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#### 1. INTRODUCTION

The application of synthetic aperture imaging techniques to sonar is currently a topic of considerable interest [1-3]. In many respects the techniques are similar to those of synthetic aperture radar, but there are some important differences, principally due to the much lower velocity of propagation. The applications for synthetic aperture sonar are mainly associated with high-resolution short-range imaging of objects on the seabed, usually at relatively high frequencies (100 kHz or more). There is also interest, however, in application to long-range surveillance, at much lower frequencies (< 1 kHz), where the principal objective would be to reduce the required towed array length for a given azimuth resolution, for reasons of operational convenience. It should be admitted that there are significant questions about the temporal and spatial stability of the propagation medium for this application.

The purpose of the work described in this paper is to examine how synthetic aperture sonar systems behave when the target is moving (particularly in the context of the low-frequency long-range active systems), and to discuss how moving targets may be detected, and the target motion parameters estimated to set up the appropriate matched filter to image the moving target correctly.

#### 2. SYNTHETIC APERTURE IMAGING

2.1 Synthetic aperture imaging of stationary targets Consider a sonar carried by a platform moving in a straight line along the x-axis with velocity  $\nu$  (Figure 1). The instantaneous range r to a point target is given by:

$$r = (r_0^2 + x^2)^{1/2}$$

where  $r_0$  is the range at closest approach, which occurs when x = 0.

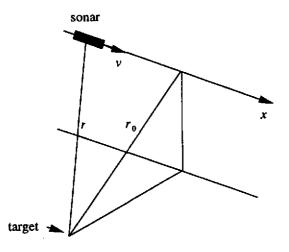


Figure 1. Synthetic aperture sonar geometry.

Equation (1) can be expanded:

$$r = r_0 + \frac{x^2}{2r_0} - \frac{x^4}{8r_0^3} + \dots$$
 (2)

The phase history of the sequence of echoes from the point target (making the 'stop-start' approximation) is:

$$\phi(x) = -\frac{2\pi}{\lambda} \cdot 2r(x)$$

$$= \phi_0 - \left(\frac{2\pi x^2}{r_0 \lambda} - \frac{\pi x^4}{2r_0^3 \lambda} + \dots\right) \qquad \dots (3)$$

where  $\lambda$  is the sonar wavelength and  $\phi_0 = -4\pi r_0/\lambda$ 

If the length of the synthetic aperture is restricted such that the third and subsequent terms of the expansion of equation (2) are negligible (which will usually be the case in practice), this is a quadratic function of x (Figure 2a), and also of time t, since  $x = \nu t$ . It is also useful to think in terms of the Doppler history of the sequence of echoes, which may be obtained by differentiating the phase history with respect to time:

$$f_D = \frac{1}{2\pi} \frac{d}{dt} \left( \phi_0 - \frac{2\pi v^2 t^2}{r_0 \lambda} \right) = -\frac{2v^2 t}{r_0 \lambda} = -\frac{2vx}{r_0 \lambda} \qquad (4)$$

which represents a linear-varying Doppler history (Figure 2b).

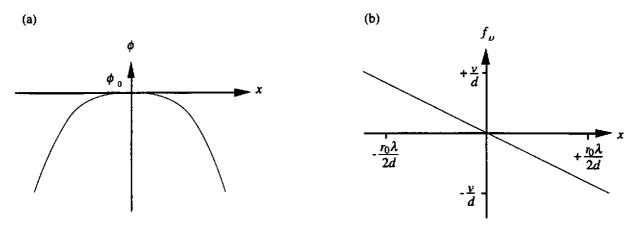


Figure 2. (a) phase history, and (b) Doppler history of echoes from a point target.

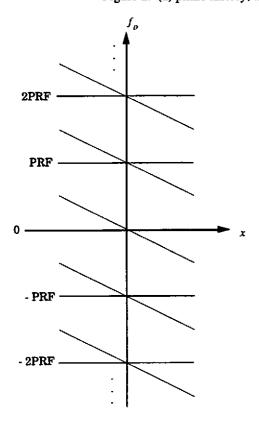


Figure 3. The Doppler history is sampled at the pulse repetition frequency (PRF).

The Doppler is band-limited (though not perfectly) by the transducer azimuth beamwidth, which is approximately  $\lambda/d$  (where d is the along-track dimension of the transducer array):

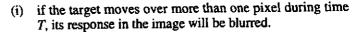
$$-\frac{\nu}{d} \le f_D \le \frac{\nu}{d} \qquad \qquad \dots (5)$$

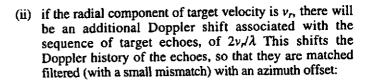
Since the Doppler is sampled at the PRF rate, there are additional versions of the Doppler history, centred on multiples of the PRF (Figure 3). Consequently, in order to avoid aliasing:

$$PRF > \frac{2\nu}{d} \qquad \qquad \dots (6)$$

The process of aperture synthesis is one of matched-filtering the Doppler history of the sequence of echoes. The maximum achievable azimuth resolution is d/2. Equivalent range resolution is provided by means of a narrow pulse, or more usually by pulse compression of a waveform of bandwidth  $\Delta f$ , such that the range resolution is  $c/2\Delta f$ , where c is the velocity of propagation.

2.2 Moving targets If the target is moving, there are several effects. The problem has been examined for the case of synthetic aperture radar by Raney [4]. For simplicity, assume initially that the target moves with uniform velocity during the time  $T = r_0 \lambda v d$  taken to synthesise the aperture.





$$\Delta x = \frac{2v_r}{\lambda} \cdot \frac{r_0 \lambda / d}{2v / d} = \frac{v_r r_0}{v} \qquad \qquad \dots (7)$$

This is a well-known effect in synthetic aperture radar; for example, a satellite SAR image of a ship can appear displaced from the head of its wake. In a sonar system, for which the magnitudes of  $\nu_r$  and  $\nu$  may be comparable,  $\Delta x$  can be very large indeed.

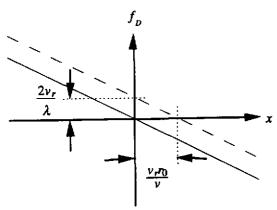


Figure 4. A target with radial velocity  $v_r$  is matched-filtered with an azimuth shift  $\Delta x = v_r r_0 / v$ .

(iii) if linear FM pulse compression is used, a radial component of target velocity also results in a range error:

$$\Delta r = \frac{c}{2} \left( \frac{2\nu_r}{\lambda} \cdot \frac{\tau}{\Delta f} \right) \qquad (8)$$

where  $\tau$  is the uncompressed pulse length. This effect is usually quite small.

Thus in general the image of a moving target will suffer from blurring and displacement from its true position. Figure 5 shows some examples of simulated responses of targets with various values of velocity in the azimuth direction  $(v_{az})$  and range direction  $(v_f)$ .

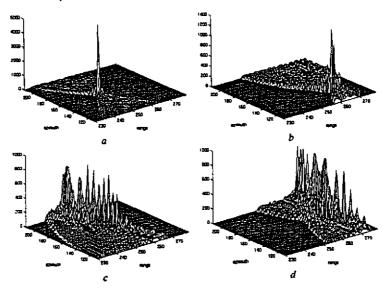


Figure 5. Simulated synthetic aperture sonar images, without compensation for target motion: (a)  $v_{az} = 0$ ,  $v_r = 0$ , (b)  $v_{az} = 0$ ,  $v_r = 2$ m/s; (c)  $v_{az} = 5$ m/s,  $v_r = 0$ ; (d)  $v_{az} = 5$ m/s,  $v_r = 2$ m/s.

In addition, though, in a sonar system the magnitude of the Doppler shift  $2\nu/\lambda$  due to target radial motion may be much greater than the limits of Doppler history  $\pm \nu/d$  from a stationary target, and hence this 'moving target' Doppler may be badly undersampled. This contrasts with the case with a synthetic aperture radar, and means that in a synthetic aperture sonar, any attempt to estimate target velocity must be done at pulse level rather than in the image.

It also means that, to correctly image a moving target it is necessary to correct for the target migration over a number of pixels (in order to properly superimpose the echoes), and then to apply the correct phase weights so that the echoes add coherently.

#### 3. PULSE-LEVEL PROCESSING

3.1 <u>DPCA processing</u> A scheme for detecting moving targets can be devised, based on an array composed of two transducers (or subarrays), displaced along-track. In radar parlance this scheme is known as DPCA (Displaced Phase Centre Antenna) processing [5].

A pulse is transmitted and received from the forward transducer, then a second pulse transmitted from the rear transducer when it is in the same spatial location as the first one was (Figure 6). The two echoes are subtracted; if the target scene has not changed, this subtraction should be perfect, but echoes from any moving target will give a non-zero result. Thus echoes from stationary targets are suppressed, but those from moving targets are not.

This requires, of course, that the pulse repetition frequency is accurately controlled as a function of platform velocity  $\nu$ , so that the spatial location of the samples coincide properly. In practice, the degree of suppression will be limited by imperfect matching of the transducer gains and patterns.

The interval  $\Delta t$  between the samples depends on the maximum unambiguous range of the sonar, and represents the interval at which the target Doppler is sampled. For a short-range system this may be acceptable, but for a long-range system  $\Delta t$  may be a hundred seconds or more, so the Doppler shift due to target motion is still severely undersampled, as before. DPCA processing is therefore not suitable for long-range sonar systems.

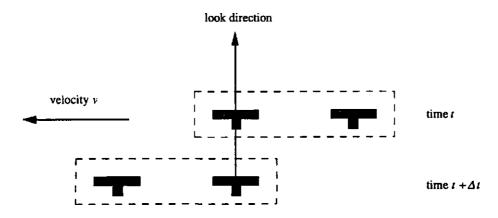


Figure 6. DPCA processing (after ref.[4]).

3.2 <u>Time-frequency processing</u> A second approach, also used in synthetic aperture radar, is to use processing techniques such as the Wigner-Ville transformation to estimate the target Doppler history [6,7]. This works because the energy is highly concentrated around the instantaneous frequency. The Wigner-Ville Distribution W(t, f) of a continuous signal  $s(\tau)$  is defined as:

$$W(t,f) = \int_{-\infty}^{\infty} s\left(t+\frac{\tau}{2}\right) \cdot s^{*}\left(t-\frac{\tau}{2}\right) \exp\left(-j2\pi f\tau\right) d\tau \qquad (9)$$

For a sampled signal this becomes:

$$W(n,f) = 2T \left[ S(n)S^*(n) + \sum_{k=1}^{\infty} S(n+kT)S^*(n-kT) + S(n-kT)S^*(n+kT) \right] \qquad ... (10)$$

in which  $S(m) = s(m) \exp(-j2\pi fkT)$ .

From the time-frequency representation, the frequency history and hence the phase history  $\phi(t)$  can be extracted:

$$\phi(t) = 2\pi \int_0^t f(\tau) d\tau + f(t_0) \qquad \qquad \dots (11)$$

However, this suffers from the same problem as the DPCA approach, in that the moving-target Doppler is potentially badly undersampled. To illustrate this, a simulation was carried out to derive the WVD of the frequency history of the sequence of echoes from a moving target. The result is shown in Figure 7, in which the effects of aliasing can clearly be seen.

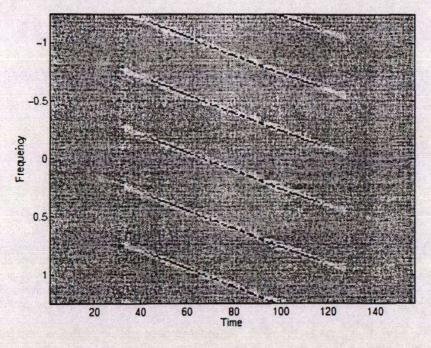


Figure 7. Simulated Wigner-Ville Distribution (WVD) of sampled Doppler history of a moving target.

3.3 <u>Single-pulse processing</u> If it is not possible to determine unambiguously the target Doppler on the basis of multiple pulses, there remains the possibility of single-pulse processing. In a long-range system, some form of pulse compression would most likely be used, with a pulse bandwidth  $\Delta f$  and uncompressed duration  $\tau$ . The problem thus reduces to designing a waveform with appropriate range and Doppler resolution. The range resolution of a single pulse is  $c/(2B\Delta f)$ ; the Doppler resolution is approximately that frequency at which there is one cycle of phase slip over the pulse duration, which corresponds to a resolution of  $1/\tau$ .

The range and (unambiguous) Doppler can therefore be estimated for each pulse, and on the basis of this information, the appropriate matched filter set up to image the target correctly. Single-pulse processing of this kind also removes the constraint of having to assume uniform target motion, so in principle manoeuvering or accelerating targets can be handled. The signal-to-noise ratio at pulse level for a point target will be a factor  $10\log_{10}N$  dB lower than at image level (where N is the number of pulses used to synthesise the aperture), but since N is relatively low (a few tens at most) this loss is not large.

#### 4. MULTIPLE PARALLEL CHANNEL APPROACH

Rather than estimate the target motion parameters and set up a matched filter for a particular moving target, it is also possible to adopt a 'brute force' approach and form several images in parallel, each assuming a different set of target motion parameters (say a total of 4 velocities  $v_n$  and 8 directions  $\theta_m$ ) and calculating the migration and focusing parameters accordingly. Once a moving target is detected in one of the images, the matched filter may be iteratively refined by trying a further set of motion parameters centred about those of the first pass through the algorithm. The computational load is reduced if it can be assumed that only approaching targets are of interest. Figure 8 shows a block diagram of the scheme.

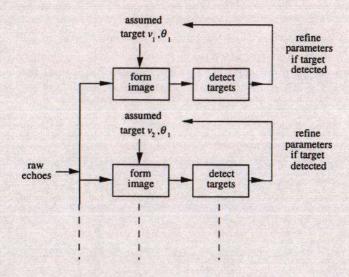


Figure 8. Block diagram of multiple parallel channel scheme.

#### 5. CONCLUSION

We have shown that the problem of detection of moving targets in synthetic aperture sonar images is different in several respects from that in synthetic aperture radar. Two conventional radar techniques have been shown not to work, but two other techniques look more promising, either (i) processing individual pulses to obtain the target range and Doppler history, and then forming the matched filter to focus the image of the target, or (ii) forming several images in parallel on the basis on a number of different assumptions about target velocity and direction.

The first of these depends for its success on the ability to find a form of pulse modulation able to provide good discrimination in both range and Doppler from a single pulse, and this problem is currently under investigation. The ability to cope with non-uniform target motion is likely to be an important advantage.

The second is potentially more sensitive, since the full benefit of the processing gain of the synthetic aperture is realised. The basic scheme assumes uniform target motion (though in principle further images could be formed in parallel under the assumption of accelerating or manoeuvering targets).

Both techniques now require further work to verify and refine the basic concepts, and to test them with real sonar data. As noted in the introduction, there are other quite fundamental factors that remain to be explored to establish whether or not aperture synthesis is applicable to low-frequency long-range systems.

### 6. ACKNOWLEDGEMENTS

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## SYNTHETIC APERTURE SONAR IMAGING OF MOVING TARGETS

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