DESIGN OF A WIDE-BAND VIBRATION NEUTRALISER TO CONTROL FLEXURAL WAVE ON AN INFINITE BEAM

H Salleh Institute of Sound and Vibration Research, University of Southampton, UK
M J Brennan Institute of Sound and Vibration Research, University of Southampton, UK

1 INTRODUCTION

There are many practical applications that involve flexural wave motion in beam-like structures. Thus, there is a need to control this type of wave. One simple passive method is to use an auxiliary system consisting of a mass-spring-damper system called a vibration neutraliser. However, vibration control using such a device will only completely suppress a flexural wave at a single excitation frequency, which is called the tuned frequency. The bandwidth over which there is significant vibration reduction is very limited[1]. Outside this bandwidth, the vibration can be amplified. In practice, although many machines operate at a single frequency, very often there is a drift in excitation frequency. Thus, there is a need to design a wide-band vibration neutraliser to attenuate vibration over a reasonably wide frequency range.

Some work has been done in relation to a *tunable* vibration neutraliser, which is a device that can track a particular forcing frequency [2]. In this case the stiffness or mass of the neutraliser is adjusted to tune it so that its natural frequency matches the excitation frequency; it can be considered to be a tunable narrow-band vibration control device. A mass supported by an air mount whose natural frequency is adjusted by varying the air pressure is an example of a tunable vibration neutraliser[3]. Vibration neutralisers have also been used for pipe vibration suppression[4]. Brennan[5] has further described the tuning parameters involved when using a tunable vibration neutraliser to control the transmission of a flexural propagating wave on an infinite Euler-Bernoulli beam.

One way of improving the limited bandwidth is to use an array of neutralisers with slightly different natural frequencies. In this design the neutralisers collectively have a high impedance over a range of frequencies and can attenuate the vibration of a structure over a wide frequency range[6.] However, to authors' knowledge, a wide-band neutraliser has not been used to control wave propagation on a beam. This paper thus describes an investigation into the design of such a device to control flexural waves on a beam.

2 MULTIPLE NEUTRALISERS ATTACHED TO AN INFINITE BEAM

The effect of using a single neutraliser in controlling the flexural wave on an infinite beam is discussed in the Appendix. It is shown that using a single undamped neutraliser, such that it exerts a translational force on the beam, can only completely suppress wave propagation at a single frequency. The frequency at which this occurs is called the tuned frequency. It should be noted that the neutraliser has to have stiffness-like impedance at this frequency, and thus the tuned frequency is greater than the natural frequency of the neutraliser.

Brennan[6] has studied the characteristics of a wide-band neutraliser by using an array of neutralisers and this section further investigates the application of such a device to flexural wave suppression on an infinite beam. Figure 1 shows an infinite Euler-Bernoulli beam with multiple neutralisers attached to it at a single point via a mass-less rigid link and that only a force (not a moment) is transmitted to the beam. The neutralisers collectively form a dynamic stiffness that will influence a propagating wave, A_i, that is incident upon it. In this section, the way in which the neutraliser affects the transmission ratio, τ, which is square of the ratio of the modulus of the transmitted wave, A_t, to the modulus of the incident wave, is investigated. The transmission ratio is given by [7]

$$\tau = \left|\frac{A_t}{A_i}\right|^2 = \left|1 + \frac{j\varepsilon_T}{4 - \varepsilon_T - j\varepsilon_T}\right|^2 \tag{1}$$

where ε_T is the non-dimensional dynamic stiffness of the neutraliser, which for a single device, is given by equation (A5). For n neutralisers, the non-dimensional dynamic stiffness is given by

$$\varepsilon_T = \frac{\psi_1 \Omega^{0.5}(1+j\eta)}{\Omega^2 - 1 - j\eta} + \frac{\psi_2(\Omega - \delta)^{0.5}(1+j\eta)}{(\Omega-\delta)^2 - 1 - j\eta} + \ldots\ldots \frac{\psi_n\left\{\Omega - (n-1)\delta\right\}^{0.5}(1+j\eta)}{\left\{\Omega-(n-1)\delta\right\}^2 - 1 - j\eta} \tag{2}$$

which is the sum of the normalised dynamic stiffness for each of the neutralisers. The normalised frequency $\Omega = \omega/\omega_1$, is the ratio of the excitation frequency to the undamped natural frequency of the first neutraliser. The natural frequency of each of the other neutralisers, ω_n, is related to the natural frequency for first neutraliser, ω_1, by

$$\omega_n = \omega_1\left\{1 + (n-1)\delta\right\} \tag{3}$$

where δ is the spacing between the natural frequencies of the neutralisers, and ψ_n is the mass ratio evaluated at the natural frequency of the nth neutraliser. The mass ratio of the nth neutraliser is related to the mass ratio of the first neutraliser by,

$$\psi_n = \psi_1\left\{1 + (n-1)\delta\right\}^{0.5} \tag{4}$$

According to Brennan[6], multiple neutralisers with a non-dimensional frequency spacing of at most equal to their loss factor, η, can produce a wide-band vibration neutraliser when attached to a structure. Figures 2(a) and 2(b) shows the modulus and phase of the normalised dynamic stiffness given by equation (2), when using ten neutralisers. This device is attached to an infinite beam and the corresponding transmission ratios for small and large frequency spacing are shown in figures 3(a) and (b). Figure 2(a) shows that the system has a high dynamic stiffness across a reasonably wide frequency range. However when it is used on an infinite beam, it can significantly attenuate the flexural wave only at a single frequency. At other frequencies, the attenuation is very small as shown in figure 3(a). This is due to the phase characteristics shown in figure 2(b). As mentioned in the appendix, a single neutraliser attached to apply a translational force to the beam has to have a stiffness-like characteristic at its tuned frequency. This implies a specific magnitude *and* phase characteristic of the dynamic stiffness of each neutraliser is required. It appears that although the configuration of a neutraliser with closely spaced natural frequencies may have the correct dynamic stiffness magnitude, it does not have the correct phase characteristic over a wide frequency band. However, if the neutraliser tuned frequencies are reasonably well-spaced, each neutraliser is reasonably effective as shown in figure 3(b).

3 MULTIPLE NEUTRALISERS ATTACHED TO THE BEAM VIA WITH A MOMENT ARM

3.1 Model

It was shown in the previous section, that multiple neutralisers attached so as to exert a force only to the beam at a single point, cannot effectively increase the frequency range over which a flexural wave is suppressed. In the appendix, a neutraliser that exerts both a force and moment has been shown to improve the tuned frequency range. Thus, multiple neutralisers with this configuration are considered in this section. Figure 4 shows the arrangement of a wide-band neutraliser using multiple neutralisers connected to an infinite beam via a moment arm, so as to exert both a force and a moment. The equation that describes the non-dimensional dynamic stiffness for this configuration is equation (A1), which is written below for clarity

$$\tau = \left| 1 + \frac{j\varepsilon_T}{4 - \varepsilon_T - j\varepsilon_T} + \frac{j(2\pi a/\lambda)^2 \varepsilon_T}{4 + (2\pi a/\lambda)^2 \varepsilon_T - j(2\pi a/\lambda)^2 \varepsilon_T} \right|^2 \tag{5}$$

where ε_T is given in equation (2), a is the length of the moment arm, λ is the wavelength of a flexural wave in the beam. The effect on the transmission ratio of this type of neutraliser array that contains ten neutralisers is shown in figure 5. Also plotted in this figure are the effects of a neutraliser array that applies a force only and a neutraliser array that applies a moment only. The advantage of attaching the neutraliser array via a moment arm is evident. A much wider frequency range over which a flexural wave can be suppressed is achieved. The effects of the design parameters on the suppression of a flexural wave namely the number of neutralisers, moment arm, loss factor and mass ratio are discussed below.

3.2 Design Parameters

As discussed previously, using one neutraliser connected to the beam via a moment arm can suppress the transmission of a flexural wave at two distinct frequencies. More than one neutraliser with slightly different natural frequencies attached in the same way can suppress the wave at more than two distinct frequencies. Figure 6 shows the effect on the transmission ratio of using one neutraliser and a neutraliser array with each configuration having the same total mass. It can be seen that one of the non-dimensional tuned frequencies is always unity. It also shows that for moment arm with a length greater than the critical length (defined in the Appendix), that increasing the number of neutraliser will shift the second non-dimensional tuned frequency nearer to unity and shifts the operating frequency range of the device to a higher frequency. The bandwidth of the device depends on the natural frequency spacing of the neutralisers, δ.

The effect of the normalised length of the moment arm, a/λ_1, on the transmission ratio is plotted in figure 7, where λ_1 is the wavelength of the flexural wave in the beam at the lowest neutraliser natural frequency, ω_1. It shows that increasing the length of the moment arm can increase the bandwidth of the device. The non-dimensional tuned frequency, at which the transmission ratio is the smallest, can be shifted to be greater or less than unity by adjusting the length of the moment.

Decreasing the neutraliser loss factor is can increase the attenuation at a single frequency[5]. However to achieve a wide-band attenuation with less ‘rippling’ effect, higher damping is desirable. Figure 8 shows the effect of changing the loss factor for an array of ten neutralisers on the transmission ratio. The non-dimensional frequency spacing, δ, should be at most equal to the loss factor to reduce the ‘rippling’ effect. Thus, it should be noted that the reduction in damping is limited to the desired non-dimensional frequency spacing of a particular design. If greater attenuation is required than a larger mass ratio should also be considered.

Figure 9 shows the effect of the changing the mass ratio of an array of ten neutralisers attached via the moment arm on the transmission ratio. The mass ratio for each of the neutralisers is defined in equation (4). As the mass ratio increases the maximum reduction in the transmission ratio also increases, and the tuned frequency is shifted away from the lowest natural frequency of the neutraliser array. In summary, adding more mass to the neutraliser array can improve the attenuation, as well as improving the bandwidth of device.

4 CONCLUSIONS

This paper has discussed the design of a wide-band vibration neutraliser to control a flexural wave on a beam. Although an array of neutralisers with slightly different natural frequencies can have high impedance over a reasonably wide frequency range, attaching this type of device such that it exerts a force only on the beam does not increase the frequency range over which a flexural wave can be suppressed. Multiple neutralisers attached to the beam via a moment arm such that the array can suppress both the translational and the rotational motion of the beam, does however, result in wide-band attenuation of a flexural wave. Design parameters, such as the number of neutralisers, spacing of the neutraliser natural frequencies, loss factor, mass ratio and the length of the moment arm need to be considered to design an effective wide-band neutraliser. In general, spacing of the non-dimensional natural frequencies of the neutralisers at most equal to their loss factors is needed to reduce the ‘rippling’ effect on the transmission ratio. A large mass ratio and long moment arm are preferable to obtain a wide working frequency range.

REFERENCES

1. D.J. Mead, Passive Vibration Control. Chichester. John Wiley and Sons. (1998).
2. A.H. Von Flotow, A. Beard, and D. Bailey. ‘Adaptive Tuned Vibration Absorbers: Tuning (Laws, Tracking Agility, Sizing and Physical Implementations’. In *Proceedings of Noise-Con 94*, Fort Lauderdale Florida, 437-454. (1994).
3. M.J. Brennan, ‘Vibration Control using a Tunable Vibration Neutralizer. Proceedings of the Institution of Mechanical Engineers’, Journal of Mechanical Engineering Science (Part C). 211: 91-108. (1997).
4. P.Clark, Devices for the Reduction of Pipeline Vibration. PhD Thesis. Institute of Sound and Vibration, University of Southampton. (1997)
5. M.J. Brennan, ‘Control of Flexural Waves on a Beam Using a Tunable Vibration Neutraliser’ Journal of Sound and Vibration. 222(3): 389-407. (1998).
6. M.J. Brennan, ‘Characteristics of a Wide-band Vibration Neutralizer’, Noise Control Eng. J. 45(5) 201-207. (1997).
7. B.R. Mace, Wave Reflection and Transmission in Beams. Journal of Sound and Vibration, 97(2): 237-246. (1984).

APPENDIX: EFFECT OF A SINGLE NEUTRALISER ON THE PROPAGATION OF FLEXURAL WAVES ON AN INFINITE BEAM

This appendix discusses the effect of different neutraliser configurations such as those shown in figures A1, A2 and A3 on flexural wave propagation on an infinite beam. The general equation for the transmission ratio, which is the square of the ratio of the modulus of the transmitted wave, A_t, to the modulus of the incident wave, A_i, for a discontinuity attached to an infinite beam, has been derived by Mace[7]. For a discontinuity attached via a mass-less link of length *a*, forming a moment arm, is given by

$$\tau = \left|\frac{A_t}{A_i}\right|^2 = \left|1 + \frac{j\varepsilon_T}{4-\varepsilon_T - j\varepsilon_T} + \frac{j\left(2\pi a/\lambda\right)^2 \varepsilon_T}{4+\left(2\pi a/\lambda\right)^2 \varepsilon_T - j\left(2\pi a/\lambda\right)^2 \varepsilon_T}\right|^2 \quad \text{(A1)}$$

where a is the moment arm length, λ is the wavelength of a flexural wave in the beam, and ε_T is the non-dimensional dynamic stiffness of the discontinuity which is given by.

$$\varepsilon_T = \frac{K_d}{EIk^3} \quad \text{(A2)}$$

where E and I is the Young's Modulus and second moment of area of the beam respectively, k is the flexural wave number, and K_d is the dynamic stiffness of the discontinuity. The second and third terms in equation (A1) relate to the translational and rotational components of the force applied by the discontinuity to the beam respectively. The dynamic stiffness of a neutraliser is given by[5]

$$K_d = \frac{-\omega^2 m(1+j\eta)}{1-\Omega^2+j\eta} \tag{A3}$$

where ω is the angular excitation frequency, m is the mass of the neutraliser, η is the loss factor of the hysteretically damped spring, $\Omega = \omega/\omega_1$, is the ratio of the excitation frequency to the neutraliser undamped natural frequency. A mass ratio parameter, ψ, can be introduced, which is defined as the ratio of 2π times the neutraliser mass to the beam mass in one wavelength, and given as

$$\psi = \frac{2\pi m}{\rho A\lambda} \qquad \text{A4)}$$

where A and ρ are the cross sectional area and density of the beam respectively. It should be noted that the mass ratio is frequency dependent as $\lambda \propto \omega^{0.5}$. Combining equations (A2), (A3) and (A4), the non-dimensional dynamic stiffness for a neutraliser can be written as

$$\varepsilon_T = \frac{\psi(1+j\eta)}{\Omega^2-1-j\eta} \tag{A5}$$

Consider a neutraliser attached to an infinite beam as shown in figure A1, such that the neutraliser exerts only a translational force on the beam. The third term in equation (A1) is zero. Assuming the neutraliser has negligible damping, the transmission ratio given in equation (A1) can now be written as

$$\tau = \frac{(4\Omega^2-\psi-4)^2}{2\psi^2-8\psi(\Omega^2-1)+16(\Omega^2-1)^2} \tag{A6}$$

The tuned frequency is defined as the frequency at which the amplitude ratio of the transmitted propagating wave normalised by the incident-propagating wave, $\left|A_t/A_i\right|^2$ is minimum. From equation (A6), by setting the numerator to zero, the tuned frequency, Ω_t, for a small mass ratio, is found to be [5]

$$\Omega_t = \left(1+\frac{\psi_t}{4}\right)^{1/2} \tag{A7}$$

where ψ_t is the mass ratio evaluated at the tuned frequency.

Figure A2 shows a neutraliser is attached to the beam such that it exerts a bending moment only. Thus the second term in equation (A1) is zero. Assuming the neutraliser has negligible damping, the transmission ratio in equation (A1) can be written as

$$\tau = \frac{\left((2\pi a/\lambda)^2\psi+4(\Omega^2-1)\right)^2}{2(2\pi a/\lambda)^4\psi^2+8(2\pi a/\lambda)^2\psi(\Omega^2-1)+16(\Omega^2-1)^2} \tag{A8}$$

Setting the numerator to zero, the tuned frequency is found to be

$$\Omega_t = \left(1 - \frac{\psi_t}{4}(2\pi a/\lambda_t)^2\right)^{1/2} \tag{A9}$$

where λ_t is the wavelength evaluated at the tuned frequency. Consider a neutraliser that transmits both a force and moment to the beam is shown in figure A3. This is the combination of two previous cases, and thus from equation (A1), with an assumption of negligibly small damping, the transmission ratio is given by

$$\tau = \frac{4\left(\Omega^2-1\right)^2\left\{(2\pi a/\lambda)^2\psi - \psi + 4\left(\Omega^2-1\right)\right\}^2}{\left\{\psi^2 - 4\psi\left(\Omega^2-1\right) + 8\left(\Omega^2-1\right)^2\right\}\left\{(2\pi a/\lambda)^4\psi^2 + (2\pi a/\lambda)^2\psi\left(\Omega^2-1\right) + 8\left(\Omega^2-1\right)^2\right\}} \tag{A10}$$

Setting the numerator to zero, two frequencies at which wave propagation is completely suppressed are found. The first frequency is when the excitation frequency is equal to the natural frequency of the neutraliser $(\Omega_t = 1)$ and the second frequency is given by

$$\Omega_t = \left[1 + \frac{\psi_t}{4}\left\{1 - (2\pi a/\lambda_t)^2\right\}\right]^{1/2} \tag{A11}$$

The transmission ratios for all cases are plotted in figure A4. Figure A4 shows the effect of these three devices on the transmission ratio. It can be seen that a neutraliser of moment-force (combination) type can attenuate the transmission ratio over a wider frequency range than the other two. Equation (A11) also implies that if the moment arm parameter $2\pi a/\lambda_t < 1$, then the second tuned frequency is greater than the neutraliser natural frequency and the neutraliser has a stiffness-like impedance at its tuned frequency. If $2\pi a/\lambda_t > 1$, then the second tuned frequency is smaller than the neutraliser natural frequency, and the neutraliser has a mass-like impedance at its tuned frequency. The critical arm length, a_c, which is the length at which the transition between this mass-like and the stiffness-like region occurs is given by

$$a_c = \frac{\lambda_t}{2\pi} \tag{A12}$$

This means that the neutraliser can be designed to attenuate vibration either at the higher frequency region if the arm length is larger than the critical value or at lower frequency region if the arm length is smaller than the critical value. This shows that the combination of a translational and rotational neutraliser can improve the bandwidth of the device due to its ability to react to the translational and rotational motion of the beam.

FIGURES

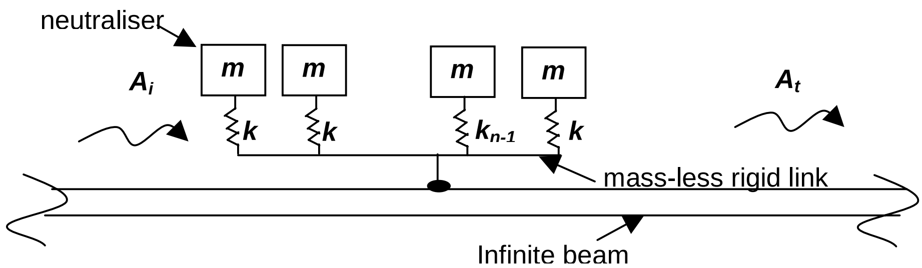
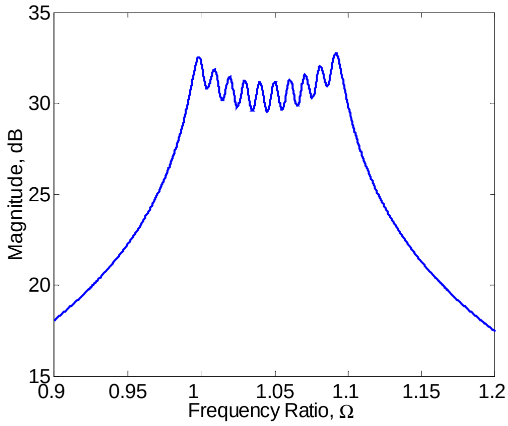
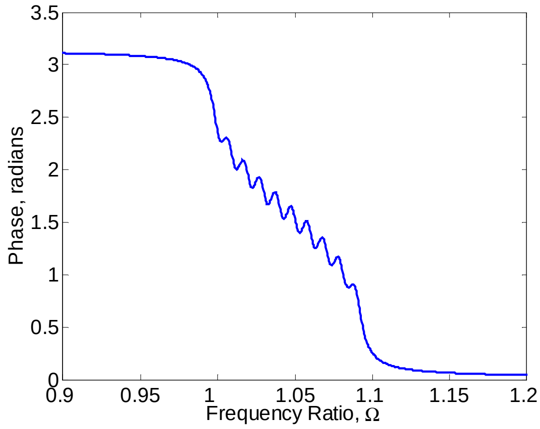


Figure 1. Multiple neutralisers attached to an infinite beam

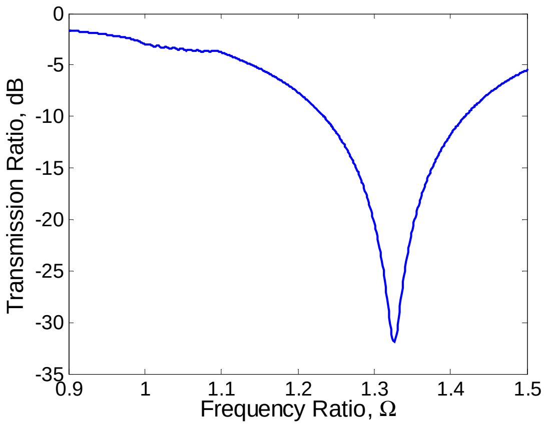


(a) Modulus

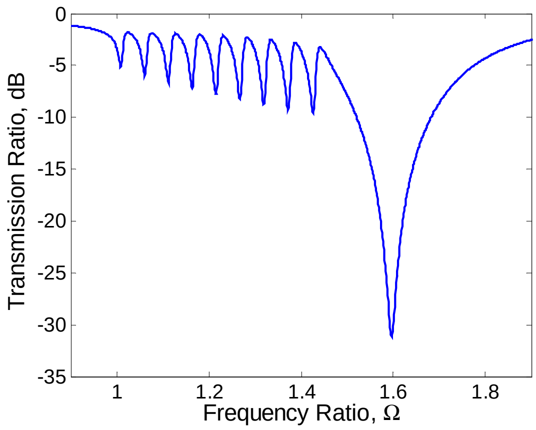


(b) Phase Angle

Figure 2. Modulus and the phase of the dynamic stiffness, ε_T , for ten neutralisers.
$\psi_1 = 0.02$, $\eta = \delta = 0.01$



(a) Small Frequency Spacing
$\psi_1 = 0.02$, $\eta = \delta = 0.01$



(b) Large Frequency Spacing
$\psi_1 = 0.02$, $\eta = 0.01$, $\delta = 0.05$

Figure 3. Transmission ratio for ten neutralisers attached to an infinite beam

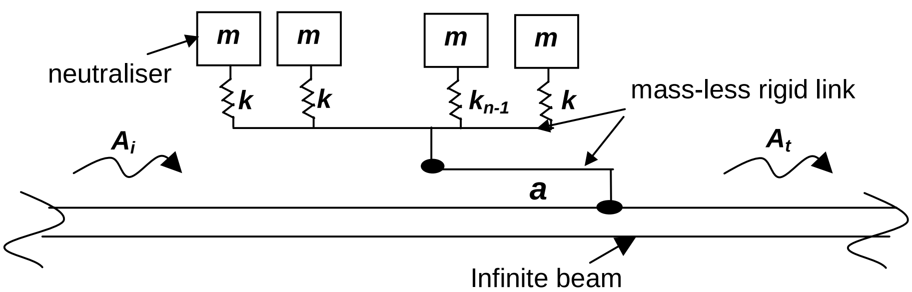


Figure 4. Multiple neutralisers with a moment arm, of length *a*, configured to exert a force and a moment on the beam

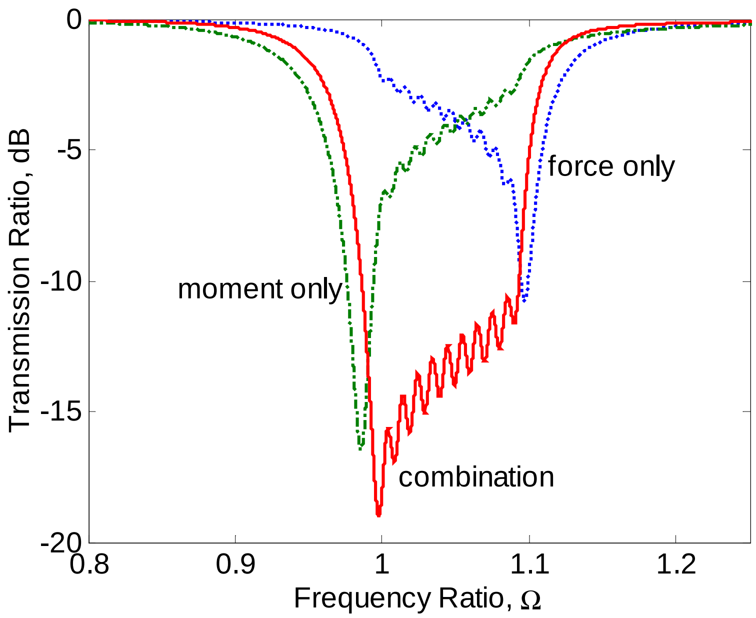


Figure 5. The effect of a neutraliser connected to the beam via a moment arm (combination) on the transmission ratio (ten neutralisers where $\psi_1 = 0.02$, $a/\lambda_n = 0.2$, $\delta = \eta = 0.01$). A neutraliser array that applies a force only and a moment only are plotted for comparison.

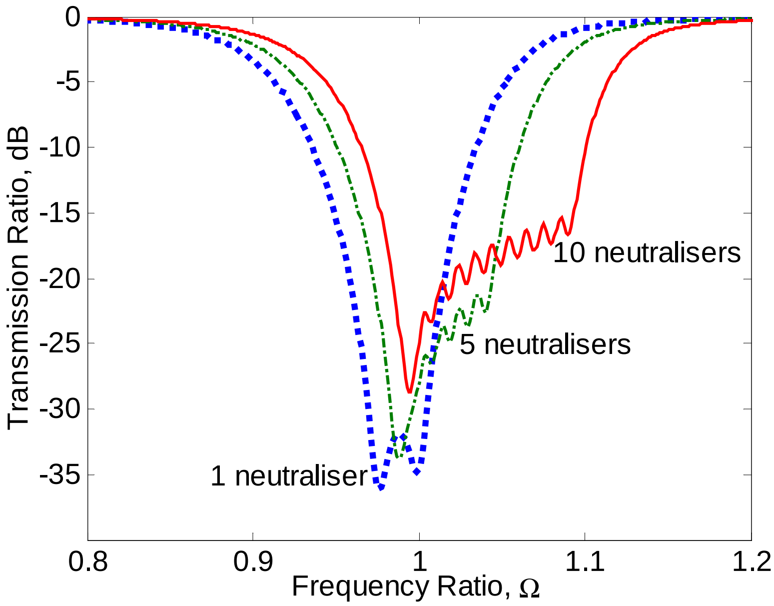


Figure 6. Effect of number of the neutralisers on the transmission ratio ($a/\lambda_n = 0.2$, $\delta = \eta = 0.01$, $\psi_1 n = 0.4$, where n is the number of neutraliser). The neutraliser applies both a force and moment to the beam.

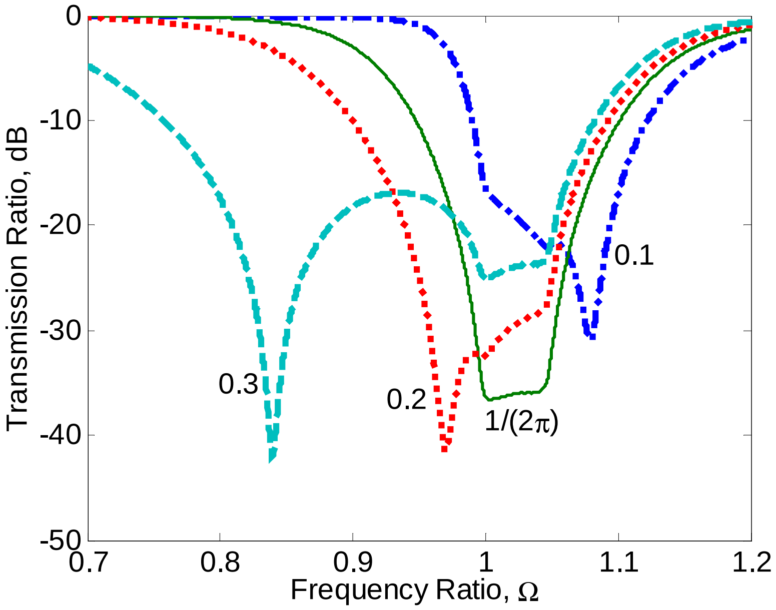


Figure 7. Effect of moment arm length, a/λ_n, for ten neutralisers on the transmission ratio ($\psi_1 = 0.07$, $\eta = 0.01$, $\delta = 0.005$)

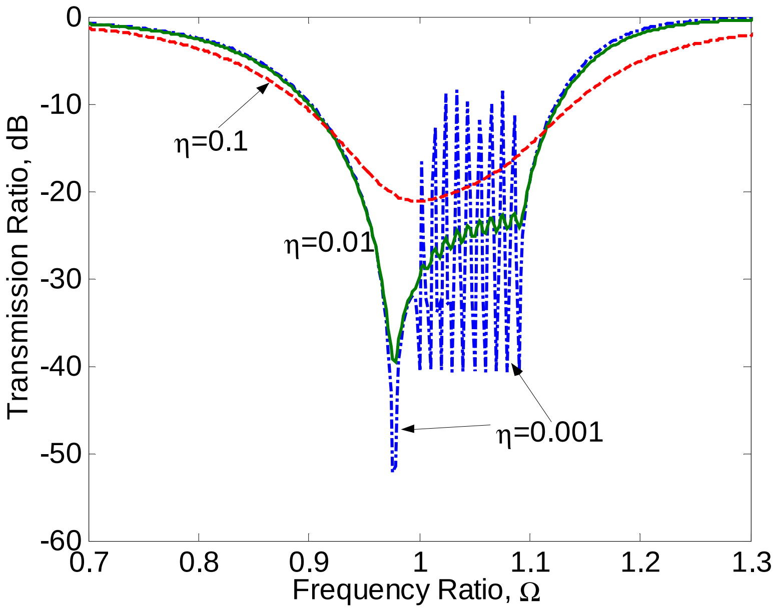


Figure 8. Effect of loss factor for ten neutralisers on the transmission ratio ($a/\lambda_n = 0.2$, $\psi_1 = 0.07$, $\delta = 0.01$)

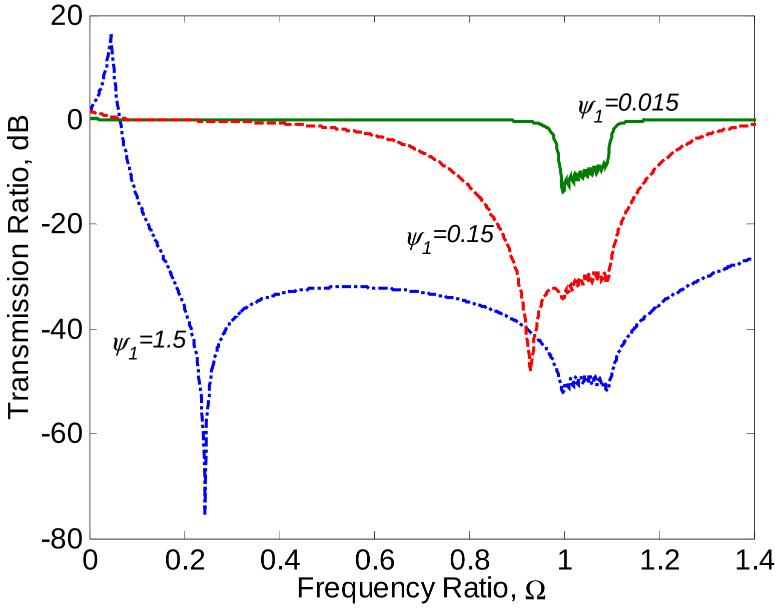


Figure 9. Effect of mass ratio for ten neutralisers on the transmission ratio ($a/\lambda_n = 0.2$, $\eta = 0.01$, $\delta = 0.01$)

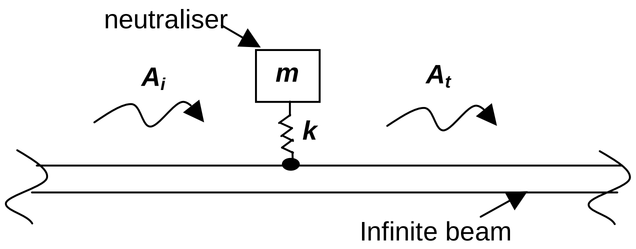


Figure A1. A neutraliser attached on an infinite beam to exert a translational force only

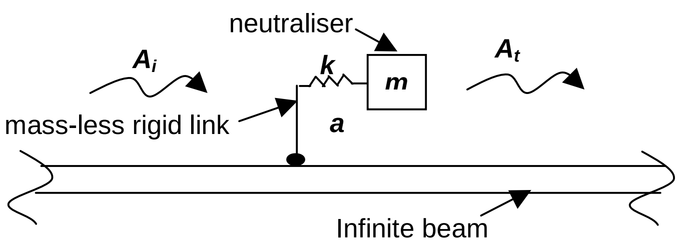


Figure A2. A neutraliser attached on an infinite beam to exert a bending moment only

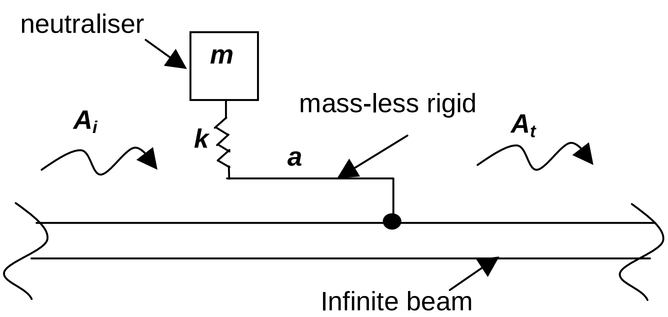


Figure A3. A neutraliser attached on an infinite beam to exert both a force and bending moment

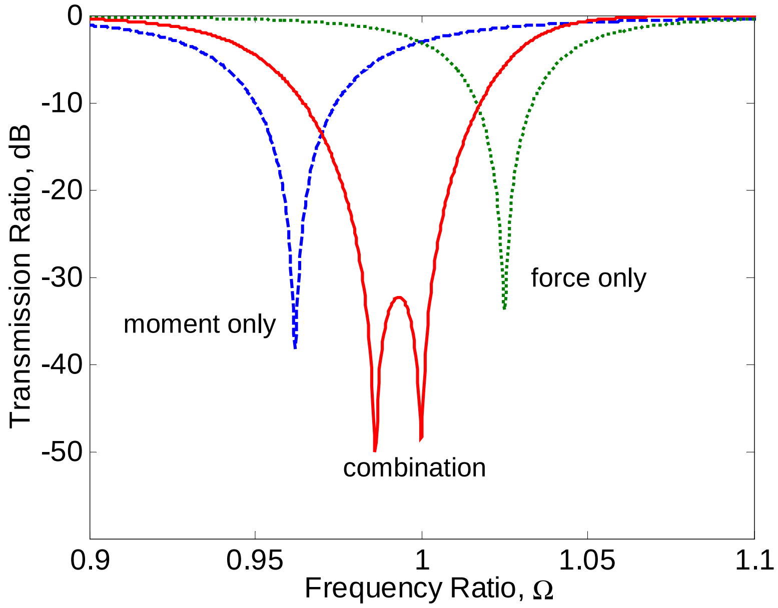


Figure A4. The effect of a neutraliser discontinuity that applies a force only, a moment only and a combination of the two on the transmission ratio ($\psi_1 = 0.2$, $\eta = 0.001$, $a/\lambda_n = 0.2$, where λ_n is the wavelength evaluated at the natural frequency of the neutraliser)