

# Spatial Fourier transform method incorporated with Johnson-Champoux-Allard model to determine absorption coefficient of materials

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## 1 INTRODUCTION

This paper describes spatial Fourier transform method which is based on calculation of complex pressure distributions on two parallel surfaces and decomposing the complex pressure distributions into plane-wave components by using two-dimensional spatial Fourier transform which is used to separate the incident and reflected plane wave components. Johnson-Champoux-Allard model is utilized to predict effective density and bulk modulus of the air in the material, which are used to calculate wave number and characteristic impedance. Consequently, they are used to determine reflection coefficient of the porous materials at a range of angle of incidence.

## 2 THEORY OF SPATIAL FOURIER TRANSFORM

### 2.1 Reflection coefficient

A material with an infinite thickness is considered in this work. The reflection coefficient of surface of the test material at a given angle is determined using following equation [1, 2].

$$C_r(k_r) = \frac{Z_1 \cos \theta - Z_0 [1 - (k_0/k_1)^2 \sin^2 \theta]^{1/2}}{Z_1 \cos \theta + Z_0 [1 - (k_0/k_1)^2 \sin^2 \theta]^{1/2}} \quad (1)$$

Where  $C_r(k_r)$  is the reflection coefficient of the surface at an angle of incidence,  $Z_1$  is the characteristic impedance in the test material,  $k_1$  is the complex wave number in the test materials,  $k_r = k_0 \sin \theta$ ,  $Z_0$  is the characteristic impedance of air, and  $k_0$  is the wave number of air, and  $\theta$  is the angle of incidence.

The characteristic impedance and the complex wave number in the test materials are calculated using Johnson-Champoux-Allard (JCA) model [3].

### 2.2 Complex sound pressure distribution

A monopole sound source is located at a point (0,0,  $z_0$ ) above the centre of the test material. The complex pressure distribution of this monopole source at a point ( $r$ ,  $z$ ) could be calculated using the following equation.

$$P(r, z) = \exp(ik_0 R_1) / 4\pi R_1 + \int_0^\infty C_r(k_r) i \frac{\exp[ik_z(z_0+z)]}{4\pi k_z} x J_0(r k_r) k_r dk_r \quad (2)$$

Where  $r = (x^2 + y^2)^{1/2}$ ,  $R_1$  is the length of the direct path,  $R_1 = [r^2 + (z_0 - z)^2]^{1/2}$ ,  $k_r = (k_x^2 + k_y^2)^{1/2}$ ,  $k_x$  and  $k_y$  are the wave vectors in  $x$  and  $y$  direction respectively,  $k_z$  is the wave vector in  $z$  direction,  $J_0$  is the Bessel function of the first kind for order 0.

The complex pressure distribution over a surface of the material given in equation 2 is numerically calculated from repeated application of the trapezoidal integration formula [4].

### 3 THEORETICAL RESULTS

A sound source was assumed to be located at 200mm height above the centre of the material. Two circular planes with a radius of 1000mm and with heights of 10mm and 30mm are considered for calculations in this work. Complex sound pressure that was applied to the surface of the material at 1000 Hz is given in Figure 1. Complex sound pressure reduces with moving towards outer of the circular planes.

The spatial spectrum of the complex sound pressure is shown in Figure 2. Spatial spectrum becomes smoother when  $k_r/k_0$  ratio is higher than 1.2 while it fluctuates when  $k_r/k_0$  ratio is lower than 1.2.

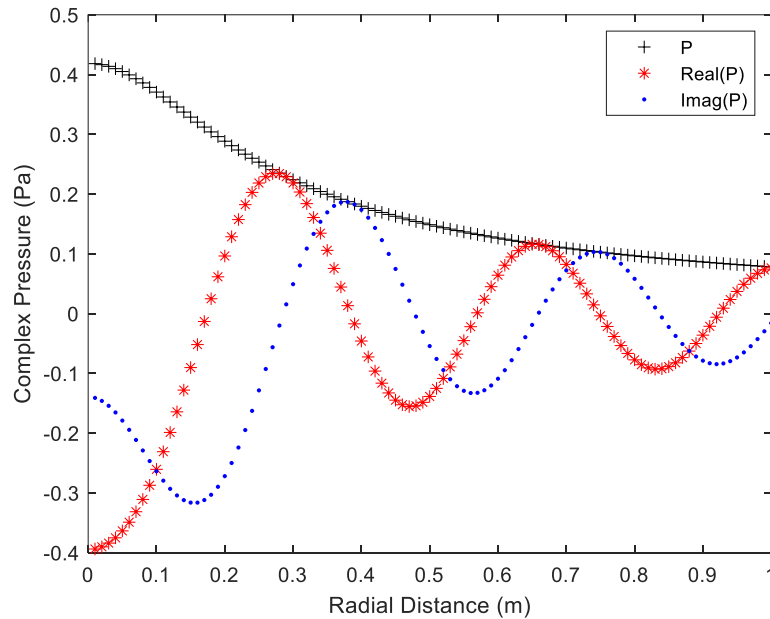


Figure 1: Complex sound pressure distribution on a plane versus radial distance.

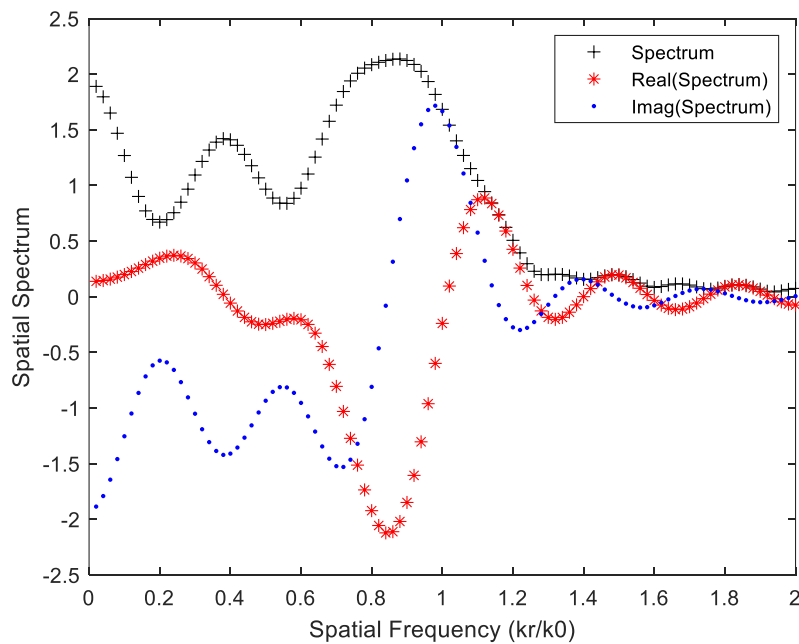


Figure 2: Spatial spectrum of the complex pressure distribution.

The reflection coefficients obtained from equation (1) are shown in Figure 3a and Figure 3b for different flow resistance values of 5460 Pa/m<sup>2</sup>.s and 100 Pa/m<sup>2</sup>.s, respectively. The discrepancy is attributed to the fact that the reduction of the sound pressure towards outer of surface is slower for material with higher flow resistance value.

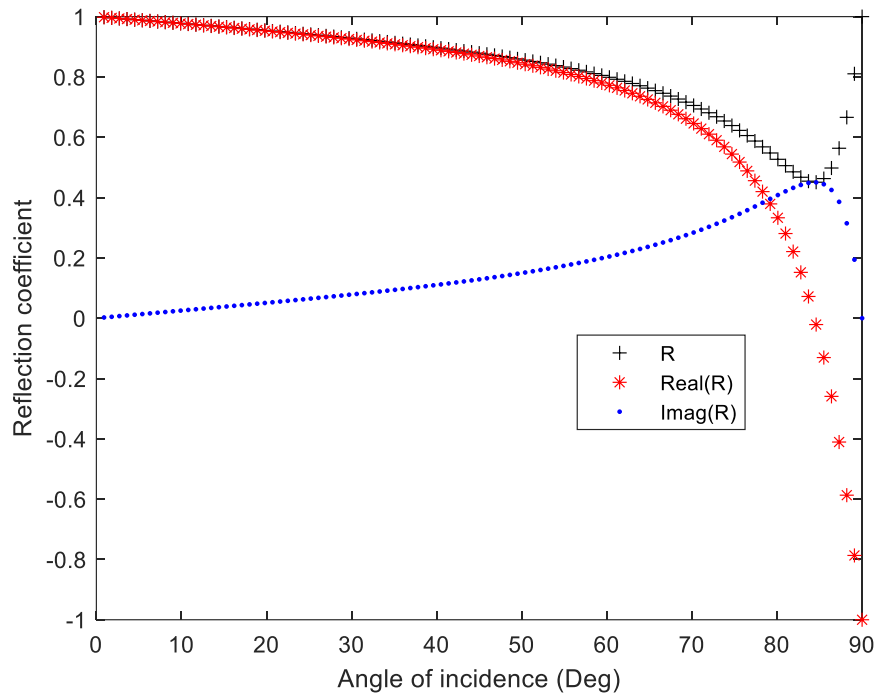


Figure 3a: Reflection coefficient versus angle of incidence for flow resistance value of 5460 Pa/m<sup>2</sup>.s at 1000 Hz.

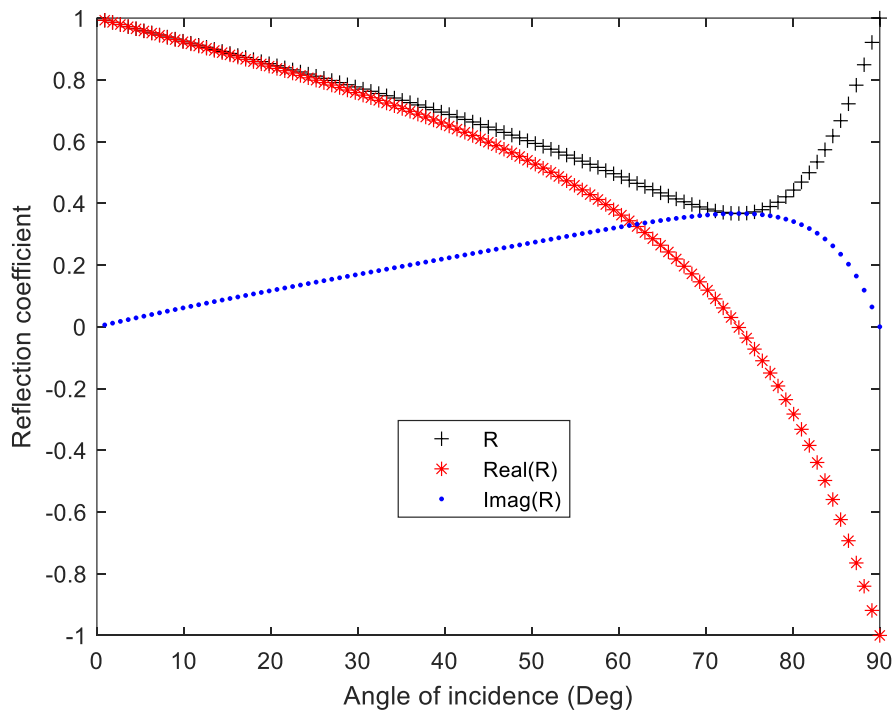


Figure 3b: Reflection coefficient versus angle of incidence for flow resistance value of 100 Pa/m<sup>2</sup>.s at 1000 Hz.

## 4 CONCLUSION

A spatial Fourier transform method is used to determine the reflection coefficient at oblique angles of incidence. The complex parameters such as wave number and characteristic impedance are predicted using Johnson-Champoux-Allard model.

## 5 REFERENCES

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