

GROUND-BORNE VIBRATION FROM UNDERGROUND RAILWAYS: SOME COMMONLY-MADE MODELLING ASSUMPTIONS AND THEIR ASSOCIATED INACCURACIES AND UNCERTAINTIES

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1 INTRODUCTION

Concern over carbon emissions and increasing urban congestion are two driving forces for the development of underground rail networks in cities. One major environmental issue arising from the development of these rail networks is the propagation of ground-borne noise and vibration into nearby buildings. The low-frequency noise and vibration produced by underground railways is disturbing to occupants and disruptive in the day-to-day operation of vibration-sensitive premises. For these reasons, accurate models of the underground environment that can predict surface noise and vibration levels are greatly sought after. However, the number of parameters involved in describing the underground environment makes formulation of a comprehensive model of vibration from underground railways a virtually-impossible task.

All of the available prediction models are based upon simplifying assumptions that are usually decided based upon available computational power or engineering intuition. These simplifying assumptions introduce inaccuracy and uncertainty into the predictions, and in many cases these inaccuracies and uncertainties remain unquantified. This paper addresses five commonly-disregarded aspects of the underground railway environment: track with discontinuous slabs, the second (twin) tunnel, soil inhomogeneity, irregular contact at the tunnel-soil interface and the presence of nearby piled foundations. For each of these features, modelling predictions that quantify the influence of the assumption on the vibration levels are presented. For the sake of brevity, full details of these models are not presented here, but the interested reader is referred in each section to relevant previous papers by the authors.

2 DISCONTINUOUS SLAB TRACK

One of the effective means to reduce vibration from underground railways is the use of floating-slab tracks [1-3], as shown in Fig 1. These tracks are constructed by supporting the rails on a slab with a large mass. The slab is supported via soft resilient elements on the track bed. The slab can be cast in-situ resulting in a track with continuous slab and it can be constructed in discrete pre-cast sections leading to a track with a discontinuous slab. Examples of floating-slab tracks are the 1.5 m slab in Toronto, the 3.4 m Eisenmann track in Munich and Frankfurt, the 7 m slab in New York subway and the WMATA continuous slab system in Washington DC.

A track with continuous slab can be modelled by solving the equations of motion of a continuous track coupled to a tunnel embedded in a full space [4] or a half-space [5]. The computations are optimised by performing the calculations in the wavenumber-frequency domain using the 2.5D Pipe-in-Pipe (PiP) model. A track with discontinuous slab can be modelled using the periodic-infinite structure theory to couple a periodic track to a continuous model of a tunnel embedded in the ground [6].

Vibration due to a single wheel-set of a train moving with a constant velocity on a track with continuous and discontinuous slabs is presented in ref. [7]. A white noise roughness is considered

for the two tracks. For a given wavelength of the rail roughness, a moving wheel-set on a continuous track results in a dynamic force at a single frequency unlike for a wheel-set on discontinuous track which results in a sum of forces with different frequencies.

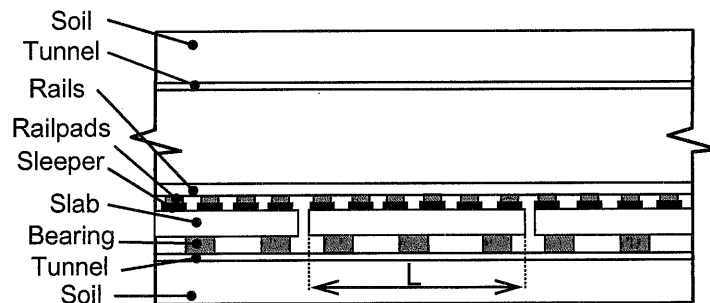


Figure 1. Floating Slab Track.

Fig 2 shows a sample result of the power spectral density of the vertical displacement at a point in the free surface of a homogeneous ground due to a single wheel-set moving on two types of tracks: a floating-slab track with continuous slab; and a floating-slab track with discontinuous slab. A white noise rail roughness is used and both tracks are supported on an underground tunnel at 20m depth. Typical parameters for tracks, tunnel and soil are used [7]. A floating-slab track with a discontinuous slab results into more vibration at the resonance frequencies of the slab. These frequencies can be calculated using the equation of the natural frequencies of a free-free beam. A difference of 10dB can be observed at some frequencies. This example demonstrates a shortcoming of using a continuous track model to account for a track with discontinuous slab.

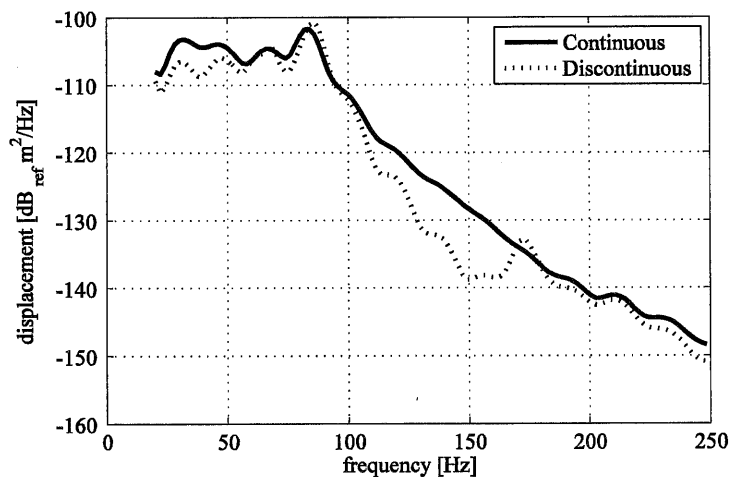


Figure 2. Power spectral density for continuous and discontinuous floating slab track.

3 TWIN TUNNELS

Many underground railway lines around the world, including those in London, Copenhagen, Taipei, Bangkok and Washington D.C. consist of two tunnels of identical construction (known as 'twin-tunnels'): one for the outbound direction and one for the inbound direction. In most cases these tunnels are located side-by-side, but occasionally, such as in the case of the Chungho Line in Taipei the tunnels are piggy-back, with one on top of the other [8]. Whilst it is possible to incorporate both the inbound and outbound railway lines into one large tunnel, this would result in a larger volume of excavated material and more involved construction techniques than that required for twin tunnels [9]. To date, the only evidence in the literature of a dynamic model which accounts for the

vibration interaction between neighbouring tunnels is the coupled wavenumber FE-BE model [10]. This model compares the response of a large, single-bore double-track tunnel and a pair of single twin-track tunnels embedded in an elastic halfspace. All other models for underground railway vibration ignore the effect of the twin tunnel.

A novel solution for two parallel tunnels of circular cross-section embedded at varying depths in a homogeneous, elastic soil fullspace has been developed by the authors [11]. The two tunnels are modelled as thin-walled cylinders, and known excitation forces are applied at each of the tunnel inverts. The known forces acting at each tunnel invert can be written as the sum of two force contributions: those forces acting on the invert of a single tunnel and those forces induced by the neighbouring tunnel. Both of these force contributions can be determined by considering a one-tunnel system. The forces induced by the neighbouring tunnel are calculated using the equations for the one-tunnel model to determine the tractions around the virtual surface of the second tunnel. These tractions are partitioned into symmetric and asymmetric contributions which are determined using Fourier series decomposition. This relationship between the forces acting on the first tunnel and the tractions produced on the second tunnel is combined with the forces acting on the tunnel invert of a single tunnel to produce a set of simultaneous equations. These equations are solved to calculate the relative force contributions then the principle of superposition is used to combine the displacement fields from each of the tunnels in the two-tunnel system. Methods exist for adapting this model to obtain vibration predictions in a homogeneous halfspace or layered halfspace without significant loss of computational efficiency [12].

Fig 3(a) & (b) shows the vertical displacement field (presented in dB ref. 1m) produced at 60Hz by a unit vertical point force applied to a tunnel invert calculated using a single tunnel model and a side-by-side twin-tunnel model. The difference between these two displacement fields is shown in Fig 3(c), the insertion gain, which represents the change in dB at any given location when the twin-tunnel model is employed. In this figure, in the region above the right-hand tunnel the twin-tunnel model predicts vibration levels in the order of 20dB greater than those predicted by the single-tunnel model.

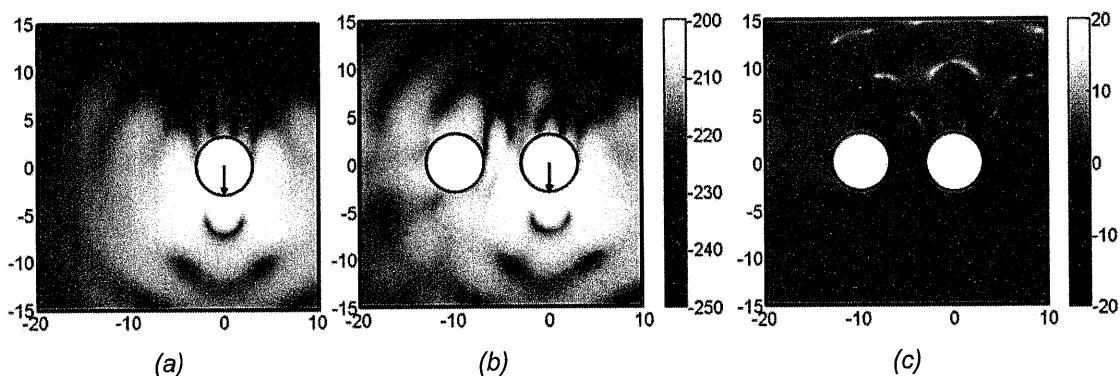


Figure 3. The vertical displacement field (dB) calculated using (a) a single-tunnel model, and (b) a twin-tunnel model. The difference between these two displacement fields is shown in (c), the insertion gain.

Further investigations of the vibration fields produced over a range of frequencies, tunnel orientations and tunnel geometries indicates that the interactions between neighbouring tunnels are highly significant, at times in the order of 20dB. The vibration response is observed to be highly dependent on frequency, position, material properties and the underground environment. This demonstrates that a high degree of inaccuracy exists in any surface vibration prediction model that includes only one of the two tunnels. It is recommended that all future models predicting vibration levels from underground railways include the interactions between neighbouring tunnels.

4 SOIL INHOMOGENEITY

The dynamic properties of soils are difficult to measure and can vary significantly over the area of interest. Experimental testing shows that dynamic soil properties can easily vary $\pm 10\%$ for test sites just meters apart [13]. This suggests that assuming homogeneous soil properties may not be sufficient for accurate representation of wave propagation from underground railways.

To investigate these issues a semi-analytical model was developed which employs hyperelements to mesh the region of interest [14,15]. Hyperelements can be finite or infinite in size as they utilize the analytical solution for wave propagating horizontally while assuming vertical displacements vary linearly through the thickness of the element; this formulation negates aspect ratio restrictions common to FE modeling thus a halfspace can be accurately modelled using relatively few elements. Vertical discretization must be sufficient to capture the highest frequency of interest; five elements per half shear-wavelength is appropriate. Two situations have been considered using this model where the homogeneous soil assumption may not be valid: inhomogeneous soil regions and inclined soil layers.

Predictive models often assume the soil region is a homogeneous halfspace to simplify the modeling process. However coefficients of variation (COV = standard deviation divided by the mean) for soil elastic modulus have been reported to vary from 15-65% [13]. In a previous paper the authors simulated ground vibrations in an elastic halfspace due to an underground railway [14]. The elastic modulus of the soil was given a COV of 25% using a 2D exponential covariance kernel. This resulted in the 95% confidence interval shown in Fig 4. These predictions suggest a relatively small amount of uncertainty in the soil properties can result in a 2.5dB (rms, ref m/s) range in the surface vibration results.

A different sort of soil homogeneity assumption is commonly used when considering layered soils. Soil layering is generally assumed to be horizontal since this minimizes modelling effort and because it is difficult to experimentally determine the exact lithology of an area. However, in reality layers are often inclined by up to 10° relative to each other or the surface. The hyperelement model is used to simulate ground vibrations in a layered halfspace due to a buried tunnel where the layer is inclined relative to the surface horizon [15]. The results from the predictive model (Fig 4) shows that neglecting to account for the soil layer inclination can result in peak-particle velocity (PPV) uncertainty of 10dB or more.

Research into the effect of soil inhomogeneity is continuing, but these preliminary results suggest it is necessary to include at least a ± 5 dB uncertainty range when simulating ground vibration using simplified soil models.

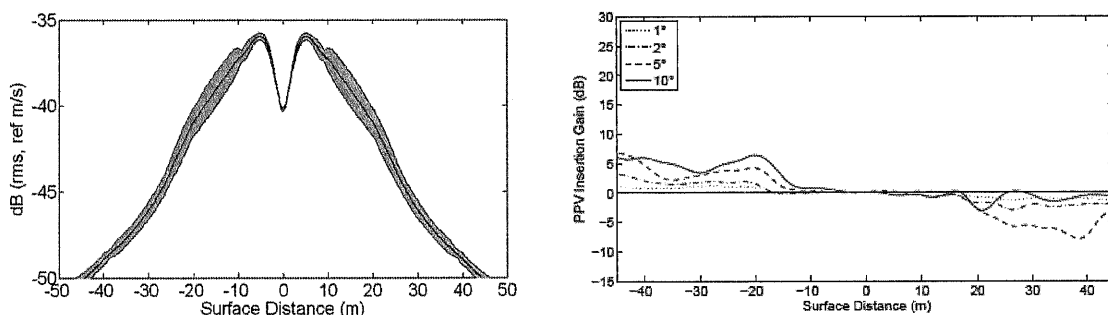


Figure 4. Effects of including soil inhomogeneity: (left) 95% confidence interval for surface rms particle velocity due to an underground railway using soil elastic modulus COV of 25%; (right) surface peak-particle velocity insertion gain for inclined soil layers compared to a horizontal layer.

5 IRREGULAR CONTACT AT THE SOIL-TUNNEL INTERFACE

Another simplifying assumption frequently made is that the soil is in continuous contact with the tunnel (ie. no voids or gaps at the tunnel-soil interface). Subsidence and frost-heave have been shown to cause significant tunnel movement [16, 17]; under such large-scale motion it is likely that a void will form over a section of the tunnel, disrupting the perfect bond at the tunnel-soil interface. Voids may also develop during construction of new buildings in close proximity to the underground tunnels due to pile-driving, excavation, landscaping, etc. The extent of voidage is difficult, if not impossible, to quantify but the existence of voids is not in doubt.

Development of a fully analytical model to investigate the effect of a void at the tunnel-soil interface on surface vibration was deemed intractable thus a semi-analytical approach is adopted. The tunnel-soil interface is discretized and the PiP method (described in Section 2) is used to determine the discrete transfer functions for both the tunnel and the surrounding soil nodes. The transfer matrices are coupled using continuity and equilibrium conditions; the void is simulated by uncoupling the appropriate tunnel and soil nodes, inhibiting the transfer of forces between the two subsystems over a finite patch. Empirically derived rail-roughness is used to compute the random-process force input to the tunnel invert. Full details on the modelling process can be found in Ref [18].

The effect of a void on near-field and far-field rms particle velocity is shown below for the example of a 2m x 90° void centered on the top of the tunnel; a schematic showing the tunnel, observation plane, load location and void placement is presented on the right of Fig 5. For this example the tunnel is 3.25m in radius and buried in London Clay. At a height of 5m there is significant variation between the voided and void-free models (upper curves in Fig. 5), with peak differences of (+4.6, -1.4)dB over a 8m span. The discrepancy between the models is less pronounced at a height of 20m, (+1.2, -0.9)dB, but is still noticeable over a 15m span.

The findings from this study suggest that the uncertainty associated with assuming a perfect bond at the tunnel-soil interface in an area with known voidage can reasonably reach +/-5dB and thus should be accounted for in the design process.

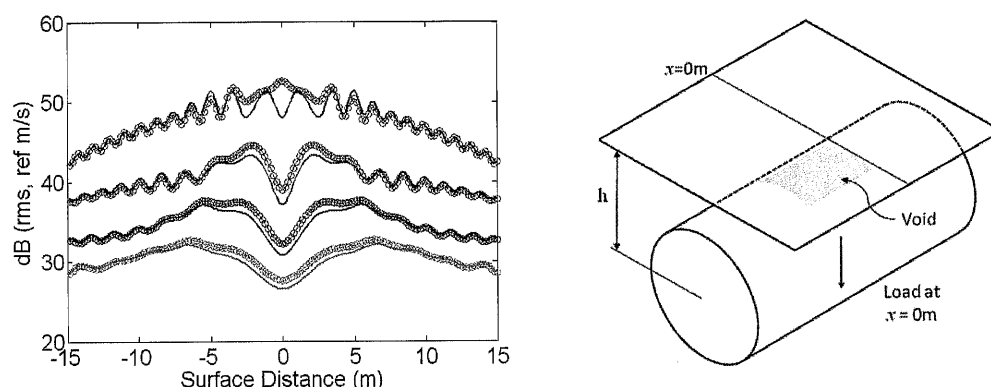


Figure 5. Particle velocity at $x=0m$ in dB(rms, ref 1 m/s) for void-free model (solid curve) and model with 2m x 90° void (circles); model schematic on right. Upper curve: $h=5m$; mid-upper curve: $h=10m$; mid-lower curve: $h=15m$; lower curve: $h=20m$

6 PILED FOUNDATIONS

A final commonly-made assumption is to ignore the effects of nearby piled foundations. Due to the close proximity of piles and underground railways, it is expected that piled foundations will have a significant influence on the transmission of ground-borne vibration into buildings.

Whilst many well-established models (for example [19-21]) exist for the dynamic behaviour of piled foundations, either due to an incident seismic wavefield or an inertial loading, there is no evidence in the literature for a comprehensive model of piled foundations subject to a wavefield generated by an underground railway. It is thought that the lack of such a model is due to the complexity of modelling such a system, particularly the large number of elements required (and subsequent increase in computation time) when using numerical finite element or boundary element models.

An efficient semi-analytical model of pile groups subject to vibration from an underground railway has been formulated as a useful design tool [22]. This model is an uncoupled source-receiver model, as the coupling of subsystems is used to join one subsystem, the underground railway model, to the other subsystem, the pile group model. The uncoupled source-receiver model gives a good approximation of the dynamic behaviour of the system when the distance separating the source and receiver is large compared with the longest wavelength in the soil. The incident wavefield produced by the underground railway is generated using the PiP model, and the displacements and tractions produced at the pile locations by this incident wavefield is used as the input excitation for the pile group model.

The response of the pile group to excitation from an underground railway is determined by calculating the response of each individual pile to the incident wavefield, and accounting for pile-soil-pile interaction (PSPI) by multiplying the response of each pile by the relevant interaction factor. The interaction factor is a measure of how the excitation of one pile affects the pile-head displacement of a neighbouring pile. Interaction factors are determined using the coupling of subsystems, where the displacements and tractions produced by the excitation of one pile are used as the input excitation for the neighbouring pile.

To illustrate how the presence of piled foundations can affect surface vibration predictions, Fig 6 compares the vertical vibration levels as a function of frequency at the pile-head for a four-pile foundation, and at the corresponding location when there is no foundation present. It can be seen that in this case, the inaccuracy inherent in a prediction that ignores the presence of piled foundations is highly dependent on frequency, and can range from -15dB to +8dB. Once again, this represents a significant margin for a prediction model of vibration from underground railways.

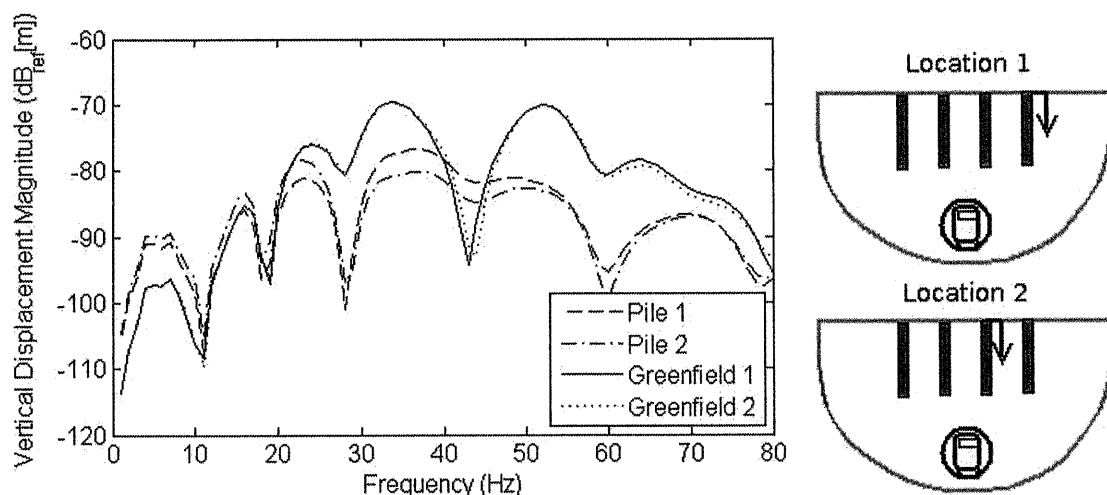


Figure 6. Surface vibration levels as a function of frequency for a four-pile foundation, with comparison to the greenfield site.

7 CONCLUSIONS

Accurate modelling of ground vibration due to underground railways is of great interest for designers who must adhere to the noise and vibration standards for developments near the subway

lines. Simplifying modelling assumptions are often used as the number of parameters involved in describing the underground environment makes formulation of a comprehensive model intractable; these simplifying assumptions introduce inaccuracy and uncertainty into the predictions. This paper quantifies the effect of five commonly-disregarded aspects of the underground railway environment: track with discontinuous slabs, the second (twin) tunnel, soil inhomogeneity, irregular contact at the tunnel-soil interface and the presence of nearby piled foundations. Results suggest that accounting each of these simplifying assumptions can result in predictions which vary from the simplified cases by at least ± 5 dB and potentially up to 20 dB. This is a significant level of uncertainty and should be considered when estimating the predictive accuracy of numerical models using simplifying assumptions.

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