A MODEL TO FIT THE STATIC STRUCTURAL COMPRESSIONAL MODULUS OF FIBROUS MATERIALS

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Fibrous materials are very likely to behave nonlinearly when they are subjected to static compression. This non-linearity has not yet been successfully represented in a single equation throughout the entire range of strain. In the present work, a transversely isotropic structure comprising elastic rods has initially been considered as a linear model, which has been extended to the nonlinear equivalent by utilising the relation between the compressive strain and the change of porosity. Subsequently, this nonlinear model has been extended by adding the effect of rotational flexibility at the ends of the fibre links. Thus, an equation with three parameters has been developed to represent the structural stress-strain behaviour of fibrous media. These parameters are: the Young's modulus of the solid material, the porosity, and a constant, which is believed to be related to the flexibility of the fibre link. This equation exhibits considerable sensitivity to the porosity, and better predictions are achieved when the equation is fitted to the data, with the three parameters acting as adjustable constants.

1. INTRODUCTION

Bulk fibrous media are composed of interwoven fibres. It is known that their structural motion can strongly influence both their acoustic and mechanical properties. In this work, an idealised model has been developed to represent this structural behaviour of fibrous media. For this purpose, a fibrous material has been modelled as a structural lattice composed of elastic cylinders. This lattice is composed of layers of elastic rods which cross each other. A similar model has already been described in the "stacked-cylinder model" of Sides, Attenborough and Mulholland [1], in which the Hertzian contact or deformation was a dominant factor in the bulk elastic behaviour. Hertzian deformation is expected to take place wherever inter-fibre contact occurs. In a realistic configuration, however, its contribution is likely to be very small because the deflection is inherently restricted by the fibre diameter, which is normally only a few microns or tens of microns. Although Hertzian deflection bears a nonlinear relationship to the applied force and the bulk elastic behaviour of fibrous media is inherently nonlinear, the Hertzian deformation of fibres is unlikely to be the explanation of the nonlinear bulk elasticity. In the present work, however, the static bulk elastic behaviour of fibrous media has been re-examined in terms of bending deformation, and a modified model has been employed, which allows the elastic rods to bend.

2. NONLINEAR BENDING MODEL

2.1 LINEAR BENDING MODEL

In this research, a fibrous material has been ideally modelled as a structure of stacked cylinders. This is reasonable because most layered fibrous media may be characterised as transversely isotropic. Parallel cylinders form each layer, and successive layers are shifted by half fibre spacing with respect to each other to allow the cylinders to bend. Hertzian deformation occurs at the contact points of adjacent cylinders. But it is assumed that Hertzian deformation is negligible as compared to the bending deflection from the viewpoint of the mechanical loading. A schematic view of the model is shown in Figure 1.

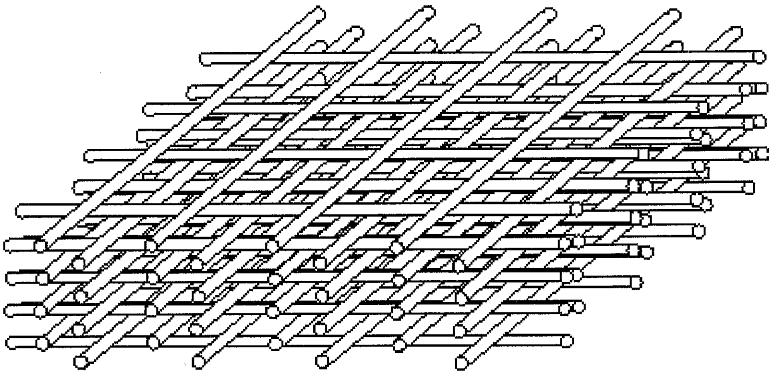

Figure 1. Idealised model for bending behaviour of fibrous material

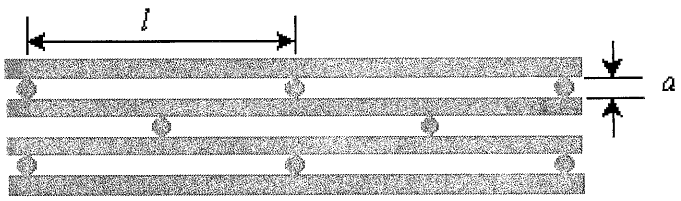

Figure 2. Side view of the idealised model, indicating the diameter and the contact points of the fibres

2.2 NONLINEARITY

It is well known that a fibrous material is deflected nonlinearly under compression. In a realistic situation, as distinct from the Hertzian model, this nonlinearity occurs dominantly by the increase in the numbers of inter-fibre contacts as the material is compressed [2]. Based on the schematic view of the bending model in the Figure 1, it is certain that there could be one more fresh contact per fibre link when the stress reaches the requisite value, but no further contacts on the basis of this particular model. Thus for this configuration, the stress-strain relation will look like a piecewise linear curve composed of only two linear regions. Under longitudinal compression, the strain increases as a function of the stress, and the porosity decreases correspondingly. It is known that this compression process causes new contacts to occur [2], which makes the length of each fibre link shorten. It should be noted that the bending model with characteristic links of length l_1 rearranges to the model with links of different length l_2. This change in the length of the fibre links occurs continuously for every infinitesimal step of strain or stress in a real fibrous medium. But this seems technically impossible in the idealised configuration, because cylinders are not allowed to move horizontally in any layer, but are assumed to deflect only vertically. Nevertheless, one can imagine an intermediate status with shortened characteristic fibre links, irrespective of how it has been achieved, and focus on the consequence of the rearrangement itself. Therefore the idealised bending model can be adopted at each stress-strain "status".

2.3 POROSITY AND STRAIN

As mentioned above, one of key mechanisms in the model is the shortening of fibre links, which is related to the porosity at each strain level. When the fibrous material is subjected to compression, the porosity and the bulk density change correspondingly. If Hertzian deformation and volume change by elongation around the bent part of fibres are neglected during the compression, the total fibre volume in the model is likely to remain unchanged, especially in a fibrous material with a high porosity. For fibrous materials which are transversely isotropic in the direction of *x* and *y* (as in the model presented here), the changing porosity Ω of the compressed material can be easily represented by

$$\Omega = 1 - \frac{1-\Omega_0}{(1+v_b\varepsilon_x)^2 \cdot (1-\varepsilon_z)} . \quad (1)$$

Here, the variable porosity is defined by the longitudinal strain ε_z (or ε if the subscript *z* is discarded) and equation (1) may be written in the simpler form

$$\Omega \approx 1 - \frac{1-\Omega_0}{1-\varepsilon} = \frac{\Omega_0 - \varepsilon}{1-\varepsilon} . \quad (2)$$

The subscript 0 denotes the values of porosity without any compression. The bulk Poisson's ratio v_b is likely to be very small in a laminar material with a two dimensional fibre arrangement and the term $v_b\varepsilon_x$ (ε_x being the lateral strain) is neglected in (2).

2.4 BOUNDARY CONDITION

The boundary conditions at the ends of each fibre link are a barometer of the flexibility at those points. Based on the schematic view of the bending model in Figure 1, the symmetry inherent in an effectively infinite stack of cylinders will prevent any rotation of structural elements at the contact points, although the actual boundary condition is simply supported. But this type of clamped boundary condition is relevant only to the ideal situation. So some modifications can be made here to give allowance for the actual structure of fibrous material. The concept of "binding force" at the ends of each fibre or at the contact point can be introduced. In Figure 3, the bending moment inside the bar for some typical boundary conditions is shown. The plot denoted by Ê is for the bar clamped at both ends, Ë is for the simply supported ends, and Ì is for both ends having applied external moments M_0.

It can be imagined that the binding force or binding restraint at the contact points is related to the rotation or flexibility there. As is well known, no rotation is allowed for clamped ends (Ê), and the ends are free to rotate in the simply supported case (Ë). One can conceive that, in certain situations, the ends may be likely to rotate more than they would in the simply supported case, especially if there are external loads at the ends. One of the possibilities for this is damage to the fibre at the ends of the links or at the contact point. The extent of bending at the contact point can be beyond the recovery point, especially when the fibre itself is brittle or the material is subjected to a heavy load. These external loads can be represented by the moment M_0 in Figure 3.

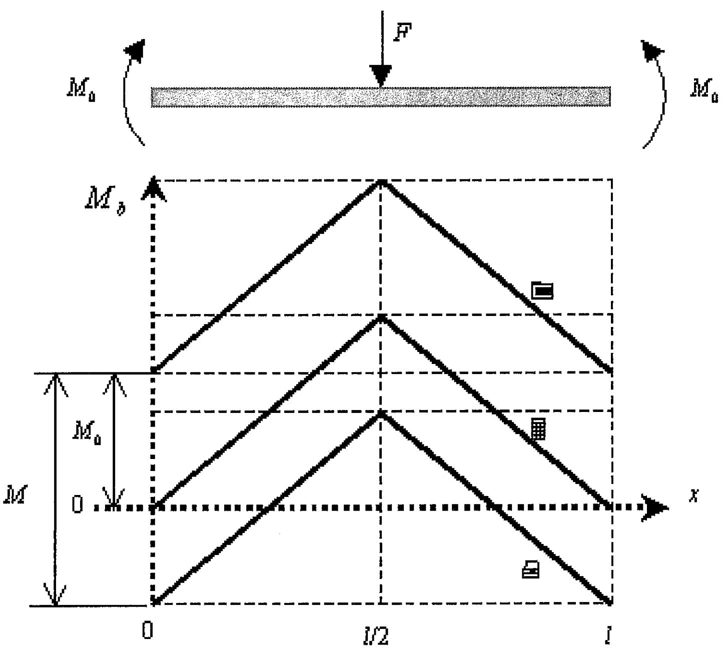

Figure 3. Diagram showing the bending moment along the fibre link.

The elastic energy of the link, created by the bending moment, may be expressed by the following equation after some manipulation, provided that the force F acts on the centre of the link [3]:

$$V = \int \frac{M_b^2}{2EI}dx = \frac{1}{2EI}\left(\frac{F^2 l^3}{192} + M^2 l\right), \qquad (3)$$

where E is the Young's modulus of the fibres. If it is assumed that M arises solely from F, the deflection of the link is straightforward to calculate by the Castigliano's theorem [3]. This assumption could be reasonable because, if the external load is related to the damage to the fibre amongst several other potential effects, the damage is sure to be related to the force F. So the external moment M can be related to the force F as follows. If damage is one of causes, as assumed, the moment M can be further related to the strain ε and one of the characteristic dimensions of the model, - the diameter d - as follows:

$$M = \gamma \cdot d \cdot F = \gamma' \cdot \varepsilon \cdot d \cdot F \qquad (4)$$

and

$$V = \frac{1}{2EI}\left(\frac{F^2 l^3}{192} + (\gamma' \cdot \varepsilon \cdot d)^2 F^2 l\right). \qquad (5)$$

The deflection δ arising from the force F is obtained from Castigliano's theorem as

$$\delta = \frac{\partial V}{\partial F} = \frac{F}{EI}\left(\frac{l^3}{192} + (\gamma' \cdot \varepsilon \cdot d)^2 l\right). \qquad (6)$$

Now if we go back to the bending model which is composed of a network of the fibre links above, the number N_l of layers in the bending model is defined by $N_l = t/d$, where t is the thickness of the model and d is the diameter of each cylinder or rod. The whole deflection Δ of the model is given by $\Delta = N_l \cdot \delta$. Recall that the rod in the model has diameter d; the second moment of the its cross sectional area I is then given by $\pi d^4/64$. From the configuration of the model, the contact force F on the link is related to the external force F_{ext} to the model by $F = F_{ext} l^2 / A$. Here, A is the surface area of the fibre network to which F_{ext} is applied to. Finally, recalling that the strain ε is defined by $\varepsilon = \Delta / t$ and the stress by $\sigma = F_{ext} / A$, the stress-strain relation for compression in the bending model will be given by

$$\varepsilon = \frac{\sigma}{3\pi E} \cdot \left(\frac{l}{d}\right)^5 \cdot \left\{1 + 192 \left(\gamma' \cdot \varepsilon\right)^2 \left(\frac{d}{l}\right)^2\right\} . \qquad (7)$$

From a simple geometric consideration, based on Figure 2, the relationship between the aspect ratio of the fibre link and the porosity is

$$\frac{l}{d} = \frac{\pi}{4\left(1-\Omega\right)} . \qquad (8)$$

Finally, the stress-strain relationship from the nonlinear bending model is obtained, if the expressions (2) and (8) are inserted to the stress-strain relation (7) above,

$$\sigma = \frac{3\pi E \cdot \left(\frac{4}{\pi} \frac{1-\Omega}{1-\varepsilon}\right)^5}{1 + 192 \left(\frac{4\gamma'}{\pi} \varepsilon \frac{1-\Omega}{1-\varepsilon}\right)^2} \varepsilon . \qquad (9)$$

The porosity Ω in equation (9) is now that for $\varepsilon = 0$, and the subscript 0 is discarded.

3. RESULTS

This nonlinear bending model has been applied to the real fibrous materials, data from two of which are shown here. They are, first a thermal insulation blanket from an aircraft fuselage, and secondly an acoustic duct-lining material. Here, they are denoted materials 1 and 2, respectively. The parameters γ and γ' can be obtained through a simple numerical iterative procedure from the measurement data. The use of the parameter γ and γ' requires the adjustment of either the porosity or the Young's modulus of the solid material. If this is done, the parameter γ is observed to increase linearly from approximately zero ($\varepsilon = 0$), as one may see in Figures 4 and 6. The dashed line in the γ-plots is a regression line. This ensures that the parameter γ' has an approximately constant value on the whole range of strain. Based on these three parameters - the Young's modulus, the porosity, and the constant γ' - the nonlinear stress-strain behaviour of the fibrous material can be reproduced, as shown in Figures 5 and 7. The value of 5×10^{10} N/m^2 is used as the Young's modulus of the solid material in both cases. The values for fitted porosity are 0.9900 and 0.9845, respectively. The constant parameter, γ', has the values 20.05 and 7.12 for materials 1 and 2, respectively. Plots of the stress-strain behaviour are in Figures 5 and 7. The solid line in the plot for bulk modulus is from the nonlinear bending model based on three parameters.

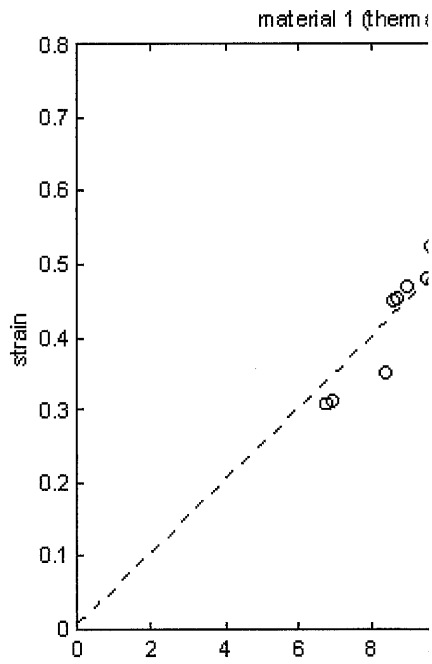

Figure 4. The parameter γ of material 1

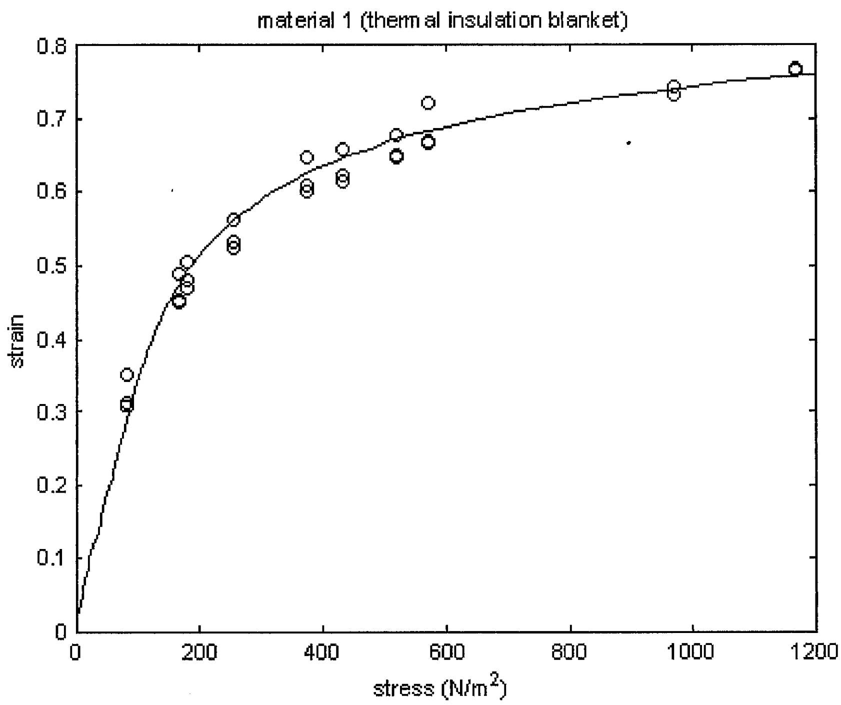

Figure 5. Stress-strain behaviour of material 1

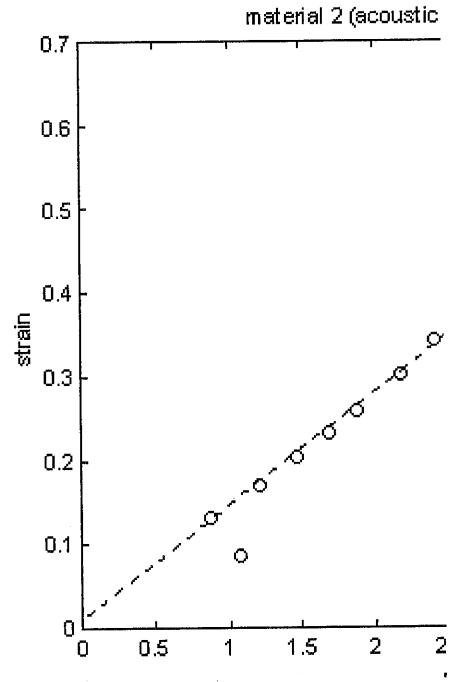

Figure 6. The parameter γ of material 2

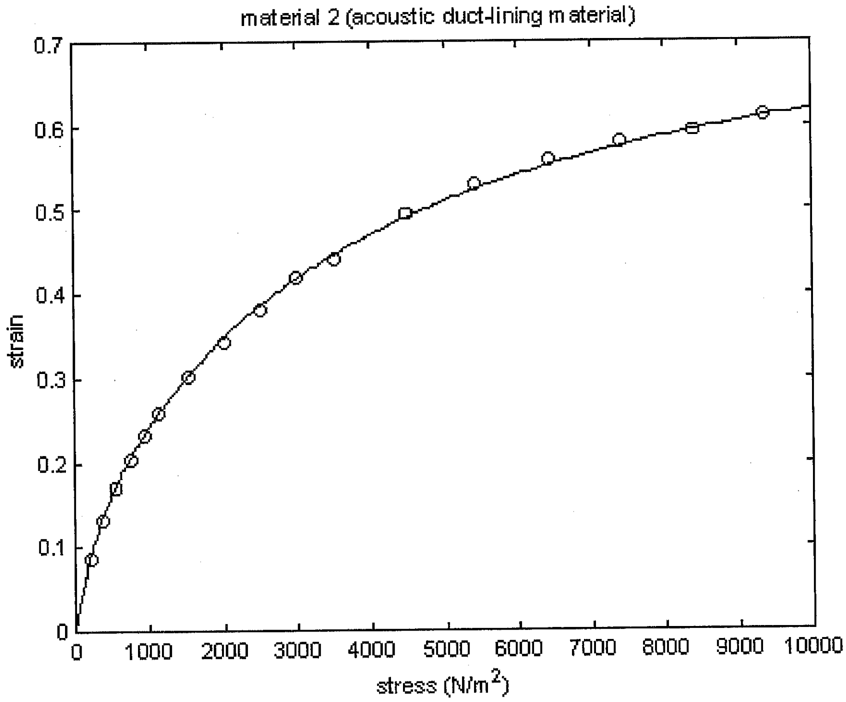

Figure 7. Stress-strain behaviour of material 2

4. CONCLUSION

A transversely isotropic structure consisting of elastic cylinders has been proposed to represent the nonlinear stress-strain behaviour of a fibrous material under compression. This structure is based on the stacked cylinder model, including staggered alternate layers to allow the components to bend. The concept of the increase in the number of contacts among fibres has been related to the shortening of fibre links, which ensures the nonlinearity of the model. This nonlinear bending model has been shown to be in good agreement with measured data, when its three parameters are treated adjustably because of its sensitivity to the porosity.

5. ACKNOWLEDGEMENT

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6. REFRENCES

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