A NUMERICAL STUDY OF INCIDENT SOUND FIELD REPRODUCTION SYSTEMS

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1. INTRODUCTION

The “incident sound field”, when used in the context of the reflection or scattering of sound, can be defined as the sound field that would be present if the scatterer or boundary were removed to allow free field propagation. Therefore, the incident sound field is not influenced by the scatterer or boundary, and so it is independent of the listener in a reproduced sound field. The incident sound field reproduction method attempts to create a virtual acoustic field by reproducing exactly the incident sound field within a control volume, where a listener is located. This paper describes the theory of the incident sound field reproduction method and numerical simulations of systems using this method. A study is also presented of the boundary surface control principle that attempts to reproduce the incident sound field within a control volume by reproducing sound pressure and its gradient on the boundary surface of the control volume.

2. BOUNDARY SURFACE CONTROL

If there is no source in a given volume *V* bounded by a surface *S*, the solution of the inhomogeneous wave equation at a single frequency reduces to the Kirchhoff-Helmholtz integral equation that is given by[1]

$$C(\mathbf{x})\,p(\mathbf{x}) = \int_S \left(g(\mathbf{x}|\mathbf{y}) \frac{\partial p(\mathbf{y})}{\partial n} - p(\mathbf{y}) \frac{\partial g(\mathbf{x}|\mathbf{y})}{\partial n} \right) dS \tag{1}$$

where **x** is a position vector, **y** is a position vector on the boundary surface *S*, **n** is the unit outward normal vector on *S*, $\partial/\partial n$ is a directional derivative in the direction of **n**, *p* is the complex acoustic pressure, $g(\mathbf{x}|\mathbf{y})$ is the free space Green’s function, and $C(\mathbf{x})$ is equal to one if **x** is within *V*, zero if **x** is outside *V*, and 0.5 if **x** is on a smooth boundary *S*. This integral equation can be solved if the boundary conditions on the boundary surface *S* are given.

The Kirchhoff-Helmholtz integral equation can be interpreted as the following boundary surface control principle[2]: The pressure field within the volume *V* can be controlled by controlling the pressure and its gradient on the surface *S*. In this case, the Green function and its gradient can be regarded as constants determined by the boundary shape. In practice, the control surface *S* is divided into *N* control points $\mathbf{x}_i$ ($i=1 \ldots N$). The pressure gradient at $\mathbf{x}_i$ can be approximately calculated from the two point pressures at $\mathbf{x}_i$ and $\mathbf{x}_i + c\mathbf{n}_i$ where $\mathbf{n}_i$ is the corresponding normal vector and *c* is coefficient that is small compared to the wavelength, such that

$$\frac{\partial p(\mathbf{x}_i)}{\partial n} = \frac{p(\mathbf{x}_i + c\mathbf{n}_i) - p(\mathbf{x}_i)}{c}. \tag{2}$$

Therefore, the sound pressures at the 2*N* the control points are recorded in the primary field and reproduced in the secondary field. Now assume for simplicity that the source generates a single frequency sound. In the primary field, the complex sound pressures at the boundary surface control points can be given by the vector $\mathbf{p}_p$. A straightforward approach[1] to reproduce the vector $\mathbf{p}_p$ is to use a vector of secondary source strengths $\mathbf{q}_s$ that produce the complex sound pressure vector $\mathbf{p}_s$ at the boundary surface control points in the secondary field such that

$$\mathbf{p}_s = \mathbf{G}\,\mathbf{q}_s \tag{3}$$

where $\mathbf{G}$ is the acoustic transfer impedance matrix relating the vector $\mathbf{p}_s$ to the vector $\mathbf{q}_s$. If the number of control points is L and the number of secondary sources is M, $\mathbf{G}$ is an $L \times M$ matrix. To replicate the primary sound field in the secondary sound field, $\mathbf{p}_s$ must be equal to $\mathbf{p}_p$. The optimal strength $\mathbf{q}_{so}$ of the secondary sources that minimises the sum of the squared differences between the sound pressure $\mathbf{p}_p$ and the sound pressure $\mathbf{p}_s$ is given by[1]

$$\mathbf{q}_{so} = \left(\mathbf{G}^{\mathrm{H}}\mathbf{G}\right)^{-1}\mathbf{G}^{\mathrm{H}}\mathbf{p}_p \tag{4}$$

where it is assumed that $L > M$.

Ise[2] suggested the virtual acoustic system based on the boundary surface control principle as shown in Fig. 1. This system controls the pressure and its gradient on the boundary surface in the secondary field so that they are identical to those in the primary field. The secondary field can be reproduced by multiple secondary sources located at arbitrary positions outside the controlled volume. The performance of this system is independent of a listener inside the controlled volume. However, a great number of loudspeakers are needed if one wishes to control the sound field at high frequencies. However, virtual acoustic systems based on the boundary surface control principle should produce a performance that is not dependent upon the listener. The objective of the current study is to evaluate the extent to which this principle might be used in practice. Of particular interest is the application where the sound field in the region of the ears of a listener is reproduced such that the field can be made independent of the geometry of the individual listener's ears. In such a case, an ear of the listener might be regarded as a scattering body inside a control volume, which the remaining part of the head of the listener can be regarded as another scattering body outside the control volume. It is then of interest to determine whether the first scattering body (the ear) influences the incident field produced by the second scattering (the head). This paper addresses this particular aspect of the incident sound field reproduction method.

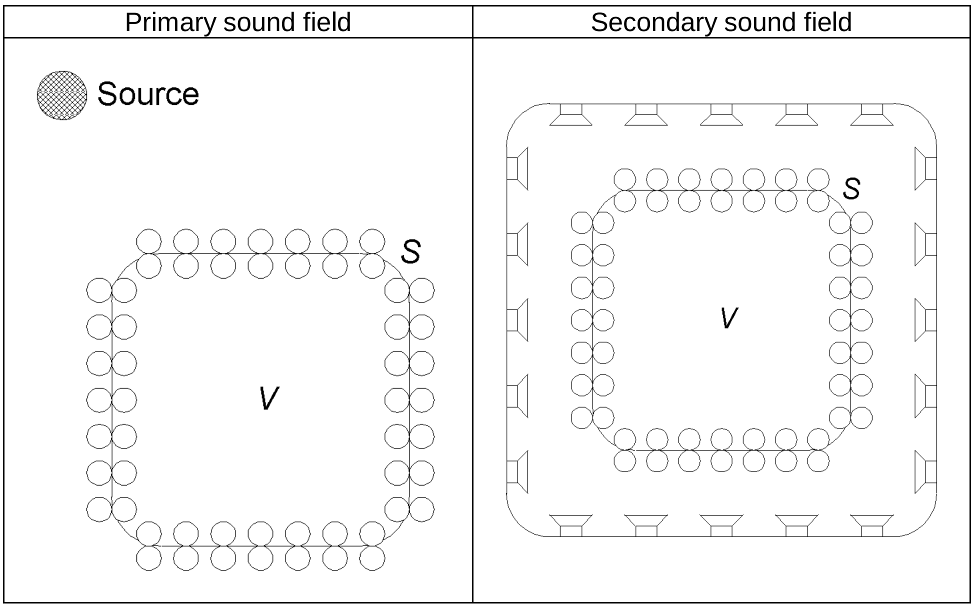


Figure 1. Sound field reproduction system based on the boundary surface control principle.

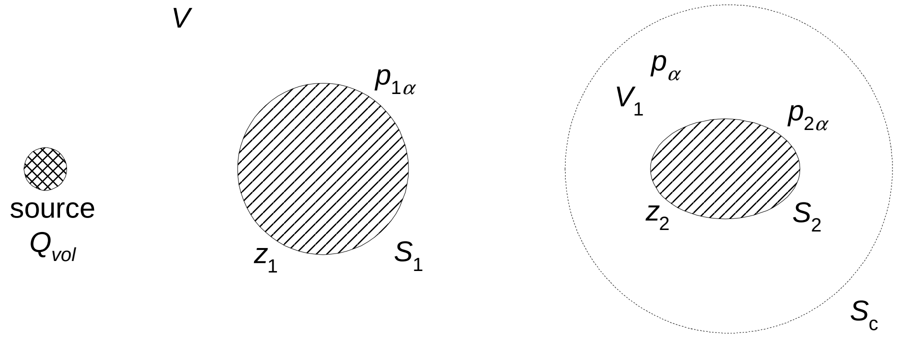


Figure 2. The sound field α in the case of two scattering bodies.

3. THE INCIDENT SOUND FIELD REPRODUCTION METHOD

3.1. Determination of the required secondary source distribution

Figure 2 shows the primary field with two scattering bodies and acoustic sources in a free field, which simulates the real acoustic environment. This is called a sound field α for simplicity in this paper. The acoustic source strength distribution Q_{vol} in an unbounded acoustic domain V outside the scattering bodies is assumed to be known and emit a single frequency sound. Each bounding surface of the scattering bodies is denoted by S_1 or S_2. All surfaces throughout this paper are assumed to be locally reacting surfaces. The specific acoustic impedance z_1 of the surface S_1 and z_2 of the surface S_2 are also assumed to be known and non-zero. Now consider the control volume V_1 bounded by the surface S_{c} and the surface S_2. The surface S_{c} is a transparent imaginary surface. The sound source distribution Q_{vol} is assumed to be located outside the volume V_1. When the vector **x** is inside the volume V_1, the sound pressure in the volume V_1 is denoted by p_α. Then, the sound pressure p_α inside the volume V_1 can be written as

$$p_\alpha(\mathbf{x}) = p_{in\alpha}(\mathbf{x}) + \int_{S_1} H_1(\mathbf{x}|\mathbf{y})\, p_{1\alpha}(\mathbf{y})\, dS + \int_{S_2} H_2(\mathbf{x}|\mathbf{y})\, p_{2\alpha}(\mathbf{y})\, dS \qquad (5)$$

where the incident sound field $p_{in\alpha}(\mathbf{x})$ from the source distribution is given by

$$p_{in\alpha}(\mathbf{x}) = \int_V Q_{vol}(\mathbf{y}_v)\, g(\mathbf{x}|\mathbf{y}_v)\, dV\ , \qquad (6)$$

and where the transfer function $H_1(\mathbf{x}|\mathbf{y})$ from sound pressure on the surface S_1 to sound pressure at the field point is given by

$$H_1(\mathbf{x}|\mathbf{y}) = -\frac{j\omega\rho_0 g(\mathbf{x}|\mathbf{y})}{z_1(\mathbf{y})} - \frac{\partial g(\mathbf{x}|\mathbf{y})}{\partial n} \qquad (z_1(\mathbf{y})\neq 0), \ (7)$$

and $H_2(\mathbf{x}|\mathbf{y})$ is the corresponding transfer function for the surface S_2. This sound pressure p_α is the desired sound field that is supposed to be reproduced in the secondary field. Note that all the boundary surfaces are assumed to be smooth boundaries for simplicity.

Figure 3 illustrates the secondary field β with the same scattering body having surface S_2 as that in the sound field α, but with different sound sources that are intended to reproduce the primary sound field α within the control volume V_1. If continuous transparent monopole and dipole source layers are placed on the surface S_c and there is no other source in an unbounded free field, the sound pressure p_β at a single frequency in the volume V_1 can be written as

$$p_\beta(\mathbf{x}) = p_{in\beta}(\mathbf{x}) + \int_{S_2} H_2(\mathbf{x}|\mathbf{y}) p_{2\beta}(\mathbf{y})\, dS \quad (8)$$

where the vector **x** is inside the volume V_1 and the incident sound field $p_{in\beta}$ produced by the source layers on the surface S_c is given by

$$p_{in\beta}(\mathbf{x}) = -\int_{S_c} j\omega\rho_0 v_{nc\beta}(\mathbf{y})\, g(\mathbf{x}|\mathbf{y}) + p_{c\beta}(\mathbf{y}) \frac{\partial g(\mathbf{x}|\mathbf{y})}{\partial n} dS\ . \quad (9)$$

where $p_{c\beta}$ is the sound pressure on the surface S_c, and $v_{nc\beta}$ is the normal particle velocity on the surface S_c. The monopole and dipole source layers on the surface S_c are intended to reproduce the same sound field as the sound field α inside the control volume V_1. If the sound field β is the same as the sound field α in the volume V_1, that is, $p_\alpha(\mathbf{x})=p_\beta(\mathbf{x})$ when the vector **x** is inside the volume V_1 and $p_{2\alpha}(\mathbf{y})=p_{2\beta}(\mathbf{y})$ when the vector **y** is on the surface S_2, then the following equation to determine the required secondary source distribution results from subtracting Eq. (8) from Eq. (5):

$$p_{in\beta}(\mathbf{x}) = p_{in\alpha}(\mathbf{x}) + p_{s\alpha}(\mathbf{x}) \quad (10)$$

where the vector **x** is in the volume V_1 and the scattered sound field $p_{s\alpha}(\mathbf{x})$ from the surface S_1 in the sound field α is given by

$$p_{s\alpha}(\mathbf{x}) = \int_{S_1} H_1(\mathbf{x}|\mathbf{y})\, p_{1\alpha}(\mathbf{y})\, dS \quad (11)$$

This equation shows that the monopole and dipole source layers on the surface S_c reproduce only the "total" incident sound field on the volume V_1 which is composed of the sound field $p_{in\alpha}(\mathbf{x})$ produced by the sound source and the sound field $p_{s\alpha}(\mathbf{x})$ scattered from the scatterer S_1 in the primary field α. The sound field $p_{s\alpha}(\mathbf{x})$ can be regarded as another incident sound field that propagates into the volume V_1. However, this additional incident sound field $p_{s\alpha}(\mathbf{x})$ is dependent on the geometry and boundary condition of the scatterer S_2. The sound field scattered from the surface S_2 inside the control volume does not need to be reproduced by the sources layers because it is determined only by the "total" incident sound field and it is reproduced or scattered by the surface S_2 itself. This will be discussed by studying another pair of primary and secondary fields.

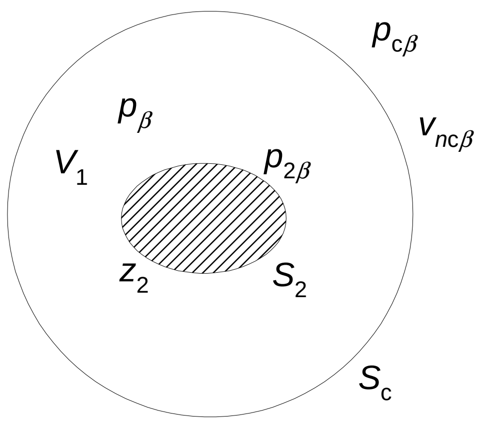


Figure 3. The sound field β in the case of two scattering bodies.

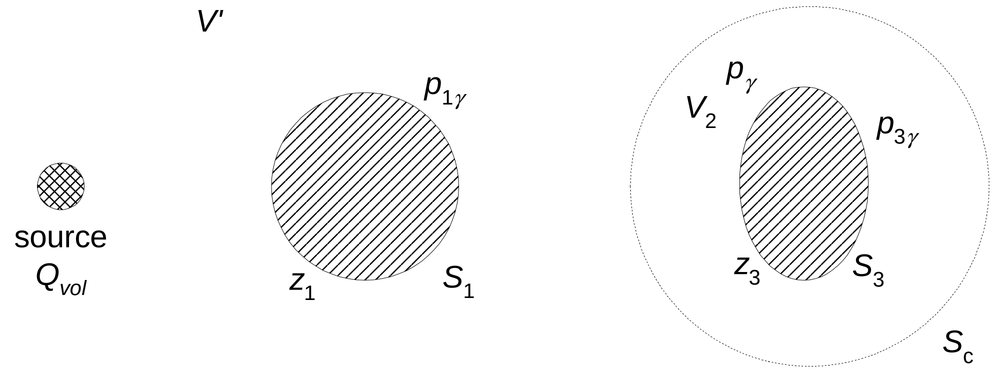


Figure 4. The sound field γ in the case of two scattering bodies.

3.2. Conditions for production of the virtual sound field

Figure 4 illustrates another primary field γ in an otherwise unbounded free field with the sound sources and one scattering body that are the same as those in the sound field α but with one scattering body that differs from that in the sound field α. The bounding surface of the same scattering body is the surface S_1 and that of the different scattering body is denoted by S_3, and this different scattering body is assumed to be within the surface S_c. The specific acoustic impedance z_3 of the surface S_3 is assumed to be known and non-zero. The acoustic source strength distribution Q_{vol} is assumed to be the same as that in the sound field α. The outer bounding surface of the control volume V_2 is the surface S_c that is the same transparent surface as that in the sound field α, and the inner bounding surface of the volume V_2 is the surface S_3. When the vector **x** is in the volume V_2, the sound pressure in the volume V_2 is denoted by p_γ. The sound pressure p_γ in the volume V_2 at a single frequency can be written as

$$p_\gamma(\mathbf{x}) = p_{in\gamma}(\mathbf{x}) + p_{s\gamma}(\mathbf{x}) + \int_{S_3} H_3(\mathbf{x}|\mathbf{y})\, p_{3\gamma}(\mathbf{y})\, dS \tag{12}$$

where the vector **x** is inside the volume V_2, and

$$p_{in\gamma}(\mathbf{x}) = \int_{V'} Q_{vol}(\mathbf{y}_v)\, g(\mathbf{x}|\mathbf{y}_v)\, dV , \tag{13}$$

the sound field $p_{s\gamma}(\mathbf{x})$ scattered from the surface S_1 in the sound field γ is given by

$$p_{s\gamma}(\mathbf{x}) = \int_{S_1} H_1(\mathbf{x}|\mathbf{y})\, p_{1\gamma}(\mathbf{y})\, dS , \tag{14}$$

and $H_3(\mathbf{x}|\mathbf{y})$ is the corresponding transfer function to the surface S_3. This sound pressure p_γ is the desired sound field that is supposed to be reproduced in the secondary field.

The sound field $p_{in\gamma}(\mathbf{x})$ is equal to the sound field $p_{in\alpha}(\mathbf{x})$ when the vector **x** is in the intersection $V_1 \cap V_2$, i.e. when the vector **x** is inside both V_1 and V_2, due to the same source strength distribution Q_{vol}. However, the incident sound field is the sound field in the absence of the scattering body and so is not influenced by the scattered sound field or the scattering body. Therefore, the incident sound field $p_{in\gamma}$ or $p_{in\alpha}$ can be extended to the domain in which either scattering body is located. Then the sound field $p_{in\gamma}(\mathbf{x})$ is equal to the sound field $p_{in\alpha}(\mathbf{x})$ when the vector **x** is in the union $V_1 \cup V_2$, i.e. when the vector **x** is inside V_1 or V_2. Therefore, the incident sound field $p_{in}(\mathbf{x})$ produced by the acoustic source strength distribution Q_{vol} in either primary sound field α or γ can be rewritten as

$$p_{in}(\mathbf{x}) = p_{in\alpha}(\mathbf{x}) = p_{in\gamma}(\mathbf{x}) = \int_{V_1 \cup V_2} Q_{vol}(\mathbf{y}_v)\, g(\mathbf{x}|\mathbf{y}_v)\, dV \qquad (15)$$

where the vector **x** is in the union volume $V_1 \cup V_2$.

Figure 5 illustrates another secondary field δ with the scattering body having the surface S_3, which is the same as that in the sound field γ but with different sound sources that are intended to reproduce the primary sound field γ within the control volume V_2 in an otherwise unbounded free field. If continuous transparent monopole and dipole source layers are placed on the surface S_c and there is no other source, the sound pressure p_δ at a single frequency in the volume V_2 can be written as

$$p_\delta(\mathbf{x}) = p_{in\delta}(\mathbf{x}) + \int_{S_3} H_3(\mathbf{x}|\mathbf{y}) p_{3\delta}(\mathbf{y})\, dS \qquad (16)$$

where the vector **x** is inside the volume V_2 and the incident sound field $p_{in\delta}$ produced by source layers on the surface S_c is given by

$$p_{in\delta}(\mathbf{x}) = -\int_{S_c} j\omega\rho_0 v_{nc\delta}(\mathbf{y})\, g(\mathbf{x}|\mathbf{y}) + p_{c\delta}(\mathbf{y}) \frac{\partial g(\mathbf{x}|\mathbf{y})}{\partial n}\, dS \qquad (17)$$

where the position vector **x** is in the volume V_2, $p_{c\delta}$ is the sound pressure on the surface S_c, and $v_{nc\delta}$ is the normal particle velocity on the surface S_c. The monopole and dipole source layers on the surface S_c are intended to reproduce the same sound field as sound field γ inside the control volume V_2. If the same monopole and dipole source strengths obtained in the sound field β are applied to the sound field δ, the following equation results from Eq. (10) and Eq. (15):

$$p_{in\delta}(\mathbf{x}) = p_{in\beta}(\mathbf{x}) = p_{in\alpha}(\mathbf{x}) + p_{s\alpha}(\mathbf{x}) = p_{in}(\mathbf{x}) + p_{s\alpha}(\mathbf{x}) \qquad (18)$$

where the vector **x** is in the union $V_1 \cup V_2$ for the same reasons as those discussed above. When the vector **x** is inside the volume V_2, the subtraction of Eq. (16) from Eq. (12) follows that

$$\begin{aligned} & p_\gamma(\mathbf{x}) - p_\delta(\mathbf{x}) - \int_{S_3} H_3(\mathbf{x}|\mathbf{y}) \left(p_{3\gamma}(\mathbf{y}) - p_{3\delta}(\mathbf{y}) \right) dS \\ &= p_{in\gamma}(\mathbf{x}) + p_{s\gamma}(\mathbf{x}) - p_{in\delta}(\mathbf{x}) = p_{in}(\mathbf{x}) + p_{s\gamma}(\mathbf{x}) - \left(p_{in}(\mathbf{x}) + p_{s\alpha}(\mathbf{x}) \right). \\ &= p_{s\gamma}(\mathbf{x}) - p_{s\alpha}(\mathbf{x}) = \int_{S_1} H_1(\mathbf{x}|\mathbf{y}) \left(p_{1\gamma}(\mathbf{y}) - p_{1\alpha}(\mathbf{y}) \right) dS \end{aligned} \qquad (19)$$

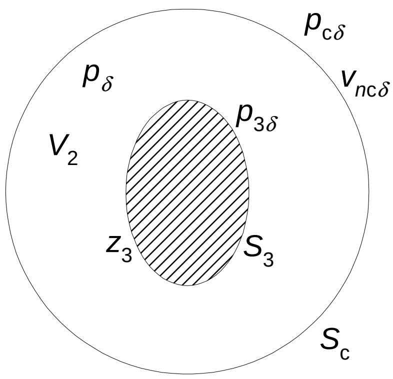


Figure 5. The sound field δ in the case of two scattering bodies.

When the vector $\mathbf{x}_3$ is on the surface S_3, equation (19) can be modified to

$$\begin{aligned}&\frac{1}{2}\left(p_{3\gamma}(\mathbf{x}_3)-p_{3\delta}(\mathbf{x}_3)\right)-\int_{S_3} H_3(\mathbf{x}_3|\mathbf{y})\left(p_{3\gamma}(\mathbf{y})-p_{3\delta}(\mathbf{y})\right)dS\\&=\int_{S_1} H_1(\mathbf{x}_3|\mathbf{y})\left(p_{1\gamma}(\mathbf{y})-p_{1\alpha}(\mathbf{y})\right)dS=p_{s\gamma}(\mathbf{x}_3)-p_{s\alpha}(\mathbf{x}_3)\end{aligned} \tag{20}$$

where the vector $\mathbf{x}_3$ is on the surface S_3. Strictly speaking, the sound field $p_{s\alpha}(\mathbf{x}_3)$ is different from the sound field $p_{s\gamma}(\mathbf{x}_3)$ on the surface S_3 since the sound field $p_{s\alpha}(\mathbf{x})$ or $p_{s\gamma}(\mathbf{x})$ depends on the geometry and boundary condition of the scatterer S_3. However, $p_{s\gamma}(\mathbf{x}_3) - p_{s\alpha}(\mathbf{x}_3)$ will be nearly zero in certain cases. In such cases it may be assumed that the sound field incident on the control volume from scatterers outside the control volume is independent of the geometry and boundary conditions of the scatterer inside the control volume. This assumption is only reasonable if one of the following requirements is met.

First, the geometry and boundary condition of the scatterer within the control volume in the sound field α is similar to that in the sound field γ, that is, $S_2 \approx S_3$ and $z_2 \approx z_3$. At low frequencies where wavelengths are much longer than the size of the scatterer, the detailed shape of the scatter cannot influence the sound field very much, so this requirement can be fulfilled. However, at high frequencies where wavelengths are similar or shorter than the size of the scatterer, the detailed shape of scatterer can influence the sound field significantly and interact with the incident waves, so this requirement may not be fulfilled. Second, the sound pressure on the surface of the scattering body outside the control volume in the sound field α is similar to that in the sound field γ, that is, the sound pressure $p_{1\alpha}(\mathbf{x}_1) \approx p_{1\gamma}(\mathbf{x}_1)$ on the surface S_1. On the other hand, if the sound field produced on the surface S_1 scattered from the surface S_2 or S_3 is sufficiently weak compared to the sound pressure on the surface S_1, then this requirement can be fulfilled. Third, the incident sound field inside the control volume produced from the scatterers outside the control volume is sufficiently weak compared to the total sound pressure inside the control field. Then, the sound pressure $p_{s\alpha}(\mathbf{x}) \approx p_{s\gamma}(\mathbf{x})$ within the volume $V_1 \cup V_2$.

In this case, the acoustic properties such as the sound pressure on the surface S_1 is not changed when the geometry and boundary conditions of the scatterer having the bounding surface S_2 is changed into those of the scatterer having the bounding surface S_3 inside the control volume. Under these circumstances, the sound pressure on the surface S_1 in the sound field α is assumed to be the same as that in the sound field γ and therefore

$$p_{1\alpha}(\mathbf{y})=p_{1\gamma}(\mathbf{y}) \tag{21}$$

where the vector $\mathbf{y}$ is on the surface S_1. This shows that

$$p_{s\gamma}(\mathbf{x}_3)-p_{s\alpha}(\mathbf{x}_3)=\int_{S_1} H_1(\mathbf{x}_3|\mathbf{y})\left(p_{1\gamma}(\mathbf{y})-p_{1\alpha}(\mathbf{y})\right)dS=0 \tag{22}$$

where the vector $\mathbf{x}_3$ is on the surface S_3. The assumption of Eq. (21) also causes that the sound field $p_{s\alpha}(\mathbf{x})$ is the same as the sound field $p_{s\gamma}(\mathbf{x})$ where the vector $\mathbf{x}$ is in the union $V_1 \cup V_2$ from Eq. (11) and (14). If the surface sound pressure difference $p_{3\gamma}(\mathbf{x}_3) - p_{3\delta}(\mathbf{x}_3)$ in Eq. (20) is substituted by $p_3(\mathbf{x}_3)$ for simplicity, equation (20) can be rewritten as

$$\frac{1}{2}p_3(\mathbf{x}_3)-\int_{S_3} H_3(\mathbf{x}_3|\mathbf{y})\,p_3(\mathbf{y})\,dS=0\,. \tag{23}$$

This integral equation is known as a homogeneous Fredholm equation of the second kind. One solution of this equation is that $p_3(\mathbf{x}_3)=0$. This can be understood by examining the numerical solution of this equation by discretising the surface S_3 into a set of boundary elements. Then equation (23) can be rewritten in matrix form such that $(\mathbf{H}-0.5\mathbf{I})\mathbf{p} = 0$. If the matrix $(\mathbf{H}-0.5\mathbf{I})$ is assumed to be non-singular, the integral equation (23) has the unique solution $p_3(\mathbf{x}_3)=0$. Then, equation (20) shows that

$$p_{3\gamma}(\mathbf{x}_3) = p_{3\delta}(\mathbf{x}_3) \tag{24}$$

where the vector $\mathbf{x}_3$ is on the surface S_1. The following equation thus results from Eq. (19):

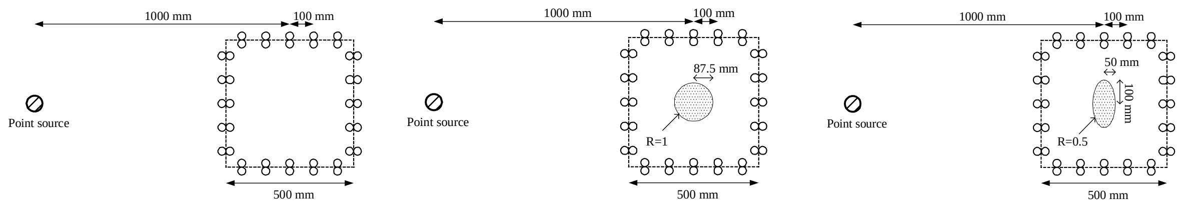
$$p_{\gamma}(\mathbf{x}) = p_{\delta}(\mathbf{x}) \tag{25}$$

where the position vector $\mathbf{x}$ is in the volume V_2. That means the secondary sound field δ is same as the primary sound field γ.

The primary sound field α or β is successfully reproduced in the corresponding secondary sound field β or δ by applying the same monopole and dipole source layers in the secondary field. This confirms that the total sound field for any scattering body within the control volume can be successfully reproduced if it is produced by the same sources that reproduce the incident sound field on a given scattering body within the control volume. It has been assumed, however, that the field scattered by bodies outside the control volume is not affected by the scatterer inside the control volume.

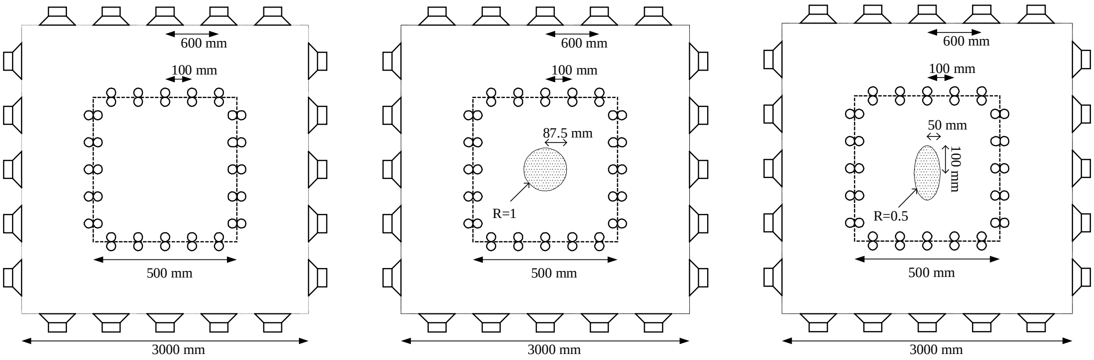
4. NUMERICAL SIMULATION

Numerical simulations are performed to simulate a sound reproduction system with a single scatterer inside the control field. The simulations in a two-dimensional space are used for illustration since three-dimensional simulations need a much greater number of sources in the secondary field. Numerical models are created by using the ANSYS software package, and the sound pressure is evaluated by using the direct boundary element method in the SYSNOISE software package, and the optimal secondary source strengths are evaluated by using the MATLAB software package. Figure 6 illustrates the two-dimensional primary sound field produced by a point source in an unbounded free field (a) without any scattering body, or (b) with a rigid cylinder, or (c) with a partly absorbent ellipsoid. The reflection coefficient of the ellipsoid is 0.5. In these simulations, the peak complex sound pressure amplitude at a cylinder of unit radius is set to be unity at all frequencies. The control field is a two-dimensional square with the side length of 500 mm. Pairs of microphones are placed on the boundary surface of the square to measure the surface sound pressure and its gradient. The frequency range of interest is set to be from 100 Hz to 3000 Hz. The distance between a pair of microphones is 20 mm and the distance between two adjacent pairs of microphones is 100 mm as shown in Fig. 6. The number of control points is 40. The point source is 1000 mm away from the centre of the control field.



(a) (b) (c)

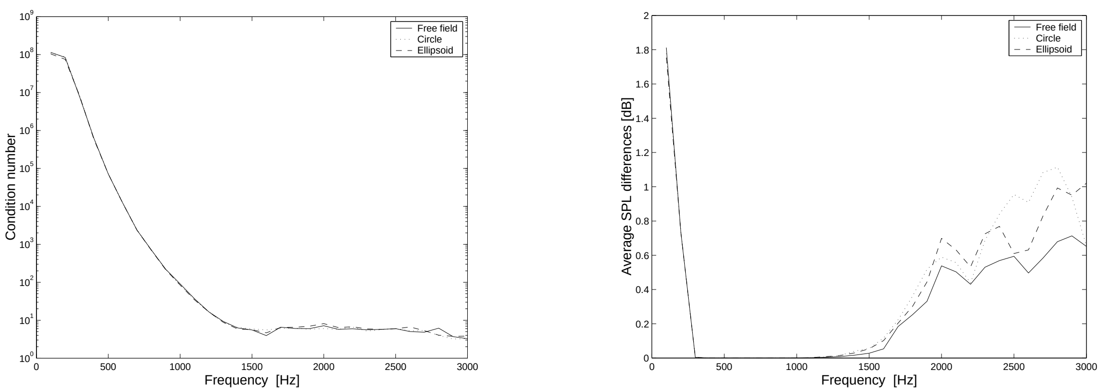
Figure 6. The 2-D primary sound field produced by a point source in an unbounded free field (a) without any scattering body, or (b) with a rigid cylinder, or (c) with a partly absorbent ellipsoid.



(a) (b) (c)

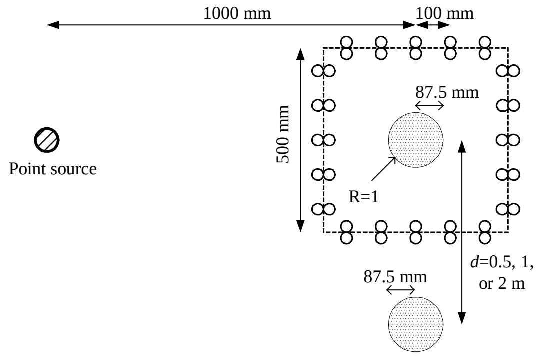
Figure 7. The 2-D secondary sound field produced by multiple point sources in a free field (a) without any scattering body, or (b) with a rigid cylinder, or (c) with a partly absorbent ellipsoid.

The complex sound pressure vector $\mathbf{p}_p$ at the control points is evaluated in SYSNOISE, and it is the desired sound pressure that we aim to reproduce in the secondary field. Figure 7 illustrates the two-dimensional secondary sound field produced by multiple point sources in an unbounded free field (a) without any scattering body, or (b) with a rigid cylinder, or (c) with a partly absorbent ellipsoid. The 20 point sources are located on a square with the side length of 3000 mm outside the control volume in the secondary field. The acoustic transfer impedance matrix **G** from 20 point sources to 40 control points is evaluated by using the method presented in the previous section. Then, the optimal secondary source strengths necessary to reproduce the primary sound field within the control field are evaluated by using the least squares method. The optimal source strengths for all three cases are nearly the same and maximum amplitude difference of the optimal source strengths for different cases is less than 0.4 %. This verifies that the secondary sources reproduce all the same incident sound field from the primary source for different scattering bodies. The optimal secondary source strengths that are evaluated for the case of the free field are applied to secondary sources for all three cases. Figure 8(a) shows the condition number of the transfer impedance matrices for various cases. The condition number is the ratio of the largest singular value to the smallest singular value of the matrix. Figure 8(a) shows the condition numbers of the matrices for all cases are nearly the same, and the system is ill-conditioned at low frequencies[3]. Figure 8(b) shows the average sound pressure level differences over the control points between the desired values and the reproduced values in various cases. It shows similar tendencies for all cases.

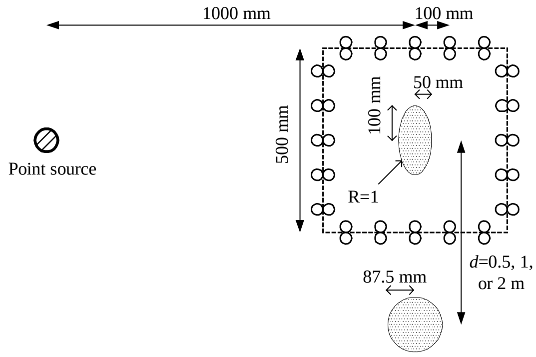


(a) (b)

Figure 8. (a) Condition number of the various transfer impedance matrices.
(b) Average sound pressure level differences over control points in various cases.



(a)

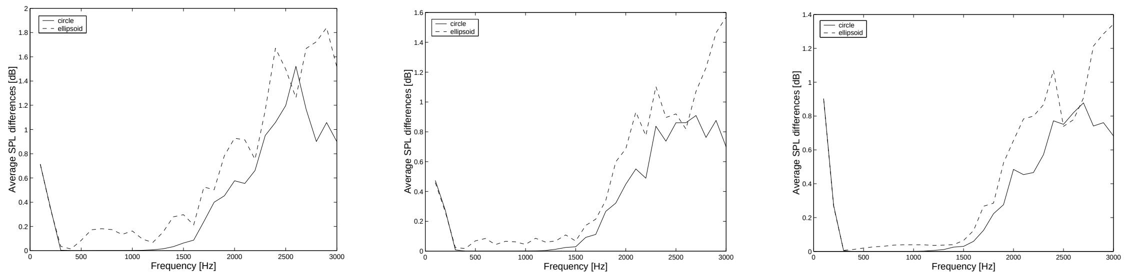


(b)

Figure 9. The two-dimensional primary sound field produced by a point source in an unbounded free field (a) with two rigid cylinders (b) with a rigid cylinder and a rigid ellipsoid.

The reproduction is successfully performed between 300 Hz and 1500 Hz for all cases. This good performance can be a practical illustrator of the incident sound field reproduction method using the boundary surface control principle. However, the primary field is not always reproduced perfectly at all frequencies in limited conditions. A large reproduction error at low frequencies below 300 Hz results from an ill-conditioned system for the matrix inversion. A large reproduction error at high frequencies above 1500 Hz results from spatial aliasing effect due to spatial sampling. The 100 mm microphone spacing corresponds to half the wavelength of a 1700 Hz wave. These high reproduction errors at low and high frequencies may be applied to other incident sound reproduction systems generally. Therefore, there may be a middle frequency range in which the incident sound reproduction systems can successfully create virtual acoustic images.

Another numerical simulation in a two-dimensional space is performed to simulate sound reproduction systems with two scattering bodies. Most numerical models and the procedure are the same as those in the case of a single scatterer. Figure 9(a) illustrates the two-dimensional primary sound field produced by a point source in an unbounded free field with two rigid cylinders. In this case, one cylinder is located inside the control volume and the other cylinder is located outside the control volume. The corresponding secondary sound field is the same as that shown in Fig. 7(b). The acoustic transfer impedance matrix **G** from 20 point sources to 40 control points is evaluated again. Then, the optimal secondary source strengths to reproduce the primary sound field within the control field shown in Fig. 10 are evaluated by using the least squares method. Figure 9(b) illustrates the two-dimensional primary sound field produced by a point source in an unbounded free field with a rigid cylinder and a rigid ellipsoid. In this case, the ellipsoid is located inside the control volume and the cylinder is located outside the control volume. The distance *d* between the two scattering bodies is set to be 0.5 m, 1 m, or 2 m. The location of the source and the control field are the same as those in the case of a single scatterer. The corresponding secondary sound field is the same as that shown in Fig. 7(c) except the boundary condition of the ellipsoid, which is rigid in this case. In this case, the optimal secondary source strengths evaluated for the case of two rigid cylinders are applied in this secondary field with the rigid ellipsoid. Figure 10(a) shows the average sound pressure level differences over control points between the desired values and the reproduced values when the distance between two scatterers is 0.5 m. The solid line represents the case of two cylinders. This shows successful reproduction between 300 Hz and 1500 Hz and the performance is similar to that shown Fig. 8(b). The dashed line represents the case of a cylinder and an ellipsoid. This shows worse reproduction than that in the case of two cylinders. The secondary sources still reproduce the “total” incident sound field recorded in the case of two cylinders when the scatterer inside the control field is changed from the cylinder to the ellipsoid. The change of the geometry of the scatterer inside the control field can change the sound field scattered from the cylinder outside the control volume. Figure 10(b) shows the average sound pressure level differences over control points when the distance between two scatterers is 1 m, and Figure 10(c) shows those when the distance between two scatterers is 2 m.



(a) (b) (c)

Figure 10. Average sound pressure level differences over control points when the distance between two scatterers is (a) 0.5 m (b) 1 m (c) 2 m.

These figures show the performance of the system between 300 Hz and 1500 Hz gets better as the distance between two scatterers gets longer. This is because the sound field scattered from the cylinder outside the control volume becomes weaker than the sound field from the source and the scatterer inside the control volume in the primary field as distance between two scatterers gets longer. This is one of the requirements of the validity of the assumption discussed in the previous section. Figure 10(c) shows the reasonably successful reproduction between 300 Hz and 1500 Hz when the distance between two scatterers is 2 m. Therefore, in this case of 2 m distance, the assumption of the theory of the incident sound field reproduction system is reasonable and this system can successfully reproduce the desired sound field. This good performance illustrates well the incident sound field reproduction method with two scattering bodies.

5. CONCLUSION

A study has been presented of the incident sound field reproduction method that creates a virtual acoustic field by reproducing exactly the incident sound field within a control volume. The incident sound field from the source in the primary field is not influenced by changes of the scattering body. A virtual acoustic system that reproduces the incident sound field within a control volume using multiple secondary sources is suggested. This system may produce any kind of virtual acoustic field that can be produced from arbitrary multiple phantom sources in full three-dimensional space of arbitrary shape and boundary condition. The theory of the system is explained by introducing the case of two scattering bodies. The secondary sound field is always the same as the primary sound field even though the geometry and boundary condition of the scatterer inside the control volume is changed if the sources in the secondary field reproduce exactly the sound field inside the control volume produced in the primary field for a given scattering body inside the control volume. All the conditions outside the control volume in both primary and secondary field remain same. It is assumed that all the incident sound field on the control volume is not changed when the geometry and boundary condition of the scattering body within the control volume is changed. However, this may change due to the presence of scattering bodies outside the control volume. The results of the numerical simulations for two-dimensional scattering bodies show the incident sound reproduction systems can successfully reproduce the field over a range of frequencies.

6. REFERENCES

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