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SURFACE MOBILITY FOR A UNIFORM FORCE OVER A RECTANGULAR EXCITATION AREA OF AN INFINITE THIN PLATE

H M Williamson & J Dai

Acoustics and Vibration Centre, School of Aerospace & Mechanical Engineering, University College, The University of New South Wales, Australian Defence Force Academy, Canberra, ACT Australia

1. INTRODUCTION

In many complex structures, machinery mounted on plate-like supports often generate high levels of vibration and noise. The design and selection of isolators to attenuate this noise and vibration should include consideration of the mobility of the supporting structure as well as the isolator characteristics and characteristics of the machine. Previously, most of investigations have dealt only with point mobility, that is mobility with respect to a force applied over a very small area. In practice, machines are connected with their supports over a considerable area. For an accurate model of vibration isolation, the influences of the contact area between machines and supports must be considered. The concept of surface mobility, defined below, will be used to extimate mobility for rectangular areas of contact. Hammer and Petersson [1] developed the concept of surface mobility and applied it to a one-dimensional strip of contact. Contact areas of circular shape were considered by Norwood, Williamson and Zhao [2]. In practical situations, many contact areas are rectangular and these are considered here. As with previous studies, the receiving structure is assumed to be an infinite thin plate. The exciting force is assumed to be a uniform conphase force distribution.

2. DEFINITION OF SURFACE MOBILITY

Generally, mobility is defined as:

$$M=V/F \qquad (1)$$

where F and V are the complex amplitudes of the exciting force and resulting velocity respectively. In most studies, mobility relating to point excitation is used, however, when the excited region covers a significant area, some of changes should be considered. At the interface of a contact area S between a machine and its support there exist a stress distribution p(x,y) and a velocity distribution v(x,y). For a linear and passive bending system, the definition of surface mobility based on complex power can be obtained from [1] as

$$M=2Q/|F|^2 \tag{2}$$

where |F| is the net force amplitude supplied by the source system on the receiver and Q is the complex power transmitted.

$$Q=\frac{1}{2}\int_S p(x,y)^* v(x,y)ds \tag{3}$$

The power injected into the plate is the real part of the instantaneous complex power, hence

$$\mathrm{Re}(M)=2\,\mathrm{Re}(Q)/|F|^2 \tag{4}$$

3. SURFACE MOBILITY OF RECTANGULAR EXCITATION AREA

To calculate complex power directly from equation (3) above is difficult because the velocity and force distributions must be known over the contact area. The approach taken here is to calculate the input power from the total power flow in the far field. Power is integrated around a closed surface located at a distance R from the center of the excitation area in order to obtain the total power. The velocity distribution in the far field caused by distributed forces acting over a rectangular area is calculated first. The excitation region, area S, length l and width w is shown in Figure 1.

According to [3], the resulting velocity dv(x,y) in the z direction perpendicular to the plate at point (x,y) in the far field caused by an infinitesmal force at (x_o,y_o), pressure $p(x_o,y_o)$ over the contact area $dx_o dy_o$ is

$$dv(x,y)=M_0 p(x_0,y_0)\sqrt{\frac{2}{\pi kr}}e^{-j(kr-\pi/4)}dx_0 dy_0 \tag{5}$$

Where r is the distance from (x_0,y_0) to (x,y); k is the bending wavenumber; and M_O is the ordinary point mobility of an infinite, homogeneous thin plate. For velocity in the far field far, the factor r in the denominator is approximately equal to $R=\sqrt{x^2+y^2}$ the distance from the centre of the excitation region and for a uniform force distribution, $p(x_0,y_0)$ can be replaced by a constant p_0. Integrating over the whole of the excitation region, the velocity at point (x,y) is then given by

$$v(x,y)=\frac{\sqrt{2}e^{j\pi/4}M_0 p_0}{\sqrt{\pi kR}}\iint_S e^{-jkr}dx_0 dy_0 \tag{6}$$

To simplify the calculation, r' is used to approximate r, See Figure 2.

$$r\approx r'=R-r''=R-\frac{xx_0+yy_0}{R} \tag{7}$$

This results in the following expression for far field velocity.

$$v(x,y)=\frac{\sqrt{2}e^{j\pi/4}M_0 p_0}{\sqrt{\pi kR}}(lw)\,e^{-jkR}\,\frac{\sin(\frac{kl}{2}\frac{x}{R})\sin(\frac{kw}{2}\frac{y}{R})}{(\frac{kl}{2}\frac{x}{R})(\frac{kw}{2}\frac{y}{R})} \tag{8}$$

Using the method outlined in [4,5], the real part of the total power passing through the surrounding cylinder surface, radius R, was calculated and by conservation of energy is equal to Re(Q), the power injected into the plate. Thus by equation (4), the real part of the surface mobility over a rectangular excitation area, in polar coordinates, is as follows.

$$\mathrm{Re}(M)=\frac{M_0}{2\pi}\int_0^{2\pi}\left[\frac{\sin((\frac{kl}{2})\cos(\theta))}{(\frac{kl}{2})\cos(\theta)}\frac{\sin((\frac{kw}{2})\sin(\theta))}{(\frac{kw}{2})\sin(\theta)}\right]^2 d\theta \qquad (9)$$

Numerical solutions to the above equation can readily were calculated for various values of l and w. Note that when kl/2 and kw/2 trend to zero, the value of the above integral trends to 2π, showing, as expected, that the surface mobility trends to the point mobility fro small areas.

4. RESULTS AND DISCUSSION

Numerical results from equation (9) are presented in Figure 3 in non-dimensional form. Width, *w*, has been chosen as the representative length dimension for rectangular contact areas, with the aspect ratio defined as $\eta = l/w$.

For a given plate, wave number increases with the exciting frequency of the input force, hence Figure 3 can be interpreted either of two ways:

- Non-dimensional mobility versus width for a constant excitation frequency (k or $1/\lambda$) for various aspect ratios, η, or
- Non-dimensional mobility versus frequency (k or $1/\lambda$) for contact regions of constant width, w, and varying aspect ratios, η.

Considering the first interpretation, it can be seen in Figure 3, that mobility generally decreases with increasing width (or area) of the rectangle. This general increase has dips (local minima) at intervals which are approximate multiples of π on the horizontal axis, that is when the width approximately equals a multiple of the wavelength. The dips become closer to multiples of π as the aspect ratio, η, increases. Considering the second interpretation, it can be seen that mobility generally decreases with increasing exciting frequency or wave number.

Of further interest is to explore the effect of contact region shape, circular, square ($\eta = 1$) or rectangular ($\eta > 1$) for contact regions of constant area. This is done in Figure 4 by plotting normalised mobility versus ka, using the circle as the basis for the plot and plotting the normalised mobility of rectangles at equivalent area values of ka. Figure 4 shows a general decrease in mobility as either the surface dimension, a or w, or the frequency of excitation increases. On this equal area basis, a square (η=1) and a circle have similar mobility characteristics except that the circular contact region exhibits deeper dips where the wavelength approximates the circle diameter.

5. CONCLUSIONS

The surface mobility of an infinite, thin plate excited in flexure by a uniform conphase force distributed over a rectangular area has been calculated. The results show significant differences between the mobilities of point and surface contact.

1. In general, surface mobility decreases in a significant manner with increasing contact area. This implies that the input power from a source of given force can be minimised by increasing the contact area.
2. Surface mobility also generally decreases with excitation frequency .
3. When plotted against Helmholtz Number, Figures 3 & 4, surface mobility exhibits dips or local minima. These dips correspond to circumstances where the width or diameter of the contact region is approximately equal to the wavelength of the bending waves.

6. REFERENCES

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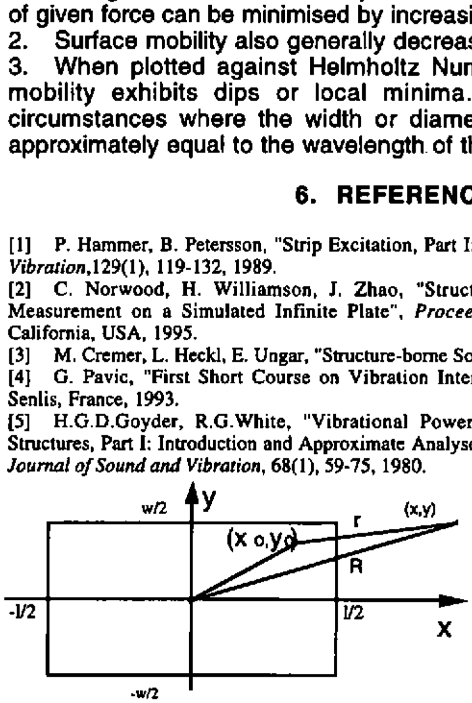

Fig.1 Force and velocity position in coordinate system

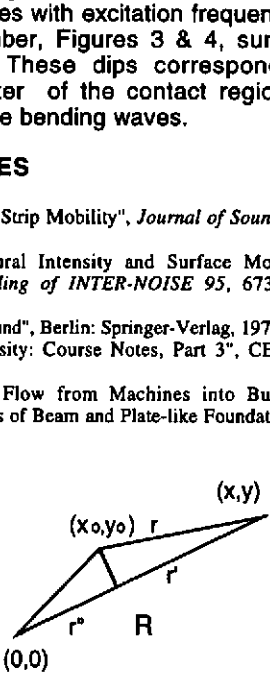

Fig.2 Relationship between r,R, r' and r''

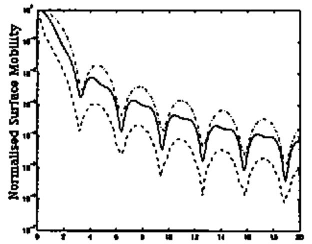

Figure 3 Normalised Surface Mobility versus Width-Based Helmholtz Number for Rectangular Contact Areas of Various Aspect Ratios: ---- l/w=1; —— l/w=2; ___ l/w=10.

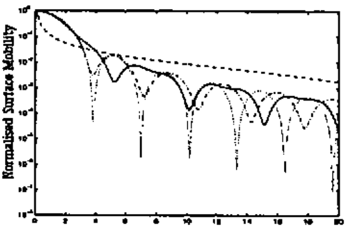

Figure 4 Normalised Surface Mobility for Constant Area: (a) dotted line-circular area; (b) dashdot line-l/w=1; (c) solid line-l/w=2; (d) dashed line-l/w=10.