inter•noise 96
25th Anniversary Congress – Liverpool

INCE: 74

A RECURSIVE ALGORITHM FOR DIGITAL FILTERS TO REDUCE THE NUMBER OF MULTIPLIERS

H Nakajima (1), M Tohyama (1) & M Tanaka (2)

(1) Kogakuin University, Room 5-603, Kogakuin University 2665-1, Nakanomachi, Hachioji-shi, 192 Tokyo, Japan, (2) NTT Corporation, Japan

1. INTRODUCTION

Sound field control requires real-time filtering. Since the filtering requires many calculations, real-time sound field control using a personal computer is still difficult. Consequently, it would be useful if the amount of calculation required for filtering could be reduced. In this article, we propose a Recursive Vector Projection (RVP) method for getting the approximate solutions of a linear equation. We can realize the use of filters having a few multipliers whose coefficients are calculated from the RVP solution. Here, filtering was formulated as a set of linear equations which do not always have exact solutions [1].

2. RECURSIVE VECTOR PROJECTION (RVP)

Recursive Vector Projection (RVP) is an algorithm to approximate the solutions of a linear equation:

$$\mathrm{A}\boldsymbol{x}=\boldsymbol{b} \qquad (1)$$

where A is a (m, n) matrix, $\boldsymbol{x}$ denotes an unknown $(n, 1)$ vector and $\boldsymbol{b}$ is a column vector. Solving a linear equation is equivalent to finding the linear combination coefficients among As column vectors which synthesize vector $\boldsymbol{b}$. RVP chooses the As column vector that has the nearest direction to vector $\boldsymbol{b}$. Always using the nearest vector, RVP reduces the norm of the error vector ($\mathrm{A}\boldsymbol{x}'-\boldsymbol{b}$) step by step, where $\boldsymbol{x}'$ is the approximate solution obtained just one step before. In every step, only one, the filter-tap coefficient corresponding to the nearest vector is updated.

The set of following equations describes the RVP algorithm.

$$\boldsymbol{x}(0)=0,\ \boldsymbol{e}(0)=\boldsymbol{b},\ u=0 \qquad (2)$$

$$\boldsymbol{p}_k=\boldsymbol{a}_k\cdot(\boldsymbol{e}(u),\boldsymbol{a}_k)/(\boldsymbol{a}_k,\boldsymbol{a}_k) \quad k=1,2,\ldots,n \qquad (3)$$

$(\boldsymbol{a}, \boldsymbol{b})$: the inner product between $\boldsymbol{a}$ and $\boldsymbol{b}$

$$m=\{k \mid (p_k, p_k)=\max\} \qquad (4)$$

max: the maximum value

$$e(u+1)=e(u)-p_m \qquad (5)$$

$$x_m(u+1)=x_m(u)+(e(u), a_k)/(a_k, a_k) \qquad (6)$$

$$x_k(u+1)=x_k(u) \quad (k \neq m)$$

$$u=u+1 \qquad (7)$$

where a_k is the k th column vector of A and x_k is the k th coefficient of x. Eq. (2) initializes the unknown vector and the error vector $e=b$-Ax.

Next, in Eq.s (3) and (4), RVP calculates the error vector's projection onto each of the column vector of the matrix A and selects one column vector which gives the longest projection. Then the projection is subtracted from the error vector (Eq. (5)). Finally, the coefficient corresponding to the column vector gains by the ratio of the norm of the column vector itself and the norm of the error vector's projection onto the column vector (Eq. (5) and (6)). The RVP repeats the Eq.s (3)-(6) increasing the counter until the error norm become small enough or the number of the non-zero coefficients reaches a given number. Because RVP always uses a nearest column vector, it reduces the norm the error vector step by step converging to LSE solution. Comparing the RVP solution with LSE's. RVP solution has smaller norm and has fewer non-zero coefficients. When realizing the solution as filter coefficients on a finite precision hardware, these characteristics enable the RVP solution to have wider dynamic range and smaller computational complexity. A fast algorithm is available for the RVP product.

3. FAST RVP (FRVP)

The following equations describe a fast RVP (FRVP) similar to Eq.s (2) to (7).

$$x=\mathbf{0},\ p=\mathrm{A}^{\mathrm{T}}b,\ \mathrm{R}=\mathrm{A}^{\mathrm{T}}\mathrm{A} \qquad (8)$$

$$m=\{k \mid p_k^2/\mathrm{R}_{kk}=\max\} \qquad (9)$$

$$p=p-R_m \cdot p_m/\mathrm{R}_{mm} \qquad (10)$$

$$x_m=x_m+p_m/\mathrm{R}_{mm} \qquad (11)$$

$$x_k=x_k \quad (k \neq m)$$

Where R_k is the k th column vector of R, R_{kk} is the k th coefficient of R_k, and p_k is the k th coefficient of p. The recurrence number u has been omitted here for simplicity. FRVP is m times faster than the RVP described by Eq.s (2) to (7). The initialization process for FRVP including $\mathrm{A}^{\mathrm{T}}\mathrm{A}$, however, takes more calculations than that for RVP.

4. OPTIMIZATION OF THE RVP SOLUTION

We can reduce the errors produced by the RVP (or FRVP) solution by newly obtaining the LSE solution of the linear equations to the RVP solution

vectors. The RVP solution can be optimized as follows.
Suppose that the RVP solution vector is

$$x_R' = [x_{k(1)}, x_{k(2)}, \ldots, x_{k(r)}]^T ,$$

where none the elements is zero.
When a matrix that consists of $a_{k(u)}$ is

$$A' = [a_{k(1)}, a_{k(2)}, \ldots, a_{k(r)}] ,$$

the optimized solution is derived from the follow equation,

$$x_L' = (A'^T A')^{-1} A' b . \tag{12}$$

This optimization process can be performed in every step of the RVP procedure.

5. AN INVERSE FILTER USING RVP

Suppose that the room transfer function is $H(z)$ and the desired response is $D(z)$. The inverse filter's transfer function $X(z)$ is obtained by solving the next equation,

$$Hx = d \tag{13}$$

where

$$H=\begin{pmatrix} h(0) & & 0 \\ h(1) & \ddots & \\ \vdots & \ddots & h(0) \\ \vdots & \vdots & h(1) \\ h(n-1) & \vdots & \vdots \\ & \ddots & \vdots \\ 0 & & h(n-1) \end{pmatrix} \quad x=\begin{pmatrix} x(0) \\ x(1) \\ \vdots \\ x(l-1) \end{pmatrix} \quad d=\begin{pmatrix} d(0) \\ d(1) \\ \vdots \\ d(l+n-2) \end{pmatrix},$$

$$H(z)=\sum_{k=0}^{n-1} h(k)z^{-k} , \quad X(z)=\sum_{k=0}^{l-1} x(k)z^{-k} , \quad D(z)=\sum_{k=0}^{n+l-2} d(k)z^{-k} ,$$

n is the length of $H(z)$ and l is the order of $X(z)$.
The inverse filter can be realized using the solution obtained by RVP.

We conducted computer experiments to investigate the differences between RVP and LSE solutions. Fig. 1 shows the room impulse response. Fig. 2 shows characteristics of the RVP inverse filter. The horizontal axis show the recurrence times u used in the RVP procedure. The number of non-zero coefficients r, the squared error $|e|^2$, and the squared norm $|x|^2$ are illustrated in Fig.s 2a, 2b, 2c respectively. We see in Fig. 2a that the increase in the number of the non-zero coefficients slows as the number of recurrences u increases. This is because the RVP procedure quite often updates the coefficients of the same taps. The bold line represents RVP solutions and the broken line (in Fig.s 2b and 2c) the optimized RVP. The errors and norms are normalized by LSE error $|e_L|^2$ and norm $|x_L|^2$ respectively. Twenty recurrences achieve error and norm below 4 dB and -6 dB respectively. The optimization effect, however, is only expected at 1 dB. The impulse response of the inverse filter by LSE x_L and by RVP x_R with 20 recurrences are shown in Fig. 3. We can clearly see that RVP is able to realize a filter with fewer multipliers than those required by LSE.

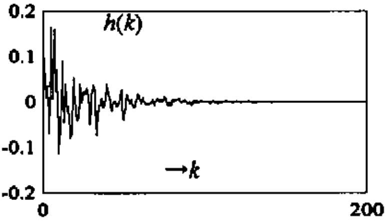

Fig. 1. Room impulse response.

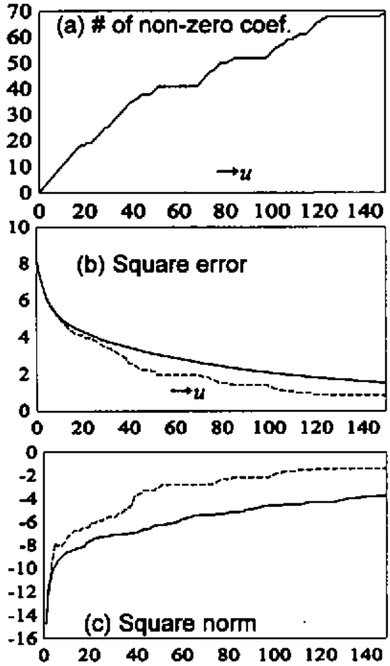

Fig. 2. The characteristics of the RVP inverse filter

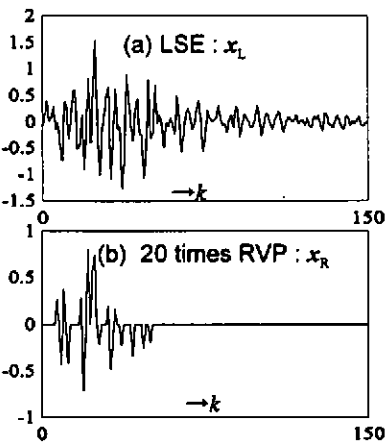

Fig. 3. Impulse response of the inverse filters

6. SUMMARY

We proposed the recursive vector projection (RVP) method as a recursive procedure solving a linear equation. By changing the recurrence times, we can obtain a solution having any number of non-zero coefficients. The fast (FRVP) algorithm allows calculation m times faster than that with RVP. The solution to RVP can be optimized by setting the least square error solution of the linear equations to RVP solution vectors, if the number of non-zero coefficients remain únchanged. We can apply RVP to inverse filtering for sound field control. The computer experiments confirmed that an RVP inverse filter can be realized through using only a few multipliers within reasonable errors.

References

[1] L. Rabiner et. al., Digital processing of speech signals, Prentice-Hall, 1978.