

SOUND FIELD CONTROL BY INDEFINITE MINT FILTERS

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1. INTRODUCTION

The Multiple Input-Output INverse Theorem (MINT)[1] showed the conditions for producing inverse filters that can achieve exact control of a sound field. According to MINT, the number of exactly controlled points must be smaller than the number of filters. The authors have been investigating the indefinite terms included in the MINT filter solutions. This article presents new MINT filter sets that use these indefinite terms. We call them Indefinite MINT filters (IMFs). The IMFs achieve approximate control of the sound field at newly introduced points, provided that the other points are exactly controlled in accordance with MINT.

2. THE LSE AND MINT FILTERS

Consider M pairs of inverse filters $X_j(z)$ and loudspeakers S_j that control the sound pressure responses at N receiving points $P_j(j=1, 2, \dots, N)$ as shown in Fig. 1.

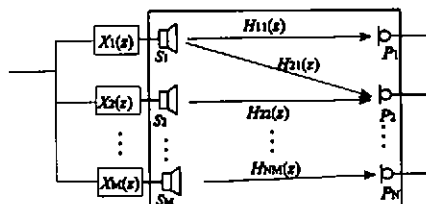


Fig. 1. A multiple-points sound field control system.

When the desired response at P_j is $D_j(z)$, the transfer function (TF) $X_j(z)$ of the inverse filter can be derived by solving the following equation,

$$Hx=d \quad (1)$$

$$H = \begin{pmatrix} H_{11} & H_{12} & \dots & H_{1M} \\ H_{21} & H_{22} & \dots & H_{2M} \\ \vdots & \vdots & \dots & \vdots \\ H_{N1} & H_{N2} & \dots & H_{NM} \end{pmatrix} \quad x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_M \end{pmatrix} \quad d = \begin{pmatrix} d_1 \\ d_2 \\ \vdots \\ d_N \end{pmatrix}, \quad H_j = \begin{pmatrix} h_{j0} & & 0 \\ h_{j1} & \ddots & \\ \vdots & \ddots & h_{jn} \\ h_{j(n-1)} & \dots & 1 \\ 0 & \dots & h_{j(n-1)} \end{pmatrix} \quad x_j = \begin{pmatrix} x_j^{(0)} \\ x_j^{(1)} \\ \vdots \\ x_j^{(n-1)} \end{pmatrix}$$

$$d_j = \begin{pmatrix} d_j^{(0)} \\ d_j^{(1)} \\ \vdots \\ d_j^{(n-1)} \end{pmatrix} \quad H_j(z) = \sum_{k=0}^{n-1} h_{jk}(k)z^k, \quad X_j(z) = \sum_{k=0}^{n-1} x_j(k)z^k, \quad D_j(z) = \sum_{k=0}^{n-1} d_j(k)z^k$$

where the TF between S_i and P_j is $H_{ji}(z)$, m denotes the size of the filter, n is the length of the TF, and H is a matrix of $N(m+n-1)$ by Mm .

When $M \leq N$, there is no solution in Eq. (1). This is because the number of columns is smaller than the number of the rows. In such an ill-posed case, the filter coefficients can approximately be given as a least square error (LSE) solution. This is called the LSE method for designing filters. If $M > N$, we can set the length of the filters $N(n-1)/(M-N)$ in order to make H square. The MINT method provides the exact solution for the filters by choosing the filter order conditions. Both the LSE and MINT solutions can be given by

$$x = H^+ d \quad (2)$$

where H^+ denotes the pseudo-inverse matrix of H . [2]

3. INDEFINITE MINT FILTERS (IMFs)

If we set a condition that $M > N$ and $m > N(n-1)/(M-N)$ in Eq. (1), the equation has many possible solutions. We can write a generalized solution using

$$x = x_M + Zy \quad (3)$$

where x_M is the minimum norm solution (MINT solution), y is an arbitrary vector, and Z is a matrix satisfying the relations,

$$HZ = 0, \quad Z = [z_1 \ z_2 \ \dots \ z_l], \quad Z^T Z = I \quad (4)$$

where Z^T is the transpose of Z , I is the identity matrix, z_k ($k=1, 2, \dots, l$) is the basis vectors in the null space of H , and l is the rank of H . The solution x given by Eq. (3) satisfies Eq. (1) as accurate as the MINT solution. Thus we achieve exact control of the sound field at P_j in Fig.1 using a general solution that includes the MINT solution. The general solution also makes possible the approximate control of the sound field at another point while maintaining precise control at P_j .

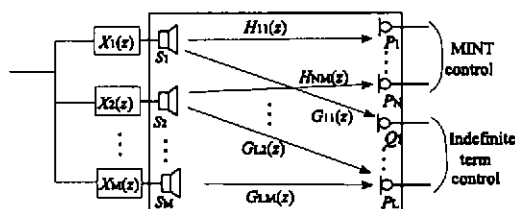


Figure 2. Multiple sound point control by the IMFs.

In the example in Figure 2, our aim is to control the sound field encompassing areas that cannot be controlled by MINT filters. Suppose that the points located in such an area are Q_k ($k=1, 2, \dots, L$), the TF between loudspeaker S_i and Q_k is $G_{ki}(z)$, and the desired response at Q_i is $D_i'(z)$. We can derive the next equation in a way similar to the way we got Eq. (3),

$$Gx = d' \quad (5)$$

where G_{ki} is a matrix similar to H_{ji} and d'_k is a vector similar to d_j in Eq.(1).

Substituting Eq.(3) into Eq.(5), we can get the equation for y :

$$GZy = d' - Gx_M \quad (6)$$

Since Eq. (6) does not produce exact solutions, we use the LSE method to derive solutions. Introducing the LSE solution y' into Eq. (3), we can get filters that can provide approximate control of the sound field in the extension of MINT-controlled areas. We call these filters Indefinite MINT Filters (IMFs), since the filters are based on the indefinite term, Zy in Eq. (3). The control errors by the IMFs produced at the newly introduced points are smaller than those by MINT, provided that the points in the MINT-controlled areas are exactly controlled.

4. SOUND FIELD CONTROL SIMULATION

We conducted computer experiments to examine the performance of our IMFs, using two filters for two-point (P and Q) control of the sound field. Figure 3 shows the desired response data of our interest and Fig. 4 the records of impulse responses between the loudspeakers and the microphones. In the original records as measured in a reverberation room, these impulse response records are exponentially truncated after 100 data points. Figure 5 illustrates the impulse responses observed at control points P and Q through the MINT filters (5a) and IMFs (5b). The size of the filters is 198 for both methods. We can confirm that when the second point (point Q) is in an area that is outside of MINT's control, the point is located in an area controllable by IMFs.

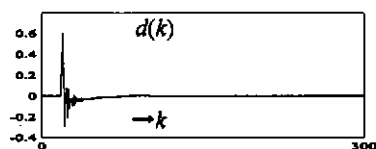


Fig. 3. Desired response.

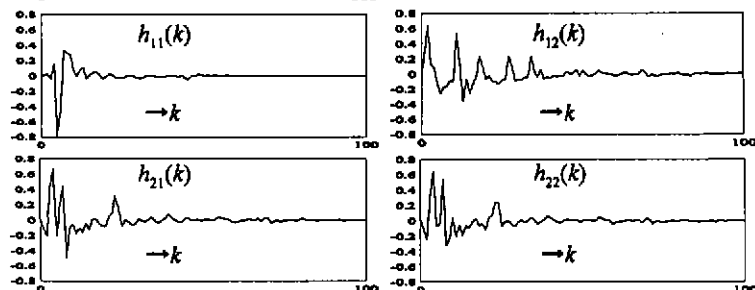


Fig. 4. Impulse responses between loudspeakers and microphones.

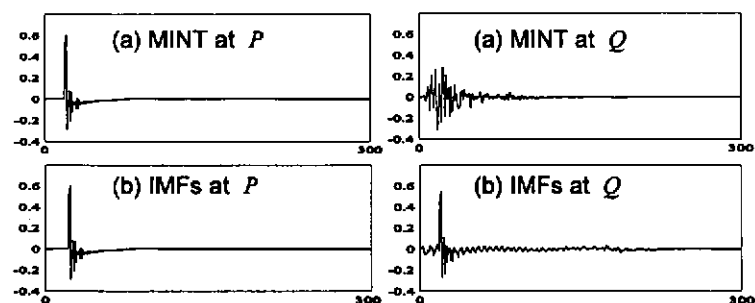


Fig. 5. Impulse responses controlled by filters.

5. SUMMARY

We proposed Indefinite MINT filters (IMFs) for approximate control of the sound field at points outside of MINT's control. The IMFs are formulated from generalized MINT filters involving with the use of some of the indefinite terms of the MINT solutions. Results of computer simulations using measured impulse response records indicate that the IMFs show promise for extending the controllable areas.

References

- [1] M. Miyoshi and Y. Kaneda, IEEE Trans. ASSP, 36, 145(1988)
- [2] G Strang, INTRODUCTION TO LINEAR ALGEBRA (Wellesley-cambridge press, Wellesley USA)