mathbf{r}_1)p_{\text{in}}(\mathbf{r}_1) \tag{1}$$

Here C is the scattering amplitude, which one would expect to be approximately a constant for a point scatterer. The function $G(\mathbf{r}_1|\mathbf{r}_2)$ is the Green's function for the steady state wave equation with a position dependant wave speed. If we assume that the wave number when no fluctuations are present is given by k, and the refractive index relative to this state may vary at the point $\mathbf{r}$ by an amount $\varepsilon(\mathbf{r})$, then G satisfies

$$\nabla^2 G(\mathbf{r} \mid \mathbf{r}_0) + k^2[1 + \varepsilon(\mathbf{r})]G(\mathbf{r} \mid \mathbf{r}_0) = -\delta(\mathbf{r} - \mathbf{r}_0) \tag{2}$$

Now, if the incident pressure field is caused by a line array with source density, $\rho(\mathbf{r})$, then

$$p_{\text{in}}(\mathbf{r}) = \int_{\text{line array}} G(\mathbf{r} \mid \mathbf{r}_0)\rho(\mathbf{r}_0)\,d\mathbf{r}_0 \tag{3}$$

and the problem reduces to that of finding the Green's function, G.

Rather than try to calculate lots of different cases, one can imagine that we have an ensemble of refractive index fluctuations corresponding to some probability distribution, $P[\varepsilon]$. One then considers quantities of interest as functionals of $\varepsilon(\mathbf{r})$ and forms ensemble averages as functional integrals, e.g., for the functional $F[\varepsilon(\mathbf{r})]$, the ensemble average is given by

$$\langle F \rangle = \int P[\varepsilon(\mathbf{r})] F[\varepsilon(\mathbf{r})] d\mu[\varepsilon] \tag{4}$$

where $d\mu[\varepsilon]$ is the measure on the space of fields $\varepsilon(\mathbf{r})$. If one then found that the standard deviation of F was small, one could consider $\langle F \rangle$ to be a "typical" value over the ensemble for the quantity F.

One quantity of special interest is the beam formed pressure, $\Gamma(\theta)$. Approximating the sums over the array sensors by integrals, the average beam formed pressure becomes

$$\begin{aligned} \langle \Gamma(\theta) \rangle &= \left\langle \int_{\text{array}} p_{\text{scat}}(\mathbf{r}) e^{i\mathbf{k}.\mathbf{r}\sin\vartheta} d\mathbf{r} \right\rangle \\ &= \int_{\text{array}} d\mathbf{r}\, e^{i\mathbf{k}.\mathbf{r}\sin\vartheta} \int_{\text{source}} d\mathbf{r}' C \{ \langle G(\mathbf{r} \mid \mathbf{r}_0) G(\mathbf{r}_0 \mid \mathbf{r}') \rangle + \langle G(\mathbf{r} \mid \mathbf{r}_1) G(\mathbf{r}_1 \mid \mathbf{r}') \rangle \}. \end{aligned} \tag{5}$$

If $\Gamma(\theta)$ is calculated using only a single scatterer normal to the array, the position, in angle, of the first minimum of the intensity formed from Γ would be the resolution of the array as defined by the Rayleigh Criterion[2]. In practice the real resolution of the array will be worse than this, since the contributions from other scatterers are coherent. Coherence effects can be studied using equation (5). In either case one must calculate the four-point function $\langle G(\mathbf{r}_o \mid \mathbf{r}_p) G(\mathbf{r}_p \mid \mathbf{r}_s) \rangle$. This is sketched out in the next section.

2.2 The four-point function

To proceed one needs to find an expression for the Green's function in the presence of an arbitrary refractive index field. Using a method first developed by Schwinger one notes that (formally at least)

$$G = i \int_0^\infty e^{-is(\nabla^2 + k^2[1+\varepsilon])} ds \equiv i \int_0^\infty g(\mathbf{r}, \mathbf{r}' \mid s)\, ds \tag{6}$$

It can be shown that g is given by the path integral

$$g(\mathbf{r}, \mathbf{r}' \mid s) = \int e^{iS[\mathbf{q};\varepsilon]} [d\mathbf{q}(\tau)] \tag{7}$$

with action functional

$$S[\mathbf{q};\varepsilon] = \int_0^s \left[\frac{1}{4} \dot{\mathbf{q}}(\tau).\dot{\mathbf{q}}(\tau) + k^2 (1 + \varepsilon(\tau)) \right] d\tau \tag{8}$$

Here τ has been used to parameterise paths with generalised co-ordinates, $\mathbf{q}(\tau)$, and the integral is to be taken over all paths that start at $\mathbf{q}(0) = \mathbf{r}'$ and end at $\mathbf{q}(s)=\mathbf{r}$.

If one assumes that the integral over s in (6) commutes with the ensemble averaging, the ensemble averaged Green's function is thus given by

$$\left\langle G(\mathbf{r} \mid \mathbf{r}') \right\rangle = i\int_0^\infty ds \int d\mu[\varepsilon] P[\varepsilon] \int [d\mathbf{q}] e^{iS[\mathbf{q};\varepsilon]} = i\int_0^\infty ds \left\langle g(\mathbf{r},\mathbf{r}';\mathbf{s}) \right\rangle, \tag{9}$$

and the four-point function by

$$\left\langle G(\mathbf{r}_o \mid \mathbf{r}_p) G(\mathbf{r}_p \mid \mathbf{r}_s) \right\rangle = \int_0^\infty ds \int_0^\infty ds' \left\langle g(\mathbf{r}_o,\mathbf{r}_p;s) g(\mathbf{r}_p,\mathbf{r}_s;s') \right\rangle \tag{10}$$

Path integrals have been used previously to analyse the propagation in random media[5,6,7]. Usually one assumes that it is possible to interchange the order of the path integral and functional integral in (9) and (10). Although in many cases this appears to give physically realistic results, this assumption cannot be justified mathematically[5]. Here we take an approach that avoids the need to interchange the order of the integrations. If it is assumed that the rate of change with position of the refractive index is small compared to a wavelength one can use a combination of perturbation theory (the gradient of the refractive index provides the "force" that bends the rays) and saddle point integration to approximate the path integrals. The functional integral over $\varepsilon(\mathbf{r})$ can then be easily performed using standard techniques. The ensemble averaged heat kernel is given by

$$\begin{aligned}\left\langle g(\mathbf{r}_p,\mathbf{r}_s;s) \right\rangle = e^{i\frac{|\mathbf{r}_p-\mathbf{r}_s|^2}{4s}+ik^2 s} \Bigg(& 1 - \frac{k^4}{2}\int_0^s d\tau \int_0^s d\tau' A(\mathbf{x}_1,\mathbf{x}_1') + 2ik^4 \int_0^s d\tau \int_0^s d\tau' h(\tau,\tau') \frac{\partial}{\partial x^\mu}\frac{\partial}{\partial x'_\mu} A(\mathbf{x}_1,\mathbf{x}_1') \\ & + ik^4 \int_0^s d\tau \int_0^\tau d\tau' \int_0^\tau d\tau'' \frac{\partial}{\partial x'^\mu}\frac{\partial}{\partial x''_\mu} A(\mathbf{x}_1',\mathbf{x}_1'') + \frac{k^4}{s}\int_0^s d\tau \int_0^s d\tau' (\tau - s)(\tau' - s) \frac{\partial}{\partial x^\mu}\frac{\partial}{\partial x'_\mu} A(\mathbf{x}_1,\mathbf{x}_1') + \ldots \Bigg)\end{aligned} \tag{11}$$

and the kernel of the ensemble averaged four-point function by

$$\langle g(\mathbf{r}_o,\mathbf{r}_p;s)g(\mathbf{r}_p,\mathbf{r}_s;s)\rangle = e^{i\frac{|\mathbf{r}_o-\mathbf{r}_p|^2}{4s}+ik^2s}\, e^{i\frac{|\mathbf{r}_p-\mathbf{r}_s|^2}{4s'}+ik^2s'}\Bigg(1-\frac{k^4}{2}\int_0^s d\tau\int_0^s d\tau'\, A(\mathbf{x}_1(\tau),\mathbf{x}_1(\tau')) \tag{12}$$

$$-\frac{k^4}{2}\int_0^{s'} d\tau\int_0^{s'} d\tau'\, A(\mathbf{x}_2(\tau),\mathbf{x}_2(\tau')) - k^4\int_0^{s} d\tau\int_0^{s'} d\tau'\, A(\mathbf{x}_1(\tau),\mathbf{x}_2(\tau'))$$

$$+2ik^4\int_0^s d\tau\int_0^s d\tau'\, h(\tau,\tau')\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial x'_\mu}A(\mathbf{x}_1,\mathbf{x}'_1) + 2ik^4\int_0^{s'} d\tau\int_0^{s'} d\tau'\, h(\tau,\tau')\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial x'_\mu}A(\mathbf{x}_2,\mathbf{x}'_2)$$

$$+ik^4\int_0^s d\tau\int_0^\tau d\tau'\int_0^\tau d\tau''\frac{\partial}{\partial x'^\mu}\frac{\partial}{\partial x''_\mu}A(\mathbf{x}'_1,\mathbf{x}''_1) + ik^4\int_0^{s'} d\tau\int_0^\tau d\tau'\int_0^\tau d\tau''\frac{\partial}{\partial x'^\mu}\frac{\partial}{\partial x''_\mu}A(\mathbf{x}'_2,\mathbf{x}''_2)$$

$$+i\frac{k^4}{s}\int_0^s d\tau\int_0^s d\tau'\;(\tau-s)(\tau'-s)\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial x'_\mu}A(\mathbf{x}_1,\mathbf{x}'_1)$$

$$+i\frac{k^4}{s'}\int_0^{s'} d\tau\int_0^{s'} d\tau'\;(\tau-s')(\tau'-s')\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial x'_\mu}A(\mathbf{x}_2,\mathbf{x}'_2)+\ldots\Bigg)$$

where $A(\mathbf{x}, \mathbf{y})$ is the correlation function of the refractive index variations. Greek indices run over the values (1,2,3) and denote the components of a vector with the Einstein summation convention assumed for repeated indices. The vector $\mathbf{x}_1(\tau)$ lies on the straight line running from the source position, $\mathbf{r}_s$, to the point scatterer at $\mathbf{r}_p$ and $\mathbf{x}_2(\tau)$ is on the straight line from $\mathbf{r}_p$ to the observation point, $\mathbf{r}_o$.

3. SIMULATION AND RESULTS

The effect of expected levels of refractive index fluctuations on the observed resolution, calculated using the Rayleigh criterion, appears to be negligible. This is perhaps not too surprising: the angular resolution $\Delta\vartheta$ of a continuous uniform line array is given by $\Delta\vartheta = \sin^{-1}(\lambda/L)$, where λ represents the wavelength and L is the length of the array[2]. A root mean squared (r.m.s) refractive index change of 5.0×10^{-9} corresponds to a wavelength change of approximately 0.007□. Although there could be significant differences to the actual propagation paths between the homogenous case and when fluctuations are present, it is the interference of neighbouring paths that affect the resolution and these changes will be small.

The fluctuations do, however, cause significant attenuation of the signal. When the range is much larger than the correlation length (for both the source and observation point at the same place), the attenuation constant is given by

$$\alpha = \frac{\Lambda}{R} \approx \frac{\pi^2}{2}k^2\sigma^2 L_\varepsilon\left(1+24\frac{\sigma_l^{\,2}}{L_\varepsilon^{\,2}}\right) \quad \text{(nepers / m)} \tag{13}$$

This corresponds to approximately 0.027 dB/m for the current case. For distributions with a constant correlation length, one sees that the attenuation constant is directly proportional to both the correlation length and the mean squared refractive index fluctuationT

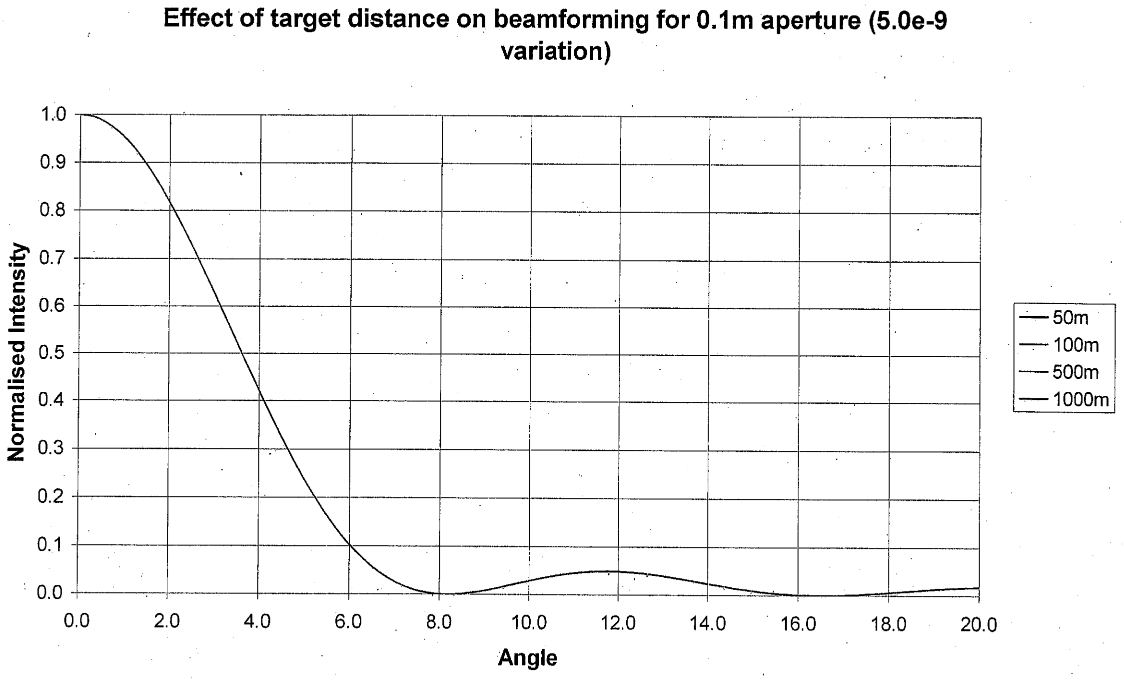

Figure 2: Effect of target distance for 0.1m array (Note all lines are overlapping.)

When the level of fluctuations or the distance to the target is increased the result is dependent on the size of the aperture. Figure 2 shows the effect of increasing the distance of the target from 50m to 1km for a 0.1m array in the presence of a 5.0×10^{-9} mean square refractive index variation (the correlation length is 0.6m). Each curve has been normalised to unity at the origin, which hides the range dependant attenuation as described above. Increasing the range appears to have little effect on the resolution.

A similar result is obtained when one increases the level of the refractive index variations. There is no significant effect on the resolution or form of the beam-formed intensity implying that the resolution of "small arrays" (i.e., less than the coherence length) is fairly insensitive to fluctuations in the media.

Things are very different when one considers a 1.0m array however. Increasing the range gives an improvement in the resolution of the array – the minimum moves to a smaller angle – but this is accompanied by an increase in the sidelobe level. The effect is much more dramatic when one increases the magnitude of the fluctuations (Figure 3). Even at 50m, once the mean square variation has risen to 1.0×10^{-6} the height of the sidelobe is over 40%□ that of the main beam. This will obviously appear as a false target.

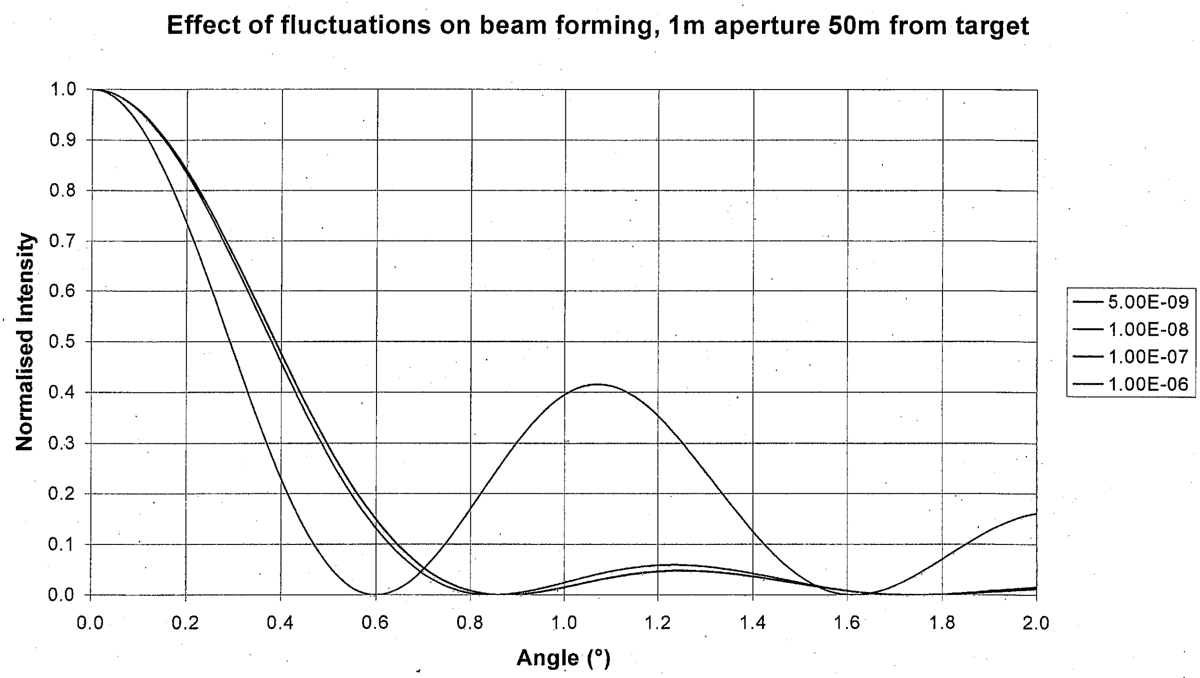

Figure 3: Effect of fluctuations for a 1m array

That introducing positional fluctuations in the wave speed of the water improves the resolution of the array seems counterintuitive at first. The primary effect of the fluctuations is to increase the apparent attenuation. However they also cause variations in both the path taken by the sound and the time taken to reach the array. The effect of this is to cause the phase to vary more rapidly with position. When one integrates over the array (i.e., beam forming) this will cause the interference minima to appear at a smaller angle, in an analogous way to a decrease in wavelength. One gets a picture where each scatterer appears slightly sharper but is surrounded by diffraction rings. The effect is akin to looking at streetlights on a foggy day.

One can also consider coherence effects and look at the ability to discriminate two scatterers at the same range but separated in angle (one must use a point source rather than a line array). Figure 4 shows the result of changing the angular separation when mean square refractive index variations of 1.0×10^{-6} are present. The presence of fluctuations drops the Rayleigh separation to 1.09°, which is the 'Resolved' case shown in blue. At this separation one can clearly tell there are two targets. When there are no fluctuations increasing the separation makes it easy to distinguish the two targets however, in this case we find that increasing the separation causes the formation of a new false target midway between the two real targets. In addition we see multiple peaks at higher angles which are sufficiently high in amplitude that they could easily be mistaken for genuine returns.

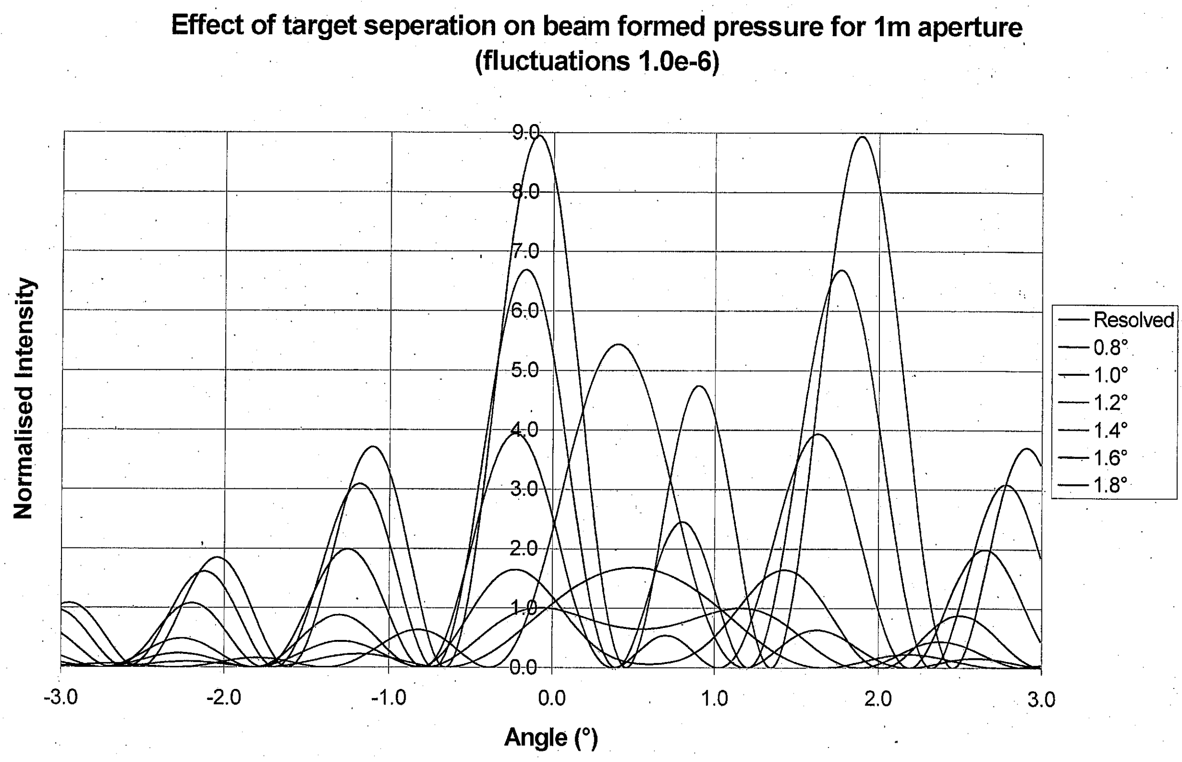

Figure 4: Discrimination of point scatterers when a mean square refractive index variation of 1.0e-6 is present.

4. CONCLUSIONS

In conclusion, the results presented here suggest that the levels of variability typically expected in the ocean[1] have a minimum effect on the array resolution and ability to distinguish targets, though they do have a significant effect on the expected signal strength (and hence signal to noise ratio). Increasing the level of variability, for arrays larger than the correlation length, tends to improve the resolution but at the expense of creating false targets due to the higher sidelobes in the beam formed intensity. Of course this does not necessarily mean that smaller arrays are better when there is variability in the environment - the targets resolved by a 1m array might not be resolved at all with a 0.1m array – however it does indicate that other factors must be considered than just the resolution when evaluating performance.

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