MATCHED FIELD PARAMETER SUBSPACE METHODS FOR SELECTIVE HIGH RESOLUTION ESTIMATION WITH REDUCED SENSITIVITY TO MISMATCH

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ABSTRACT

Sector-focused estimation applied to matched-field localization has been shown to effectively enhance range-depth resolution compared with conventional Bartlett estimation, while remaining insensitive to environmental parameter mismatch. The insensitivity to mismatch is evident in parameter space ambiguity surfaces, where the response is typically broad with low resolution. For source localization purposes this is desirable. However, for environmental inversion, high resolution in parameter space is usually required, with a corresponding insensitivity to measurement errors. This may be accomplished using sector-focusing methods by including, in the search sector, replicas derived from a set of "neighboring" environmental parameters. This study demonstrates the use of these methods to increase resolution in environmental parameter space, while mitigating against measurement uncertainty in other parameters.

1. INTRODUCTION

Since it was first discussed by Bucker and others more than 10 years ago [1-3], matched-field processing (MFP) has enjoyed a notable increase in popularity as a technique for performing underwater acoustic array processing in a waveguide environment where plane wave signal vector assumptions do not apply. The typical MFP procedure is to compare (or "match") the signal replica vectors obtained from a model of the acoustic propagation, using a series of trial source locations, with vectors derived from received signal data. When the trial source location and the true source location correspond, the two vectors should be highly correlated. One may similarly expand the search space beyond that of source range and depth and include other model input parameters in the search space. This technique, known as tomography or inversion, has been of interest recently as a means of using acoustic information to probe the acoustic waveguide.

Many methods of searching the parameter space for a match have been developed. The focalization method of Collins et al. [4-5] uses the global optimization technique known as simulated annealing. Genetic algorithms have also been employed by Gersoft.[6-7] Tolstoy et al. [8] employ perturbation methods to probe the environment. All of these search methods involve making some sort of estimate of the match between actual and modeled received energy. One of them, the so-called "conventional" or "Bartlett" [9] method, is popular for its stability against mismatch, but yields relatively low resolution results compared with the family of nonlinear estimation techniques, such as the "minimum variance distortionless response" (MVDR) [10], the "multiple constraint method" (MCM) [11], "reduced maximum likelihood" (RML) [12], and "sector-focusing" (SF) [13-15], etc.

Sector focusing has demonstrated effectiveness for enhancing range-depth resolution while minimizing the instabilities caused by environmental mismatch, as compared to other high resolution estimators such as MVDR. It has been shown [16] that a stable Bartlett response can be obtained by using a small sector in range-depth space. A larger sector can be used to give higher resolution, but sacrifices some stability to do so. However, the size, shape and dimensionality of the sector provide a wide variety of options for optimizing the high-resolution performance of the technique. This property is used to advantage when SF is applied as an MFP technique to localize acoustic sources in a waveguide, where it can be used to control the response of environmental parameter estimation, i.e., to give MVDR-like high resolution where needed, while maintaining Bartlett-like insensitivity to mismatch in other parameters.

Acoustic tomography, a topic of increasing importance in MFP, attempts to determine environmental parameters while stabilizing against measurement errors in source range and depth. Tolstoy et al. [8] point out that the broad response of the Bartlett estimator is usually sufficient to eliminate the effects of this type of mismatch. However, when a high resolution estimation method such as MVDR is used to estimate oceanographic parameters more precisely, the response is typically narrow in both range and depth, and instability is frequently encountered. In this paper the SF method is used to achieve accurate inversion of environmental parameters, while maintaining insensitivity to measurement parameter errors. Taking the case of a CW source in a shallow water channel, we describe a procedure for applying the algorithm to meet specific physical characteristics of the waveguide and stabilize the parameter estimation process. Using SF it is possible to selectively increase the resolution of some chosen parameters, while minimizing the effects of uncertainties in others. This is achieved by selecting matched field replicas over a wide sector aperture for those parameters in which high resolution is desired. For other parameters the estimator can be made to give a broad Bartlett-like response, by using a sector aperture which is small or nonexistent. High resolution estimates may also be obtained for several parameters simultaneously, while still retaining low resolution in others.

In section 2, the theoretical basis for SF is outlined. In section 3, the simulated shallow water environment and experimental procedure are described. In section 4, three examples are discussed which demonstrate the selective parameter enhancement capability of SF. Two of the examples illustrate the high resolution of environmental parameters, while retaining a broad response to measurement parameters. We end with a brief statement of conclusions from the study.

2. THEORY

The SF methodology was introduced by Byrne and Steele [13,14] and its application to matched-field processing first described by Frichter et al. [15] In this section we present a derivation of SF that is designed to demonstrate how the technique removes ambiguities and achieves stable MFP by restricting the processing to a reduced set of eigenvectors of the data covariance matrix.

The SF estimator achieves stability by restricting the processing to only those eigenvectors of the data covariance matrix that would describe the source, if it were in the sector. For source localization purposes, the sector is usually defined in range-depth space. For tomographic inversion (as in this work) the sector is defined in environmental parameter space. The data covariance matrix K consists of averaged cross-correlations between the complex acoustic pressures P_n measured on the N hydrophones of an array and may be written

$$K = \langle \mathbf{P}\mathbf{P}^{\dagger} \rangle , \tag{1}$$

where $\mathbf{P}$ is a column vector whose entries are the P_n , † denotes conjugate transpose, and <...> denotes an average over time samples (snapshots). The acoustic pressure field from which the NxN

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matrix K is derived is due to a source at range r_0 and depth z_0 from the hydrophone array, superimposed on the ambient noise field, which is due to surface waves, wind, shipping and other sources. Let e(r,z) denote the replica vectors, whose entries are the predicted complex pressures at the hydrophone locations due to an assumed source at location (r,z). In the case considered here there are more hydrophones than waveguide modes used to construct the replica vectors.

The SF estimator is constructed to restrict the processing to eigenvectors of the data covariance matrix which describe the source, in the following manner. Consider a linear filter $\mathbf{w}^\dagger K\mathbf{w}$ and require the high resolution weight vector $\mathbf{w}$ for this filter to have minimal projection outside of the subspace spanned by the eigenvectors. Let the projection matrix corresponding to the sector be labeled V. The SF estimator is obtained by optimizing the filter $\mathbf{w}^\dagger K\mathbf{w}$ for minimum variance, subject to the sector focusing constraint $\mathbf{w}^\dagger\mathbf{w} = \mathbf{w}^\dagger VV^\dagger\mathbf{w}$, together with the "distortionless look" constraint $\mathbf{w}^\dagger\mathbf{e} = 1$. To determine $\mathbf{w}$, the following dual constraint cost function must be minimized, i.e.,

$$\Gamma = \mathbf{w}^\dagger K\mathbf{w} + \lambda_1 \mathrm{Re}(\mathbf{w}^\dagger\mathbf{e} - 1) + \lambda_2 \mathbf{w}^\dagger(I - VV^\dagger)\mathbf{w}\,, \tag{2}$$

where λ_1 and λ_2 are Lagrange undetermined multipliers and I is the identity matrix. The minimization is accomplished by setting the derivative of Γ with respect to $\mathbf{w}^\dagger$ to zero, i.e.

$$\partial\Gamma/\partial\mathbf{w}^\dagger = K\mathbf{w} + \lambda_1\mathbf{e}/2 + \lambda_2(I - VV^\dagger)\mathbf{w} = 0\,. \tag{3}$$

If we pre-multiply the RHS of (3) by $V^\dagger$ and replace $\mathbf{w}$ by $V\mathbf{a}$ ($\mathbf{a}$ is any L×1 complex vector and L is the rank of V) we obtain

$$V^\dagger KV\mathbf{a} + \lambda_1 V^\dagger\mathbf{e}/2 + \lambda_2(V^\dagger - V^\dagger VV^\dagger)\mathbf{w} = 0\,. \tag{4}$$

The columns of V are generally orthonormal vectors, so that $V^\dagger V = I$. Hence, the third term on the LHS of (4) vanishes and the expression reduces to

$$\mathbf{w} + \lambda_1 V(V^\dagger KV)^{-1}\, V^\dagger\mathbf{e}/2 = 0\,. \tag{5}$$

If we pre-multiply this equation by $\mathbf{e}^\dagger$, and use the constraint $\mathbf{w}^\dagger\mathbf{e} = 1$, we obtain an expression which can be solved for λ_1, i.e.

$$\lambda_1 = -2/\mathbf{e}^\dagger V(V^\dagger KV)^{-1}\, V^\dagger\mathbf{e}\,. \tag{6}$$

Substituting this back into (5) gives the weight vector

$$\mathbf{w} = V(V^\dagger KV)^{-1}\, V^\dagger\mathbf{e}/\mathbf{e}^\dagger V(V^\dagger KV)^{-1}\, V^\dagger\mathbf{e}\,, \tag{7}$$

and further substitution of $\mathbf{w}$ into the filter $\mathbf{w}^\dagger K\mathbf{w}$ gives the SF estimator

$$SF = \mathbf{w}^\dagger K\mathbf{w} = \frac{1}{\mathbf{e}^\dagger V(V^\dagger KV)^{-1}\, V^\dagger\mathbf{e}}\,. \tag{8}$$

It is clearly seen that V pre- and post-multiplies K, and therefore projects the covariance matrix into the space of the columns of V. It similarly projects the replica vectors **e**. By this means processing is concentrated on those eigenvectors which describe sources within the sector to which V corresponds.

3. THE SIMULATED EXPERIMENT

To illustrate the effectiveness of SF to selectively resolve environmental parameter spaces, we have performed a series of simulations in a representative shallow water waveguide. The propagation calculations for these simulations were performed using the KRAKEN model.[17] Figure 1 shows the nominal environment in which the simulated experiment was performed. It consists of a range-independent, shallow-water waveguide of depth 100 m and density 1 g/cc overlying a sediment layer of 100 m thickness, density 2 g/cc and an acoustic attenuation of 0.045 db/λ at the 170 Hz CW source frequency. The water and sediment layers are underlain by an isospeed, semi-infinite basement. The nominal sound speed in the water channel decreases from 1540.0 m/s at the surface to 1525.0 m/s at the water-sediment interface. The compressional sound speed in the sediment has a positive gradient, increasing linearly from 1750.0 m/s at the water-sediment interface to 1860 m/s at the sediment-basement interface. The basement has a density of 2 g/cc and a constant compressional sound speed of 1860 m/s. Since the basement has properties matching the base of the sediment layer, reflections at the sediment-basement interface may be neglected. Shear wave propagation was not included in the problem.

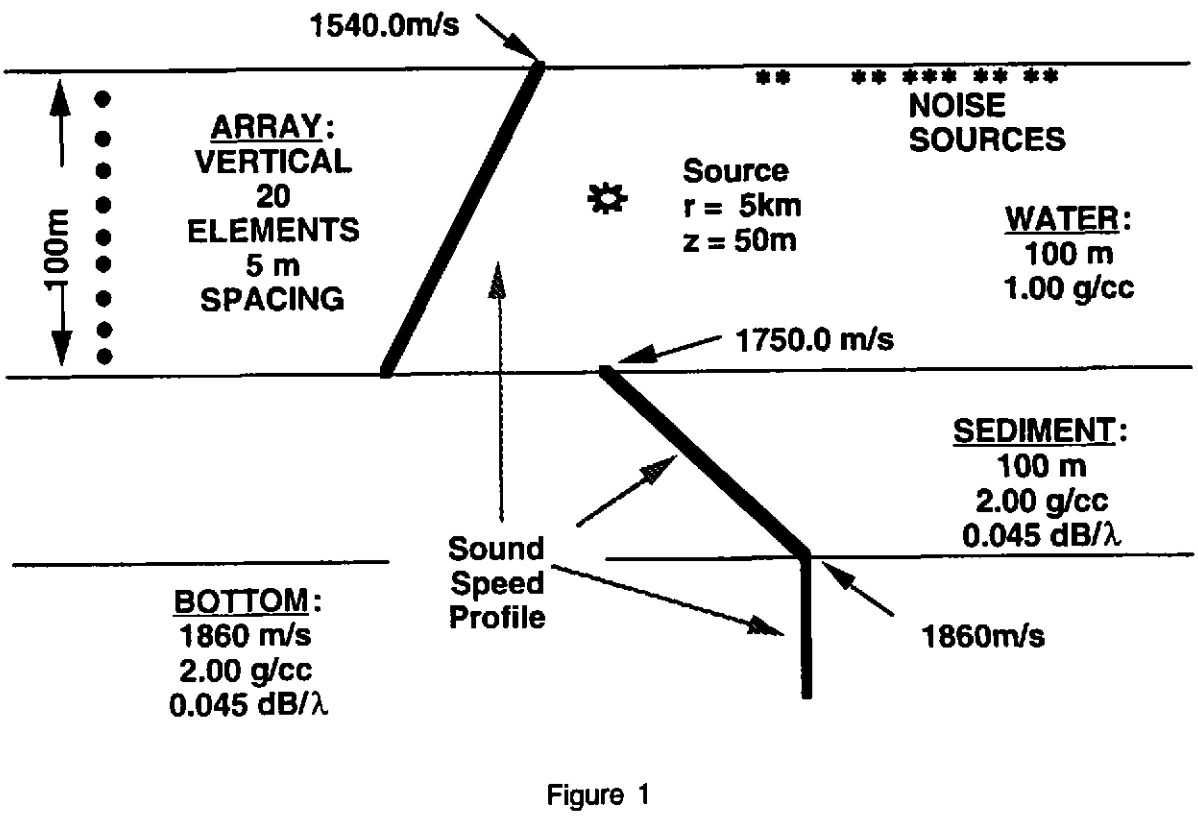

Figure 1

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A vertical array of 20 evenly spaced hydrophones spanning the water column was suspended in the waveguide and extended from 3 m to 98 m depth. The hydrophones therefore had a spacing of 5 m. A 170 Hz narrow band source was placed 5 km from the array at a depth of 50 m. Shipping noise was modeled by simulating the random movement of 40 distant sources, whose ranges were constrained to lie in a random pattern, within 10 km to 100 km of the receiving array. These noise sources were all placed at 7 m depth, and allowed to move at constant speeds of less than 12 kts between simulated snapshots taken at an interval of 1 s. Levels for the source and modal noise were 30 dB, giving a 0 dB signal to (modal) noise ratio; white noise at a level of 10 dB was also added.

For real applications it would clearly be necessary to use a more sophisticated parameterization of the environment including, perhaps, empirical orthonormal functions (EOF)s [18] (for modeling the sound speed profile in the waveguide) and bottom properties in the search space. However, the simplified waveguide model used here was deliberately chosen in order to demonstrate the environmental inversion applications of SF, and represents no loss of generality for the estimator. In the present implementation, the search space was limited to two environmental parameter space variables and two source localization variables. The parameter space variables were: γ (the sound speed gradient in the water); and c_0 (the sound speed at the water-sediment interface). The spatial variables were: r (the source range); and z (the source depth). The latter are the search variables typically used in conventional matched field localization.

It is difficult to visualize the estimator response in a multiparametric space when the number of parameters exceeds two or (at most) three. Here, for demonstration purposes, 2-D slices of the ambiguity hypersurface for the SF estimator are plotted.

After selecting the desired pair of search parameters, sectors were formed as shown in Figure 2. As the search space was scanned, the sector covariance matrix Q was formed by averaging the outer

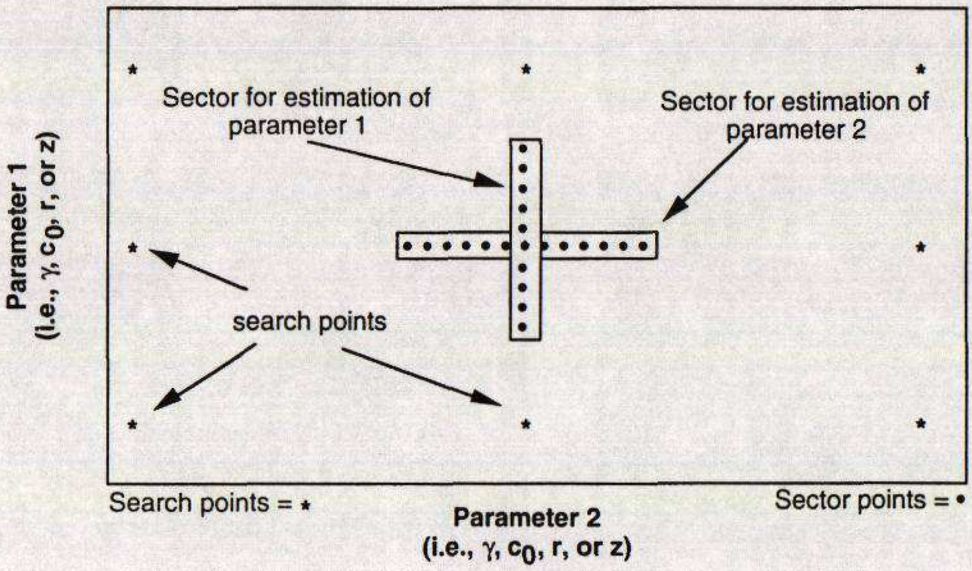

Figure 2

products of the replica vectors from the points within the sector; i.e., $Q = \langle \mathbf{e}(r, z, \gamma, c_0)\, \mathbf{e}(r, z, \gamma, c_0)^{\dagger} \rangle$. The number of points in the sector (i.e., 25) was kept constant throughout the study. This number always exceeded the number of modes supported by the waveguide and also ensured that Q, from which the projection matrix V was derived, had full rank. In order to show the resolution selectivity to best effect, all of the sectors chosen for inclusion here were "1-D", i.e., only one parameter was varied at a time to produce Q. These sectors have a single point aperture in one of the two parameters (the low resolution parameter) and 25 point aperture in the other parameter (the high resolution parameter).

4. RESULTS

The first example is a classical acoustic source localization matched field problem. It is assumed that the ocean acoustic environment is well known, and the source is placed at 50 m depth and 5 km range in the waveguide. The purpose of this example is to illustrate the selective parameter enhancement property of the SF method. Figure 3a shows the typically broad range-depth response of the Bartlett estimator. This is stable against mismatch because it also has a generally broad response in environmental parameter space, as we shall show. Figure 3b illustrates the corresponding response of the MVDR estimator. This gives sharper estimates of range and depth, but is unstable in the presence of mismatch,

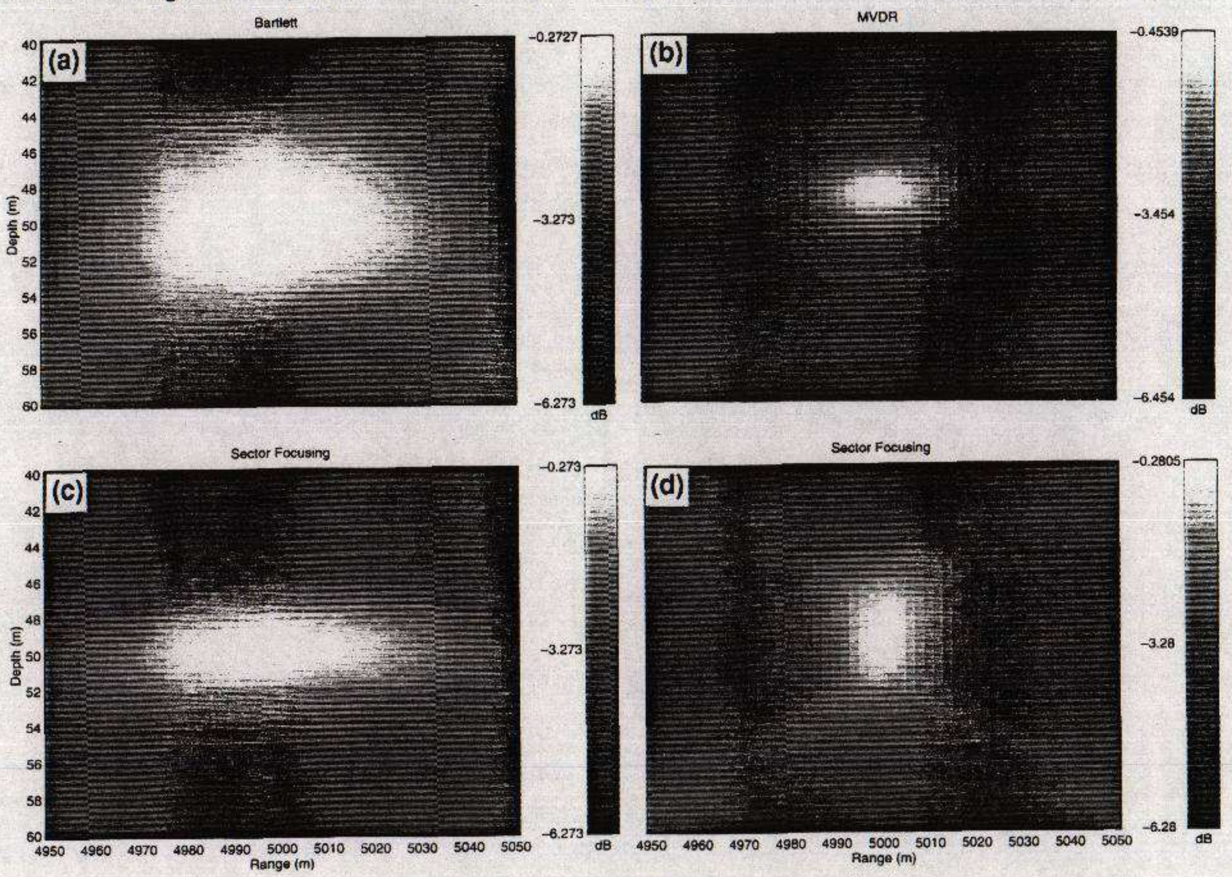

Figure 3

because it has a generally narrow response in parameter space. Figures 3c and 3d show the response obtained via conventional "range-depth parameter" SF processing.[15,16] In Figure 3c, a "depth only" 1-D sector is used. The depth aperture of the sector consists of 25 points evenly spaced between points ±2.5 m above and below each search location. The range aperture of the sector consists of the single point at each search location. Note from Figure 3c that this causes the depth response to be effectively narrowed, while a broad Bartlett like response is retained in depth. In contrast, Figure 3d shows the response obtained with a "range only" sector. This time, the range aperture of the sector consists of 25 points evenly spaced between points ±25 m on either side of each search location, while the depth aperture consists of the single point at the search location. Now the range response is narrowed, while a broad response is retained in depth. This demonstrates the capacity of SF to attain high resolution in some selected parameters while maintaining low resolution (and stability against mismatch) in others.

In the second example we show that this property can be extended into environmental parameter space. In Figure 4, the estimator response for source depth z versus c_0 (i.e., sound speed at the bottom) are shown. The Bartlett response (Figure 4a) is broad in both depth and in the environmental parameter c_0. (Recall, from Figure 3, that the source range response of this estimator is also broad.) Also, in a similar manner to Figure 3b, the MVDR response (Figure 4b) is narrow in both parameters.

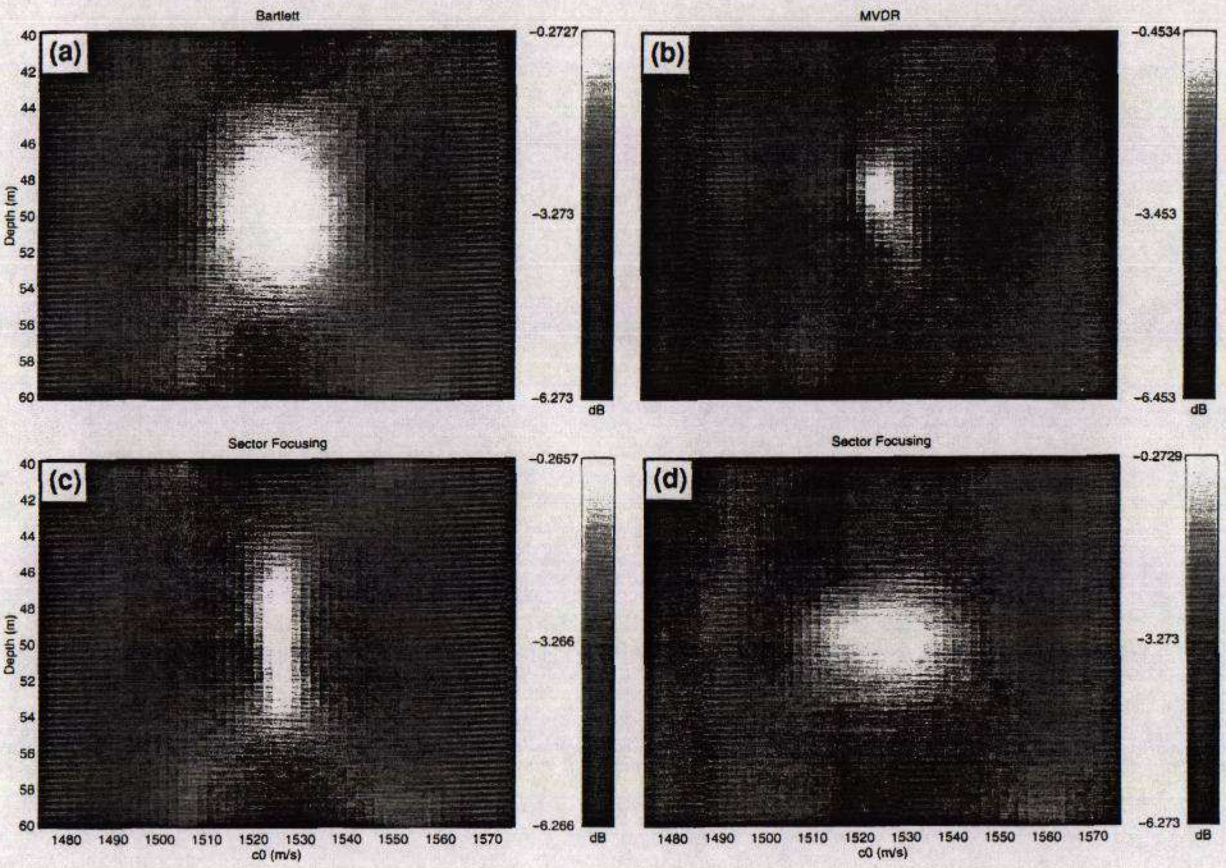

Figure 4

Taken together, Figures 3b and 4b imply that, if high resolution estimates of c_0 are to be attained with the MVDR estimator, then *any appreciable mismatch in source location* will give unstable results, since the actual source location will fall outside of the source location response pattern. The response pattern selectivity properties of SF can be used to improve this situation, and reduce the effects of poorly determined source localization parameters in the acoustic model. This is demonstrated in Figures 4c and 4d. Figure 4c shows the result of using a 1-D sector with a 20 m/s aperture for c_0, and a single point aperture in depth. The result is to retain the broad depth response seen in the Bartlett estimator (Figure 4a), while significantly narrowing the c_0 response. If the processing is intended to determine c_0, then this result shows that this implementation of SF will tolerate a high level of uncertainty in z, while still returning an accurate value for c_0. Figure 4d, in contrast, shows the result of using a 5 m sector aperture in depth, with a single point aperture in c_0. This causes the depth response to narrow, while the c_0 response remains broad. It is clear that, by carefully choosing the sector parameters and dimensions, the estimation process may be controlled. High resolution may be attained in those parameters to be found by inversion, with low resolution (and, hence, robustness) against poorly determined quantities.

In the final example (Figure 5) the SF estimator response for sound speed gradient γ versus range r is plotted. Figure 5a shows the Bartlett response. The range estimate is seen to be fairly robust

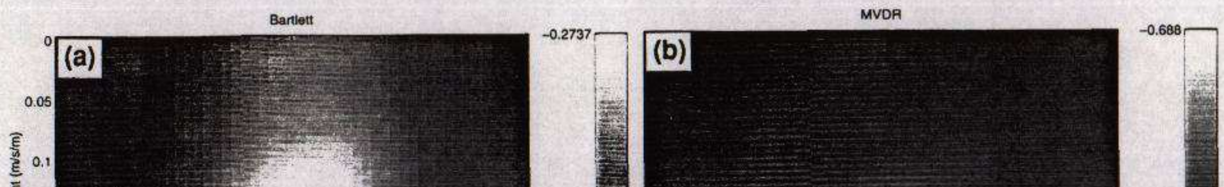

Figure 5

to sound speed gradient mismatch, but the range response pattern is quite broad. To achieve higher resolution, MVDR can be used to sharpen the response pattern, as shown in Figure 5b. Regrettably, this also causes the estimator to become significantly less tolerant to sound speed mismatch. SF can be used to great advantage in this case. By placing a 50-m-wide range only sector about the search point the range is more sharply resolved (Figure 5c), but the robustness to sound speed gradient mismatch, seen in Figure 5a, is preserved. In contrast, if the aim is to perform environmental inversion of the sound speed gradient, then SF can be used to sharpen the estimate in that parameter while remaining tolerant to source range errors. Figure 5d shows the result of using a 1-D sector with a sound speed gradient aperture of 0.1 m/s/m, but with a single point aperture in range. Here we see that the estimator returns estimates $0.2 \geq \gamma \geq 0.1$, which are very robust to source range mismatch. Note also, however, that the γ response pattern abruptly bifurcates, illustrating the nonlinear nature of extreme mismatch. This does not diminish the usefulness of the estimator, since the range of γ for which the estimator is stable falls well within the expected error bars for most provinced data.

5. CONCLUSIONS

Sector-focusing is a high resolution technique which can be flexibly applied to give selective resolution enhancement while remaining stable against mismatch. This property has been used in conventional source localization, but can also be applied to the tomographic inverse problem. In this context, it can be used to improve oceanographic parameter estimation by selectively narrowing estimator response, while remaining insensitive to errors caused by poorly determined model input parameters.

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