BENDING WAVE CONTROL IN BEAMS USING A TUNED VIBRATION ABSORBER

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1 INTRODUCTION

Tuned Vibration Absorbers (TVAs) have been used in many applications since their invention nearly a century ago by Frahm in 1911. These devices are either tuned to a problematic natural frequency of the host structure or to a troublesome excitation frequency. The use of a TVA to control flexural waves on infinite Euler-Bernoulli beam has been the subject of much research. Mead[1] has described how tuning an undamped absorber to the excitation frequency would pin the beam at the absorber's position. This will only reduce the amplitude of transmitted flexural wave by 3dB. Further control of a flexural propagating wave in a beam has been discussed by Clark[2], and Brennan[3] who considered the TVA as point translational impedance. The purpose of this paper is to expand on previous work by investigating the behaviour of the TVA if located in either the nearfield or farfield of a harmonic point force. Both the transmitted power and the net power reflected upstream from the disturbance are considered. The tuning parameters are determined and expressions for the tuning frequency for optimal energy isolation are found.

The paper is divided into five sections. Following this section, the dynamic behaviour of the TVA in controlling flexural waves is discussed. The power transmitted downstream of the TVA is given in terms of four independent parameters: the ratio of the excitation frequency to the TVA working frequency, the loss factor, the mass of the TVA compared to the mass in one wavelength of the beam and the distance between the TVA and the source of disturbance. The optimum tuning parameters of the absorber are discussed in section 3. Section 4 presents the experimental validation of the theoretical predictions. Finally, general conclusions are given.

2 WAVE REFLECTION AND TRANSMISSION OF THE TVA

2.1 Wave Model of the TVA

Consider an infinite beam with a TVA modelled as a single degree of freedom system (SDOF). The absorber is mounted at a distance l from the applied force $F\exp(i\omega t)$ as shown in Figure 1. Here m_a represents the mass of the TVA, while k_a and η represents respectively its stiffness and loss factor. The complex wave amplitudes represent positive- and negative-going propagating waves (a^+, a^-) injected by the point force, positive- and negative-going propagating waves (b^+, b^-) at a distance l from the point force and the transmitted propagating wave (c^+); the subscript N refers to evanescent waves.

The net wave propagating upstream ($a^- + b^-\exp(-ikl)$) is the superposition of the wave reflected from the TVA and the upstream wave generated by the disturbance. The waves incident on the absorber are[4]

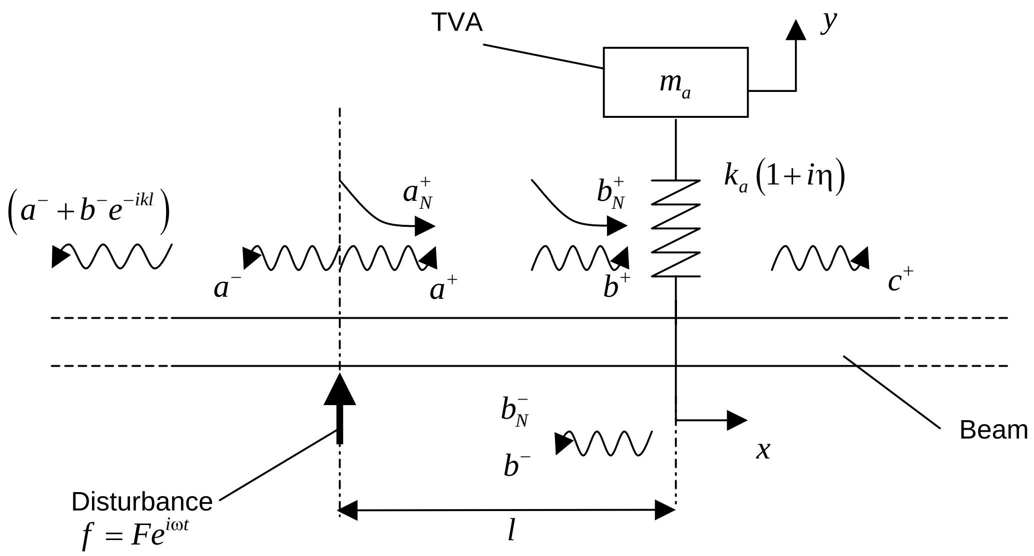


Figure 1. TVA in nearfield of point disturbance

$$\begin{Bmatrix} b^+ \\ b_N^+ \end{Bmatrix} = \begin{bmatrix} e^{-ikl} & 0 \\ 0 & e^{-kl} \end{bmatrix} \begin{Bmatrix} a^+ \\ a_N^+ \end{Bmatrix}; \quad \begin{Bmatrix} a^+ \\ a_N^+ \end{Bmatrix} = \frac{-F}{4EIk^3} \begin{Bmatrix} i \\ 1 \end{Bmatrix}, \qquad \text{(1a, b)}$$

where EI and k are the flexural rigidity and wavenumber of the beam.

The transmitted propagating wave c^+ can be found by considering the continuity and equilibrium equations of the absorber and the beam at the point where it is attached. It is given by

$$c^+ = \frac{-iFe^{-ikl}}{4EIk^3} \left[\frac{\Omega^2 - (1+i\eta)\left(1+\gamma\Omega^{1/2}\left(1-e^{-kl(1-i)}\right)\right)}{\Omega^2 - (1+i\eta)\left(1+\gamma\Omega^{1/2}(1+i)\right)} \right], \qquad (2)$$

where the dimensionless parameters

$$\gamma = \frac{\pi m_a}{2\rho A \lambda_a}; \quad \Omega = \frac{\omega}{\omega_a}, \qquad \text{(3a, b)}$$

represent, respectively, the mass ratio γ, at the absorber working frequency $\omega_a = \sqrt{k_a / m_a}$ (the frequency at which the impedance of the absorber is maximum) and the frequency ratio Ω, which is the ratio of the excitation frequency ω to the working frequency of the absorber ω_a. The parameter γ is ratio of the mass of the TVA to the mass in a length $2\lambda_a / \pi$ of the beam (λ_a is the wavelength at frequency ω_a).

The transmission ratio τ_t of the TVA is now defined to be the ratio of the power transmitted to that which would be transmitted if the absorber were absent, i.e. $\tau_t = \left|c^+ / a^+\right|^2$ and it is given by

$$\tau_t = \left| \frac{\Omega^2 - (1+i\eta)\left(1+\gamma\Omega^{1/2}\left(1-e^{-kl(1-i)}\right)\right)}{\Omega^2 - (1+i\eta)\left(1+\gamma\Omega^{1/2}(1+i)\right)} \right|^2 . \tag{4}$$

Similarly, the net wave propagating upstream ($a^- + b^-\exp(-ikl)$) is given by

$$(a^- + b^- e^{-ikl}) = \frac{-iF}{4EIk^3}\left[1 + ie^{-2ikl}\left[\frac{\gamma\Omega^{1/2}(1+i\eta)\left(1-ie^{-kl(1-i)}\right)}{\Omega^2 - (1+i\eta)\left(1+\gamma\Omega^{1/2}(1+i)\right)}\right]\right]. \tag{5}$$

The ratio of the power reflected upstream to that which would propagate upstream if the absorber were absent is defined as the reflection ratio τ_r, i.e. $\tau_r = \left|\left(a^- + b^-\exp(-ikl)\right)/a^-\right|^2$. It is given by

$$\tau_r = \left|1 + ie^{-2ikl}\left(\frac{\gamma\Omega^{1/2}(1+i\eta)\left(1-ie^{-kl(1-i)}\right)}{\Omega^2 - (1+i\eta)\left(1+\gamma\Omega^{1/2}(1+i)\right)}\right)\right|^2 . \tag{6}$$

2.2 Numerical Examples

The transmission ratio is shown in Figure 2 as a function of Ω for two different mass ratios and various locations l/λ_a of the TVA. The absorber is assumed to be undamped. Figure 2a shows that $\tau_t = 0$ at a certain frequency Ω_t for each location; this tuned frequency is defined as the frequency ratio at which the minimum power is transmitted to the right of the TVA and it equals 1 when $l/\lambda_a = 0$. In all cases, the transmission ratio asymptotes to 1 as Ω tends to either 0 or ∞. Increasing the distance of the TVA from the point force increases the tuned frequency until the

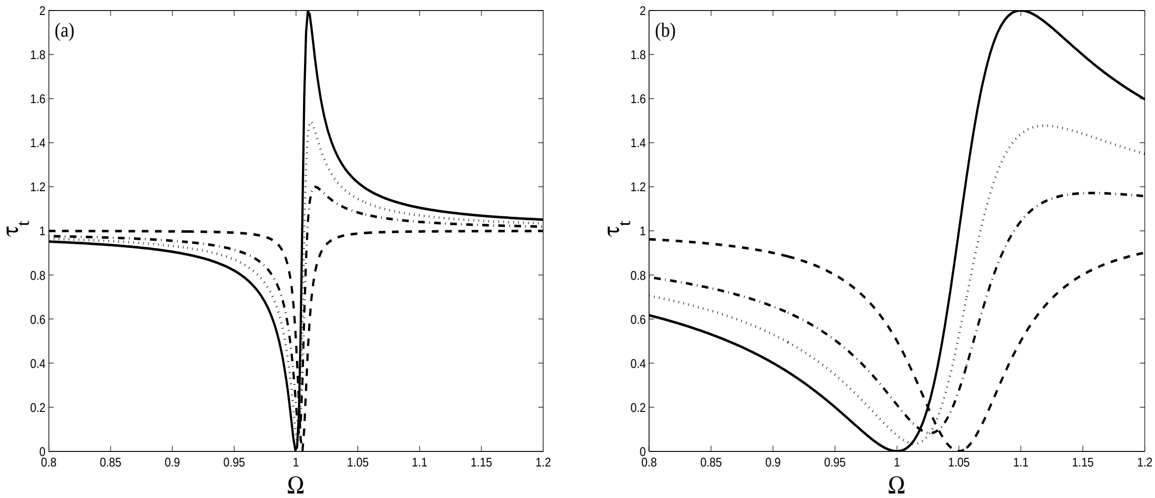
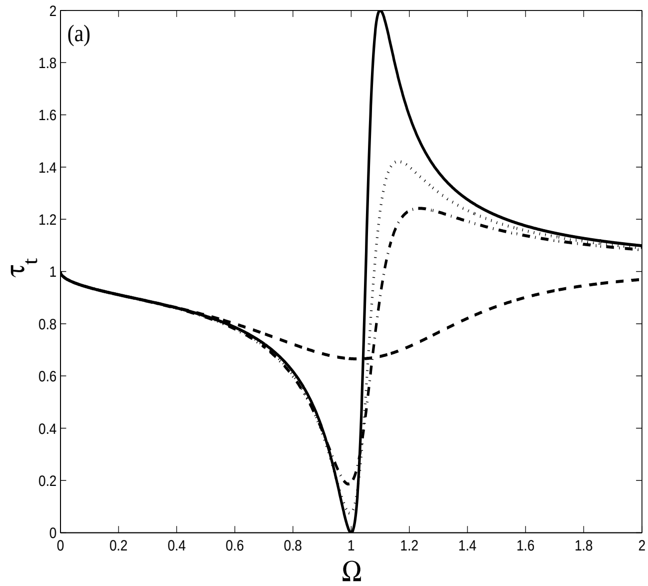


Figure 2. The transmission ratio as a function of Ω for various l/λ_a, $\eta = 0$: (a) $\gamma = 0.01$; (b) $\gamma = 0.1$;
______ $l/\lambda_a = 0$; .... $l/\lambda_a = 0.05$; _._._._ $l/\lambda_a = 0.1$; ------ $l/\lambda_a >> 1$.

incident nearfield wave becomes insignificant and the tuned frequency becomes independent of the distance l .

The absorber impedance is infinite at $\Omega_t = 1$ if $\eta = 0$, and the beam is effectively pinned when $l/\lambda_a = 0$ so that no power is input. As l increases, some power is transmitted because of rotation of the beam. Moreover, the maximum transmission ratio is 2 for the undamped TVA due to impedance matching between the TVA and the reactive part of the impedance of the beam. This maximum transmission decreases as l increases. The maximum value of τ_t reaches unity when the TVA is located in the farfield. Increasing the mass ratio increases the tuned frequency ratio as well as the width of the stop-band in which less than half the incident power is transmitted (see Figure 2b).



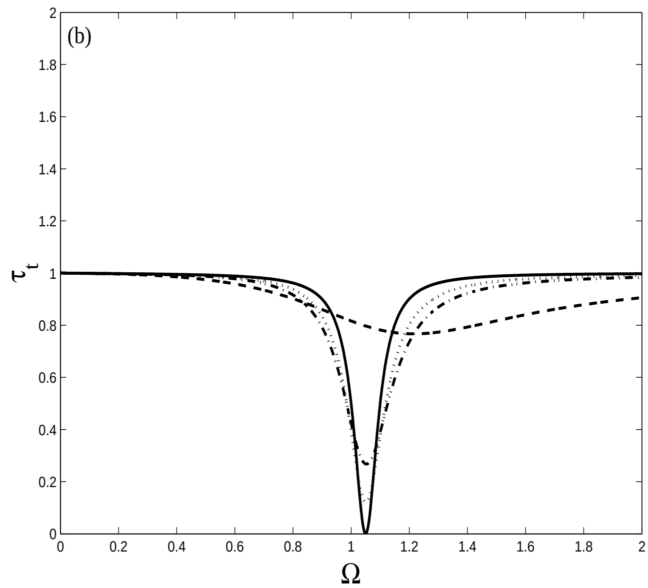


Figure 3. The effect of the damping on the transmission ratio, $\gamma = 0.1$: (a) $l/\lambda_a = 0$; (b) $l/\lambda_a >> 1$.
——— $\eta = 0$; ········ $\eta = 0.05$; -·-·-· $\eta = 0.1$; ----- $\eta = 0.8$.

Figure 3 shows the effects of damping in the TVA on the transmission ratio if the absorber is located in either the nearfield or the farfield of the point disturbance. It is shown that increasing damping reduces the degree of attenuation of τ_t at the tuned frequency, while also reducing the maximum power transmission.

The reflection ratio τ_r is highly dependent on the location of the TVA as shown in Figure 4. Minimum reflection occurs at intervals of approximately $0.5\lambda_a$ while other locations may cause high reflection ratio (maximum $\tau_r = 4$ as will be discussed later on). It is also seen that increasing the mass ratio increases τ_r at the tuned frequency as the amplitude of the reflected wave increases.

3 TUNING THE TVA

There is a strong desire to find the optimum tuning parameters of the TVA. It is desirable to minimise the power transmitted downstream of the point disturbance and possibly also that reflected upstream. Expressions from which various optimum tuning parameters can be found are given in this section.

3.1 Optimum Tuning Parameters

The transmission ratio is zero for an undamped TVA mounted at the source of disturbance when $\Omega = \Omega_t = 1$. In general, the tuned frequency must be found by a numerical solution to equation (4).

The tuned frequency for an undamped TVA located in the farfield satisfies

$$\Omega_t^2 - \gamma\Omega_t^{1/2} - 1 = 0 \,, \tag{7}$$

and the maximum transmission here occurs when

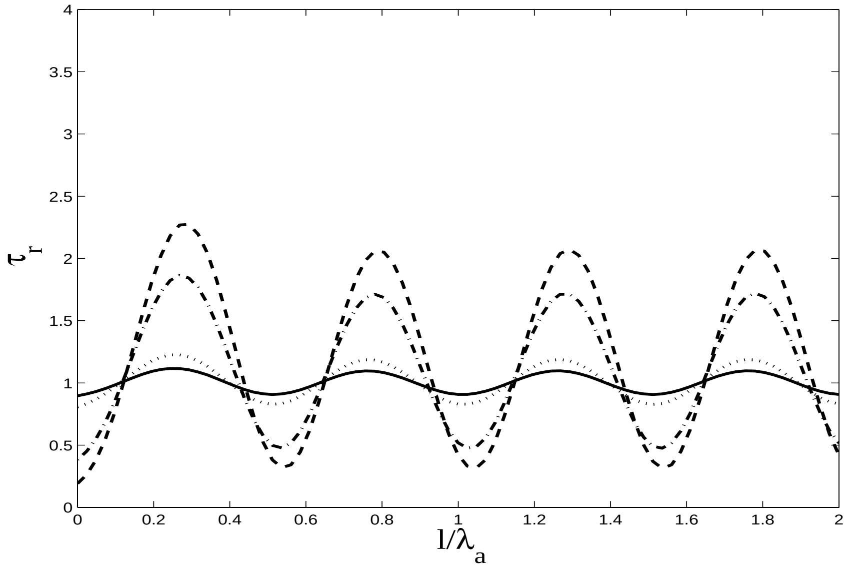


Figure 4. Reflection ratio as function of absorber location at frequency ratio $\Omega_t = 1$, $\eta = 0.1$.
——— $\gamma = 0.005$; ········ $\gamma = 0.01$; -·-·-· $\gamma = 0.05$; ------ $\gamma = 0.1$.

$$\Omega_n^2 - 2\gamma\Omega_n^{1/2} - 1 = 0 \,, \tag{8}$$

Therefore, increasing γ will also increases Ω_t as well as Ω_n.

There is no simple analytical solution for the tuned frequency, but approximations can be made when the undamped TVA is located in either the nearfield or the farfield of the point disturbance. When TVA is located in the solution to the farfield equation (7) can be approximated by

$$\begin{aligned} \Omega_t &\approx (1+\gamma)^{1/2}; \quad \gamma << 1 \\ \Omega_t &\approx \quad \gamma^{2/3} \quad ; \quad \gamma >> 1 \end{aligned} \tag{9a, b}$$

The tuned frequency ratio $\Omega_t \geq 1$ and depends on the mass ratio and the location of the absorber if nearfield waves are significant. If $l/\lambda_a << 1$, then

$$\Omega_t \approx 1 + \pi\gamma l/\lambda_a . \tag{10}$$

Figure 5 shows the variation of the tuned frequency with l/λ_a. It asymptotes to the solution given by equation (10) as $l \rightarrow 0$. When the nearfield waves become insignificant (l exceeds approximately $0.5\lambda_a$), the position of the TVA no longer affects Ω_t which then asymptotes to the constant given by equation (9). It is clear that increasing γ increases Ω_t for a given location of TVA.

The net wave propagating upstream can be written as

$$\left(a^- + b^- e^{-ikl}\right) = a^-\left(1 + re^{-2ikl}\right), \tag{11}$$

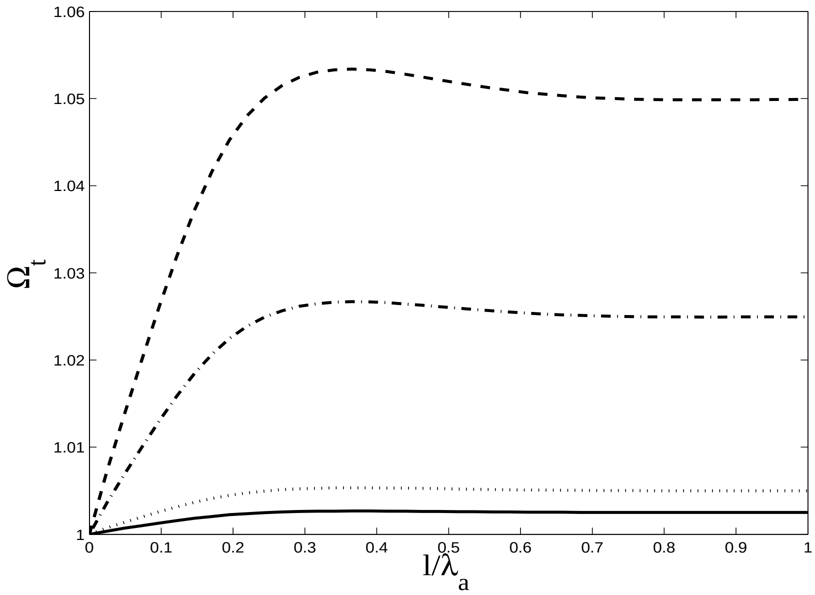


Figure 5. Optimum tuning frequency as a function of l/λ_a, $\eta = 0$:
——— $\gamma = 0.005$; ········· $\gamma = 0.01$; -·-·-· $\gamma = 0.05$; ----- $\gamma = 0.1$;

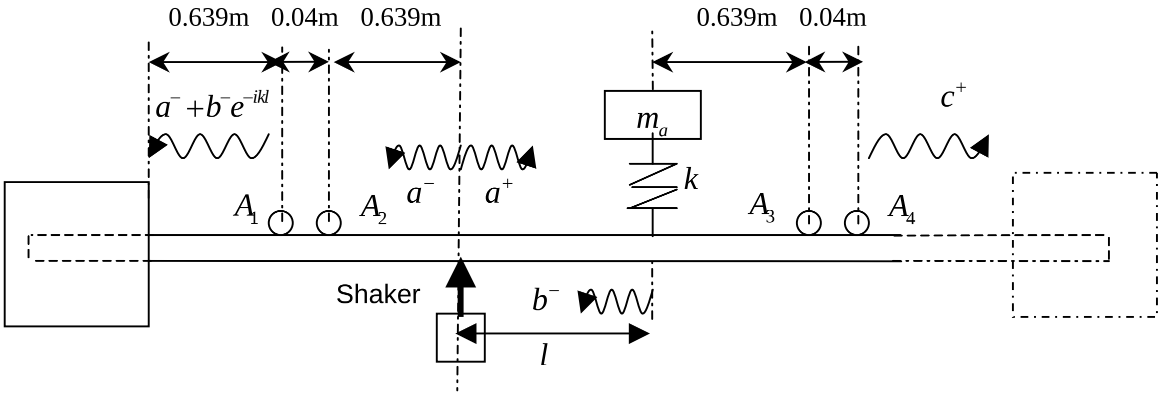


Figure 6. Experimental set-up.

where $r = b^- / a^-$ is the reflection coefficient of the TVA. If the absorber is undamped and optimally tuned then $r = \exp(-i\phi)$ where ϕ is the phase of the reflection coefficient. Therefore equation (11) becomes

$$\left(a^- + b^- e^{-ikl}\right) = a^- \left(1 + e^{-i(2kl+\phi)}\right). \tag{12}$$

If the total phase shift $2kl + \phi = 2n\pi$, where n is any integer, then the net amplitude of the superimposed waves is $2a^-$, and the maximum reflection ratio $\tau_r = 4$. In contrast, if $2kl + \phi = (2n\pi - \pi)$, then the waves interfere destructively and the amplitude of the upstream-going wave is zero. Hence, the transmitted and reflected powers can be completely suppressed at a single frequency using a single undamped TVA.

4 EXPERIMENTAL RESULTS

Figure 6 shows the experimental set-up used to validate the theoretical and numerical approaches discussed previously. The TVA was made of a steel beam ($1.7\text{mm} \times 20.5\text{mm} \times 80.4\text{mm}$) with blocks of brass ($10.3\text{mm} \times 10.2\text{mm} \times 20\text{mm}$) attached at each end. The absorber was mounted at its centre to a $6.4\text{mm} \times 50.6\text{mm} \times 5630\text{mm}$ steel beam suspended at four points along its length. The ends of the beam were embedded in sandboxes to reduce reflections. The mass ratio of the TVA was γ = 0.07, where the effective mass of the absorber was m_a = 45.7g. The absorber also contributed to a mass m_2 = 21.9g on the beam. The working frequency f_a = 343Hz. The beam was excited by a Ling V201 shaker with band limited random noise over a frequency range 50-800Hz. The propagating wave amplitudes were estimated using the wave decomposition approach.

A comparison between the experimental and numerical results is shown in Figure 7. This illustrates the effects of the location of the TVA. There is substantial attenuation in τ_t around the tuned frequency in the four chosen locations while τ_r is strongly dependent on the location of the TVA. In Figure 7a l/λ_a = 0.34, so that the TVA is well within the nearfield of disturbance. It is noticeable that there is high reflection ratio around the tuned frequency.

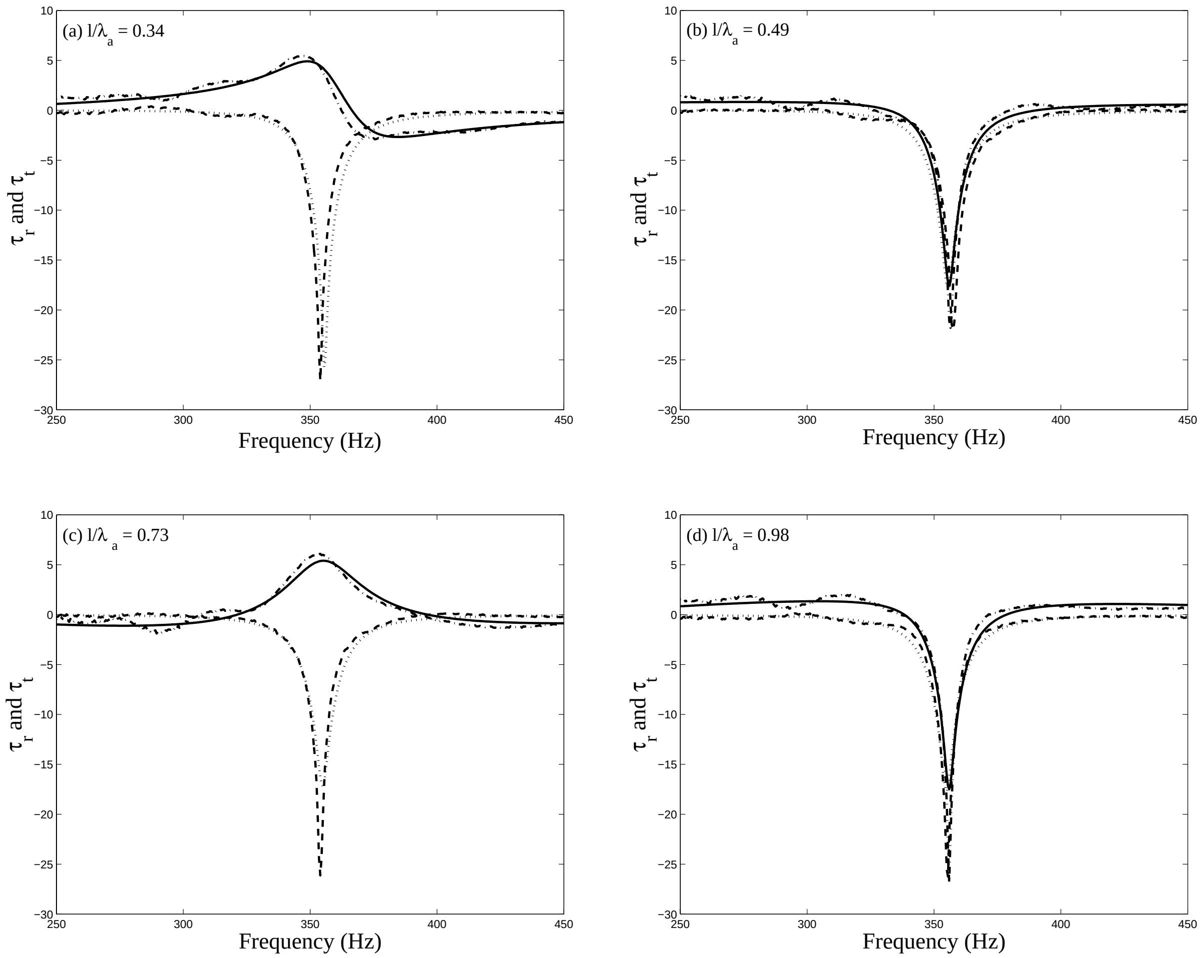


Figure 7. Power reflection and transmission:
——— τ_r , theory; -.-.-.- τ_r , experiment; .... τ_t ,theory; ------ τ_t ,experiment.

However, in Figure 7b l/λ_a = 0.49, so that the absorber is still in the nearfield. Now there is also a clear notch in τ_r , due to the interference of the two wave components propagating upstream, namely a^- and $b^- \exp(-ikl)$. Locating the absorber in the farfield at l/λ_a = 0.73 has increased τ_r by approximately 5dB as shown in Figure 7c. Nevertheless, the tuned frequency here has slightly increased to 356Hz. Figure 7d shows attenuation in both τ_r and τ_t with the tuned frequency f_t = 356Hz when the optimum location is chosen (l/λ_a = 0.98).

5 CONCLUDING REMARKS

The control of flexural waves in a beam structure using a tuned vibration absorber is discussed in this paper. Upstream and downstream propagating waves were suppressed using a single absorber. Analytical and numerical results were presented. The optimum tuning parameters, which ensure the minimum power transmission, have been found numerically. The transmitted and reflected powers depend on the mass ratio, frequency, loss factor and the distance between the absorber and the disturbance. The location of the absorber plays an important role in controlling the upstream waves. Agreement between numerical and experimental results is generally good.

6 REFERENCES

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