

VIBRATIONAL ENERGY FLOW AT JUNCTIONS OF POINT CONNECTED PLATES

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1. INTRODUCTION

The application of elastic layers as vibration isolators at junctions between walls and slabs is reported by several authors [1,2] as a key solution in reducing flanking transmission in buildings. Because of the required mechanical stability of the construction however, this type of junction is applicable only to a limited number of junctions in a building. As a consequence, vibrational energy will propagate preferably through the remaining junctions involving rigid connections [3]. This phenomenon can lead to an increase of flanking transmission for certain propagation directions. One solution to this problem is the application rigid connections at distinct points along the elastic junction line.

This paper presents a calculation model for predicting structure-borne sound transmission between thin, isotropic, semi-infinite plates coupled by a number of rigid point connections. The aim of this study is to determine the influence of the width, the density and the stiffness of the point connections on the vibrational energy flow between coupled plates.

2. CALCULATION MODEL

Basic theory. The analysis is performed using the wave approach as reported by several authors [4-7]. The junction consists of an assembly of semi-infinite, isotropic plates coupled along a common junction line. The plate motion is governed by thin plate kinematic assumptions and damping was not taken into account in the wave analysis. The excitation is modelled as a plane wave incident with oblique incidence towards the junction. This wave generates bending-, quasi-longitudinal and in-plane transverse waves propagating away from the junction on all plates. Displacements as well as forces are expressed on the plate edges, where the forces are derived from the displacements using the elementary formulae of mechanics. The equilibrium and continuity conditions at the junction line lead to a set of linear

equations the solution of which yields the amplitudes of the generated waves. The energy flow at the junction is quantified by the reflection and transmission coefficients, which are defined as the ratios of the reflected and transmitted intensities, respectively, to the intensity carried by the incident wave. The average transmission factor is calculated by integrating over all angles of incidence.

Point connections. To simulate rigid point connections, an elastic layer with space dependent elastic properties is introduced at the junction line: a point connection is modelled by a stepwise increase of the layer stiffness. The analysis is restricted to equally spaced connections, which leads to a periodic boundary condition for the coupled plates. As a consequence, the incidence of a plane wave on the junction will give rise to a scattered wavefield on all plates, for all of the wavetypes considered in the analysis. The following expression describes the resulting bending wavefield for the case of an incident plane wave characterized by a wavenumber k_i and an angle of incidence θ_i ($e^{j\omega t}$ omitted for simplicity):

$$\eta(x,z) = \sum_{n=-N}^{+N} \left(A_n e^{-jk_{zn}x} + B_n e^{-jk_{zn}x} \right) e^{-j \left(k_i \sin \theta_i + \frac{2\pi n}{L} \right) z} \quad (1)$$

Equation (1) states that the transverse displacement η is the result of the superposition of a number of plane waves travelling away from the junction. These waves are characterized by a dependency on the coordinate parallel to the junction line (z), which is imposed by the incident wave and the spatial period (L) of the boundary conditions. The wavenumbers k_{zn1} and k_{zn2} follow from substitution into the equations of motion, A_n and B_n are unknown amplitudes. The maximum wave order N is determined by testing the convergence rate of the solution. Similar expressions are derived for quasi-longitudinal and in-plane transverse waves.

Not every term in the right-hand side of equation (1) represents a travelling wave. In fact, most terms represent exponential decaying near fields. Although no energy flow is associated with these near fields, they need to be taken into account in order to satisfy the boundary conditions. The resulting reflected and transmitted intensities are calculated by averaging the energy flow over a single spatial period of the boundary.

3. NUMERICAL AND EXPERIMENTAL RESULTS

Theoretical and experimental results were obtained for a corner junction of two PVC-plates. The two identical plates have a surface area of $1.5 \times 1.14 \text{ m}^2$ and a thickness of 15 mm. The material properties are: Young's modulus $E = 3.7 \cdot 10^9 \text{ Pa}$, Poisson's ratio $\nu = 0.4$ and density $\rho = 1440 \text{ kg/m}^3$. The plates were coupled along the 1.14 m edge and the point connections were elaborated by inserting, at the junction, $15 \times 15 \text{ mm}$ aluminium plates with a thickness of 2 mm. Five different junctions were considered as listed in the

table below.

Description	Spatial period L[m]
Full 1.14 m rigid connection	/
7 point connections	0.1425
4 point connections	0.285
3 point connections	0.285
2 point connections	0.57

Angle dependence. As an example, the bending wave transmission coefficient was calculated as a function of the angle of incidence. Figure 1 shows the results for three different junctions at 500 Hz (a) and 2000 Hz (b). It can be observed that the transmission coefficient is highly dependent on the angle of incidence of the primary wave. The effect increases at high frequencies when the wavelength becomes small in comparison to the distance between points.

Experimental verification. Statistical energy analysis (SEA) was used to predict the velocity level difference between the plates. The coupling loss factors were derived from the average transmission coefficient. Reverberation time measurements were used to determine the internal loss of the SEA subsystems. Figure 2 shows the comparison between the measured (a) and predicted (b) velocity level differences between the plates, for the five junction types listed above. The results show good agreement since the average difference between measurement and calculation is less than 2 dB.

4. CONCLUSIONS

In this paper, a calculation model is presented to predict the vibrational power energy at a junction of point connected plates. The point connections were modelled by means of an elastic interlayer with periodically varying elastic properties. Comparisons between theoretical and experimental data illustrate the ability of the model to predict the structure-borne sound transmission with good accuracy.

Acknowledgments

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References

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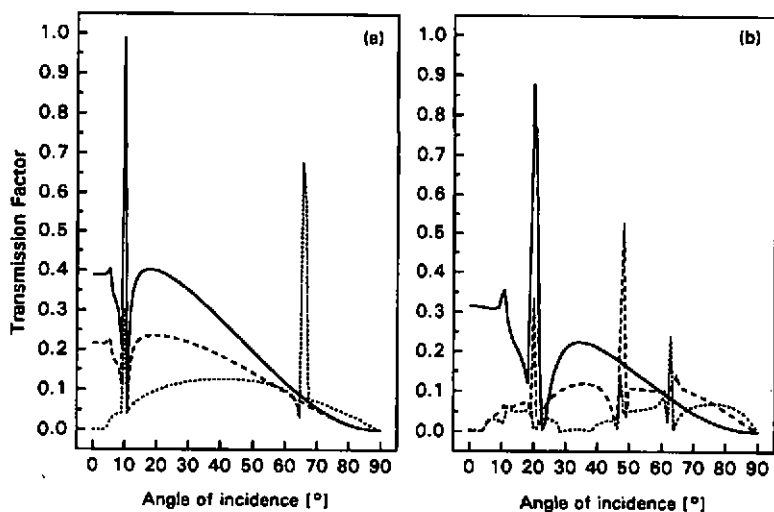


Figure 1: Transmission coefficient for incident and transmitted bending waves as a function of the angle of incidence, at 500 Hz (a) and 2000 Hz (b). Legend: Rigid (—); 7 (---) and 3 (····) point connections.

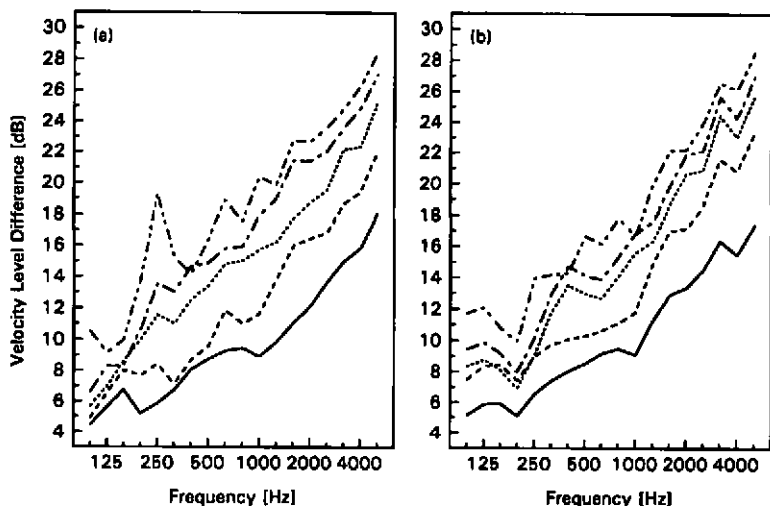


Figure 2: Comparison between measurement(a) and calculation(b) : velocity level difference for a corner junction of PVC-plates. Legend: Rigid (—); 7 (---), 4 (····), 3 (— · —) and 2 (----) point connections.