LOW SIDELOBES FOR ARBITRARY ARRAYS

I J Clarke.

DRA Malvern, Worcs. WR14 3PS. UK.

Abstract: Low sidelobes are desirable for arrays operating in noise of unknown or varying directionality. These may be achieved for line arrays of equally spaced elements by Chebyshev or Taylor weighting but for arbitrary arrays, numerical optimisation is necessary. An alternative approach based on optimum beamforming techniques and fictitious interference is discussed.

1. INTRODUCTION

Sensors arrays comprising spatially distributed microphones are used in many different types of sonar application. These range from long towed arrays of thousands of devices through static sonobuoys comprising a few tens arranged in circles or on staves and, potentially, to 2-D conformal arrays of several sensors mounted on the surface of a torpedo homing head. Such arrays are generally required for the extraction of the signal arriving from a selected direction in the presence of noise and other interference. This may involve both detection and direction estimation and, in some cases, tracking. Beamforming is the process of combining the output of each of the sensors with appropriate relative time delays, phase shifts and gain to compensate for the differential propagation delays and attenuations between an emitter and each sensor. A major design task is choosing the best weights such that the sum of the weighted sensor waveforms provides the optimum spatial filter for extraction of the wanted signals in a range of operating conditions.

For narrowband signals, the minimum width of the spatial filter or beam is constrained in inverse proportion to the spatial aperture of the array measured in wavelengths and it is not possible to generate the ideal impulse shaped response with zero everywhere except in the required direction. The optimum weighting in the beamforming summation is a compromise between one that minimises the width of the mainbeam to reject nearby interference and one that minimises leakage from emitters elsewhere and from noise. The primary objective is to maximise the ratio of the power of a wanted signal to the leakage of noise and other interference in all operating conditions. The contribution from an emission at an unwanted direction is measured in terms of the ratio of the sidelobe leakage to the mainbeam gain but, to give a valid measure of signal to noise ratio (SNR), the total power of the interference must be obtained by integrating over all directions. The lowest total leakage is obtained when the contributions from each sidelobe direction are isotropic and clearly it is desirable to steer the lowest sidelobes or nulls onto the strongest interferers. However, it turns out that it is intrinsic to the beamforming operation that reducing sidelobes in one region comes at the expense of raising the response in some other directions.

Although low sidelobe weights do not provide the optimum gain across the full range of operating conditions, the associated loss of gain may be acceptable when traded-off against the extra system cost of on-line adaptive beamforming. In some circumstances, such as rapidly varying conditions, it may not be possible to design acceptable adaptive schemes and pre-calculated low sidelobe weights may provide the only solution.

LOW SIDELOBES FOR ARBITRARY ARRAYS

Low sidelobe beams are more robust to changes in the directionality of background noise and to non-Gaussian effects such as clutter or reverberation in sidelobe directions but it is inevitable that there is a trade-off between minimising mainbeam interference and reducing the general level of sidelobes. This depends on how spatially non-isotropic the threat is expected to be.

Low sidelobes may be achieved for line arrays of equally spaced elements by established weighting formulae such as given by Chebyshev or Taylor. For arbitrary arrays, numerical optimisation is generally adopted [1] but this is susceptible to stability and convergence problems. The desired pattern may be difficult to achieve because of inherent properties of the array. An approach to the design of fixed low sidelobes weights, based on optimum beamforming techniques, is outlined in this paper.

2. LOW SIDELOBE BEAMFORMER CONCEPT

In the method presented here, low sidelobe weights are calculated for arrays of arbitrary geometry by simulating a field of fictitious uncorrelated jammers and finding the optimum beamformer for that noise field. It is also shown that, to augment the predefined low sidelobe weights, further rejection can be obtained by an additional on-line adaptive beamformer with nulls steered in specific jammer directions. This contrasts with the current approach which starts with an assumption of the desired sidelobe pattern and iteratively modifies the weights towards this goal.

The new approach results from the proposition that, by using standard adaptive cancellation techniques, an optimum beamformer can be designed to suppress any threatened distribution of jammers or reverberation (clutter). This standard approach uses a single matrix inverse rather than iteration. As stated, lowered sidelobes come at the expense of raising the leakage somewhere else. This implies that there should be a jammer-free region where higher leakage can be tolerated. In the cases presented here, the two regions either side of the mainbeam are selected and this results in increased beamwidth. This jammer-free sector we term the notch and must cover approximately the null to null directions of the broadened mainbeam. In specific applications, other sectors might be available where high sidelobe leakage has less impact. Although not presented in the same terms, super-resolution schemes have long been available that are able instead to reduce beamwidth by, in effect, distributing the artificial jammers across a mainbeam region only [2]. In these approaches resolution is improved but sidelobes are degraded.

Clearly the spacing of fictitious interferers is also an important variable but it is shown in a later section using simulation examples that a uniform spatial distribution with several artificial jammers per sidelobe is adequate to demonstrate the basic concept. By suitable prior choice of the width and depth of the jammer-free sector, an easily controlled low-sidelobe weighting scheme can, in principle, be obtained for any geometry of array.

There are both fundamental and practical limitations to the method. For example, the unweighted initial beam patterns should not have excessively high sidelobe levels because the effect of placing the artificial interferers in these directions would be similar to placing a jammer in the mainbeam. The effect of this is opposite to that required because the gain of the mainbeam is then reduced causing an upward scaling of the relative sidelobe levels elsewhere.

LOW SIDELOBES FOR ARBITRARY ARRAYS

Unlike on-line adaptive beamforming, where the statistics of interference are defined irrespective of mainbeam direction, each set of low sidelobe weights must be calculated using a different distribution of artificial interferers. Because the computational cost for many beam directions is relatively high, the method is most suited to the system design stage with the weights thereafter remaining static. The weights could however be recalculated during on-line re-calibrations. Also symmetry between directions might be exploited to reduce the computational cost.

Attempting to approximate the impossible target sidelobe response function which is zero everywhere except in the signal direction leads to weight vectors that may have lower gain than necessary [1]. The loss of gain is alleviated here by assuming a finite power within the notch that is essentially jammer-free, the finite level can be regarded as modelling the system noise close to the mainbeam direction. The ratio of the powers of the artificial jammers elsewhere to this noise level allows direct control of a target value for lowering of the sidelobes. This scheme can be called 'artificial interference sidelobe cancellation' (AISLC).

It follows that the AISLC scheme can only be expected to adequately suppress levels of localised jamming below the target assumed in the artificial model of distributed interference. It should be noted that the directions of nulls in the low sidelobe patterns are arbitrary and, in general, do not correspond to the operating directions of isolated point-like jammers. However an additional on-line partially-adaptive canceller stage can be used. In this null-steering stage, any jammers with sufficient power to breakthrough the fixed low sidelobes are then further suppressed. To avoid destruction of the desired AISLC response by the subsequent adaptive process, it is the unweighted true statistics of the interference that are estimated. By suppressing only the dominant components of this data, weight jitter is also suppressed [3].

3. THEORETICAL BASIS OF THE AISLC METHOD

For simplicity of notation, the basic principles of AISLC are described in terms of a narrowband spatial analysis using complex-valued weighting only, a similar wideband system would also require correction of the relative propagation delays between the emitter and each sensor.

In a standard adaptive beamformer, the sensitivity in jammer directions is reduced while maintaining unit gain in a chosen mainbeam direction using a normalising constraint:

$$w^H c = 1 \qquad \textit{equation 1}$$

where w denotes the adapted weight and c is termed the constraint vector. The constraint vector c can be regarded as describing the phase delay and magnitude responses of the unweighted array of sensors to a wavefront from a unit magnitude emitter impinging on the array from the required mainbeam direction. The constraint c is otherwise termed a direction or steering vector. The sensor elements of the array need not be identical. In the case of spatially uncorrelated white noise, the optimum beamformer that satisfies equation 1 is given by:

$$w_c^H = \frac{c^H}{c^H c} \qquad \textit{equation 2}$$

where w_c denotes the unweighted 'conventional' beamformer. The vector inner product $w_c^H d$

performs the required beamforming operations of scaling, phase shifting and summation of the individual sensor signals that, in this case, are defined by a data vector $\boldsymbol{d}$. The result is a scalar complex value representing an estimate of the magnitude and phase of the emitter. A measure of the gain of this beamformer is obtained by comparing the output SNR to the input SNR:

$$\frac{SNR_{out}}{SNR_{in}} = \frac{(\boldsymbol{w_c}^H \boldsymbol{c}\boldsymbol{c}^H \boldsymbol{w_c})/(\boldsymbol{w_c}^H \boldsymbol{R}\boldsymbol{w_c})}{(\boldsymbol{c}^H \boldsymbol{c})/\mathrm{Tr}(\boldsymbol{R})} = \frac{\boldsymbol{c}^H \boldsymbol{c}\mathrm{Tr}(\boldsymbol{R})}{\boldsymbol{c}^H \boldsymbol{R}\boldsymbol{c}} \quad \textit{equation 3}$$

where $\boldsymbol{R}$ denotes a spatial covariance matrix defining the expected second order correlation statistics of the noise and jamming interference. In the case of white noise, $\boldsymbol{R} = \sigma^2 \boldsymbol{I}$ and the gain relative to a single sensor is equal to the number of sensors in the array.

In the non-white case, it is well known [4, 5] that the 'adapted' weight vector which maximises the gain in SNR whilst simultaneously satisfying equation 1 is given by:

$$\boldsymbol{w}^H = \frac{\boldsymbol{c}^H \boldsymbol{R}^{-1}}{\boldsymbol{c}^H \boldsymbol{R}^{-1} \boldsymbol{c}}. \quad \textit{equation 4}$$

Although the SNR is maximised for the non-isotropic noise, it is easily shown that it is always less than for the white noise case and that there is a loss of gain. This implies that it is important not to reduce sidelobes more than is necessary to met the threat; the greater the difference between adapted vector $\boldsymbol{w}$ and the unweighted vector $\boldsymbol{w_c}$ the greater is the loss of gain.

It is the matrix inverse that leads to the suppression of jammers. For a single jammer, null depth is approximately related to the inverse of the jammer power but, in the case where a number of jammers are closely spaced, corresponding nulls cannot be generated sufficiently close together and a broad region of low sidelobes spanning the jammer directions is formed instead.

The low sidelobe approximation occurs not only when the covariance matrix $\boldsymbol{R}$ is estimated on-line from incoming sensor data with closely-spaced jammers but also if the estimate $\boldsymbol{R}$ is simulated for a perceived threat from non-white noise, clutter or interference. In the method presented here, an artificial version of $\boldsymbol{R}$ is obtained by summation over a field of distributed uncorrelated artificial interferers using:

$$\boldsymbol{R_a} = \sum_j p_j \boldsymbol{c}_j \boldsymbol{c}_j^H \quad \textit{equation 5}$$

where p_j denotes the power and $\boldsymbol{c}_j$ the constraint or direction vector defining the sensor responses to the jth interferer. Lower values of p_j can be used where the unweighted sidelobes are already low or in directions where high sidelobes are a less serious threat to performance. On moving platforms for example, it may more important to suppress the far sidelobes because the high differential doppler of clutter in this region represents a greater threat than in the near sidelobes.

The sidelobe response of the 'adapted' beam in the direction of the jth interferer is given by:

$$\boldsymbol{w}^H \boldsymbol{c}_j = \frac{\boldsymbol{c}^H \boldsymbol{R}_a^{-1} \boldsymbol{c}_j}{\boldsymbol{c}^H \boldsymbol{R}_a^{-1} \boldsymbol{c}} \quad \textit{equation 6}$$

LOW SIDELOBES FOR ARBITRARY ARRAYS

The value of the sidelobe response is not necessarily smaller than that of the unweighted beamformer (and may even be greater than at the mainbeam direction). Consider a hypothetical case where one or more of the fictitious interferers is introduced into the notch close to the constraint direction (or into an excessively high sidelobe region) then, as a result of the matrix inverse, it can be deduced that the value of the denominator is reduced more quickly than the numerator. As a consequence, the levels of the sidelobes predicted by equation 6, are scaled upward by a similar proportion. It is this observation that leads to proposal to obtain control of target sidelobe levels by varying the level of artificial noise in the jammer-free notch.

If the noise notch is centred on the direction of the constraint c, then the eigenvectors U_{amin}, corresponding to the smallest eigenvalues of R_a should correlate most strongly with the constraint vector c. Since R_a is a symmetric matrix, the eigenvectors are orthonormal and U_{amin} are necessarily orthogonal to all of the other eigenvectors. This implies that U_{amin} should be essentially uncorrelated with the direction vectors of artificial interferers outside the noise sector and that it is therefore reasonable to propose an alternative low sidelobe beamformer given by:

$$w_a{}^H = \frac{c^H U_{amin} U^H_{amin}}{c^H U_{amin} U^H_{amin} c} \qquad \textit{equation 7}$$

If sidelobes are modified then $c^H U_{amin} U^H_{amin} c \neq 1$ and there is a loss of gain.

4. NULL STEERING

It was pointed out that the adaptive beamforming, using equation 4, requires an estimated covariance matrix that is not dependent on any subsequently applied constraint. It follows that a low sidelobe weight vector can, in principle, also be used as a constraint vector, subject to maintaining unit response to a reference emitter defined by the vector c:

$$w_{aj}{}^H = \frac{w_a^H R_j^{-1}}{w_a^H R_j^{-1} c}. \qquad \textit{equation 8}$$

In this case, R_j should be an estimate of the statistics of strong jammers alone. This might be obtained, for example, by partitioning R_j into two subspaces:

a) the principal components that exceed the degree of sidelobe suppression and
b) the remaining eigen components that should already be adequately suppressed by w_a.

It follows that the covariance matrix of the principal eigen components $U_{jp}\lambda_{jp}U_{jp}{}^H$ of R_j require adaptive suppression while the remaining partition $U_{js}\lambda_{js}U_{js}{}^H$ does not. The second requirement can be met simply by replacing the smaller eigenvalues λ_{js} of R_j by unit values. Optimum suppression of the stronger interferers can be achieved by using the unweighted beamformer w_c instead of w_a on the inverse $U_{js}\lambda^{-1}{}_{js}U_{js}{}^H$ of the principal partition. Combining these two observations, we obtain an partially-adaptive low sidelobe beamformer:

$$w_{aj}^H = \frac{w_c^H U \lambda_{jp}^{-1} U^H + w_a^H U_{js} U_{js}^H}{[w_c^H U \lambda_{jp}^{-1} U^H + w_a^H U_{js} U_{js}^H] c}. \qquad \textit{equation 9}$$

Since the partitioning threshold on the eigenvalues of R_j should equate approximately to the target improvement in the sidelobe levels of w_a relative to w_c, the inverted eigenvalues of the principal partition are likely to be relatively small. Setting them to zero by ignoring the principal components and satisfying the normalisation constraint, we obtain:

$$w_{aj}^H = \frac{w_a^H U_{js} U_{js}^H}{w_a^H U_{js} U_{js}^H c}. \qquad \textit{equation 10}$$

This merely steers deeper nulls or zeroes on the strong interferers that are essentially the same width and in the same position. The overall loss of SNR for the resulting 'sub-optimum' beamformer is usually negligible. Indeed, in many important potential applications, the pdf of threatened jamming or clutter may be significantly long-tailed (super-Gaussian) and deeper nulls would, in practice, improve robustness. Because adaptation is prevented outside the null regions, weight jitter is also suppressed [2].

It is well known that eigen decomposition, needed for partitioning of R_j in equation 9 or equation 10, is computationally more demanding than an inverse of the same matrix. Any alternative implementation that avoids decomposition is therefore of interest. This is provided by the well-established regularisation technique of diagonal loading the matrix R_j (equation 8):

$$w_{aj}^H = \frac{w_a^H (R_j + R_d)^{-1}}{w_a^H (R_j + R_d)^{-1} c}. \qquad \textit{equation 11}$$

In this case, the values of the full-rank diagonal matrix R_d must be of sufficient magnitude to dominate the weaker components of R_j thus preventing adaptation except on the strong interferers. The diagonal values of R_d should therefore be approximately equal to the level of sidelobe suppression. Although all eigenvalues of the artificial interference matrix are increased by the diagonal loading, the larger values are increased by a smaller proportion and the nulls steered on the stronger components by $(R_j + R_d)^{-1}$ alone would be somewhat less deep than is optimum for 'ideal' Gaussian conditions. However the low sidelobe weighting more than compensates and the product $R_a^{-1}(R_j + R_d)^{-1}$ should give over-deep nulls that can lead to better robustness if there is a threat from pulsed-type interferers with a high peak to mean power ratio (super-Gaussian). Computationally-efficient partially-adaptive versions of AISLC are then given by:

$$w_{aj}^H = \frac{c^H R_a^{-1} (R_j + R_d)^{-1}}{c^H R_a^{-1} (R_j + R_d)^{-1} c} \quad \text{or} \quad \frac{c^H U_{amin} U_{amin} (R_j + R_d)^{-1}}{c^H U_{amin} U_{amin} (R_j + R_d)^{-1} c}. \qquad \textit{equation 12}$$

LOW SIDELOBES FOR ARBITRARY ARRAYS

5. EXAMPLES FROM SIMULATION

5.1 Regular spaced linear array.

To demonstrate that the AISLC low sidelobe beamforming methods outlined here are indeed capable of generating acceptable low sidelobe weight vectors, an initial simulation was undertaken for a linear regularly-spaced array. The result can then be compared directly with a standard Chebyshev weighted version as in Figure 1.

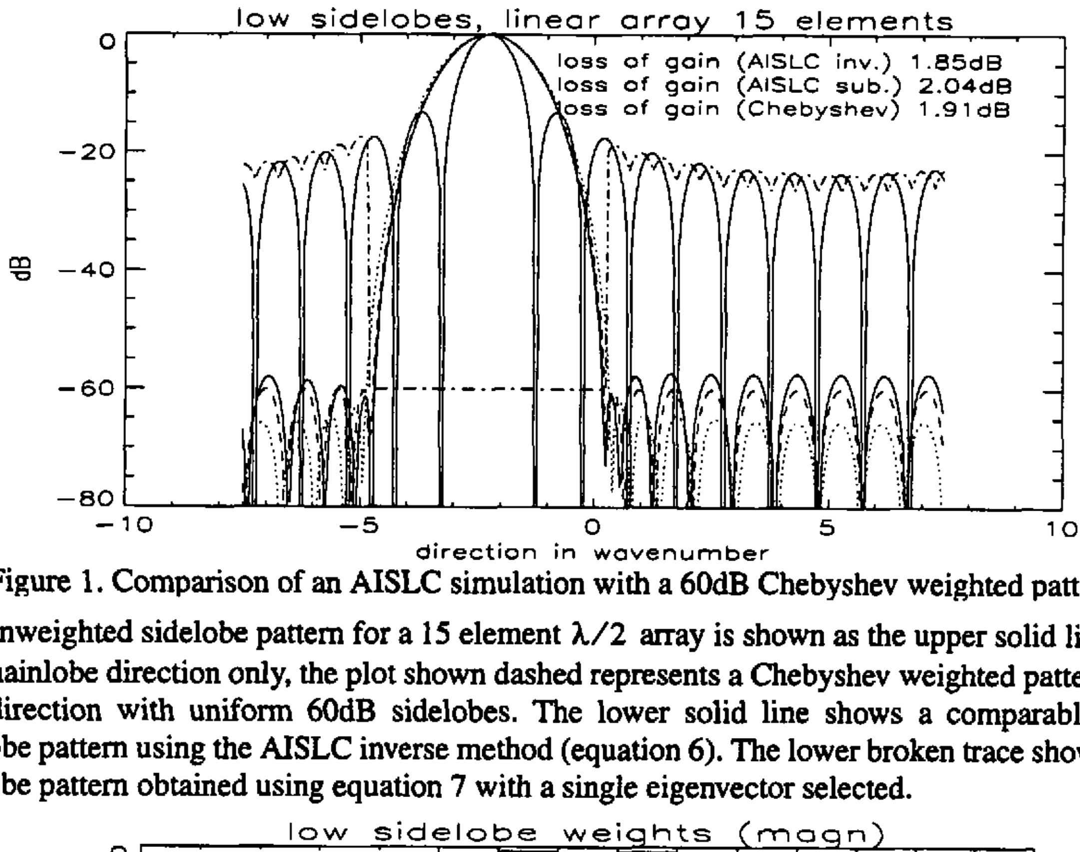

Figure 1. Comparison of an AISLC simulation with a 60dB Chebyshev weighted pattern.

The unweighted sidelobe pattern for a 15 element $\lambda/2$ array is shown as the upper solid line for one mainlobe direction only, the plot shown dashed represents a Chebyshev weighted pattern for that direction with uniform 60dB sidelobes. The lower solid line shows a comparable low sidelobe pattern using the AISLC inverse method (equation 6). The lower broken trace shows the sidelobe pattern obtained using equation 7 with a single eigenvector selected.

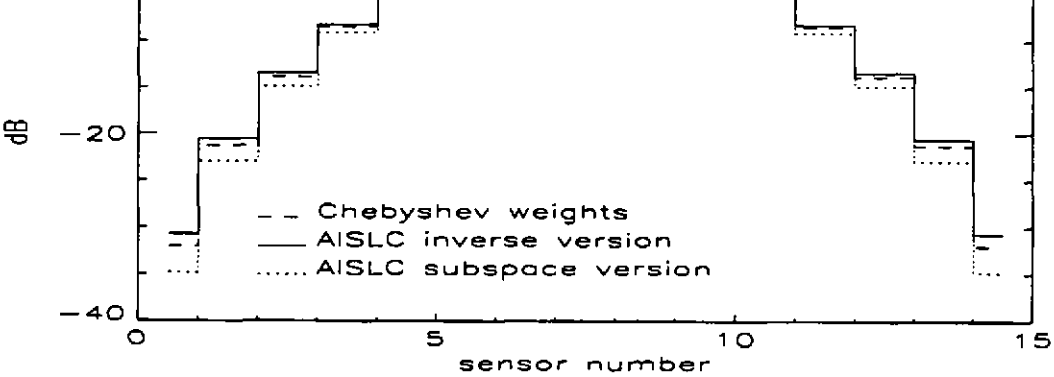

Figure 2. Comparison of the magnitudes of the beamformer weights

Figure 1 also shows (the upper broken line) the directional weighting of uniformly distributed artificial jammers used with equation 5 to generate the corresponding 'artificial' covariance matrix $\boldsymbol{R}_a$. Note the mainbeam notch and the higher weighting in the directions of higher sidelobes. The loss of array gain shown in the top right is calculated, using equation 3, for the low sidelobe weighting applied to white noise with respect to the unweighted beamformer also with white noise.

The magnitudes of the 15 sensor weights for the three low sidelobe beamformers are compared in Figure 2. It can be seen that, for regular linear array, the AISLC technique (without iteration) provides weights that are almost identical to a Chebyshev equivalent. This occurs despite the fact that Chebyshev weights are normally applied through a diagonal matrix while the matrix $\boldsymbol{R}_a^{-1}$ is not diagonal.

5.2 Circular array (omni-directional sensors).

Figure 3 shows low sidelobe patterns simulated for a circular 15 element array of omni-directional sensors (one wavelength radius). Sidelobes of approximately 35-40 db are evident as opposed to 10-15dB sidelobes in the unweighted pattern. In the broken low sidelobe pattern, one null, marked by the vertical dashed line, is steered using the diagonal loading method (equation 11). It can be seen that partial adaptation avoids destroying the low sidelobes elsewhere. The weighting used for the artificial jammers (uniform) and for the noise notch are again shown together with the unweighted conventional beam pattern. The loss of 12.6dB gain in isotropic noise relative the unweighted beamformer is a potentially serious concern.

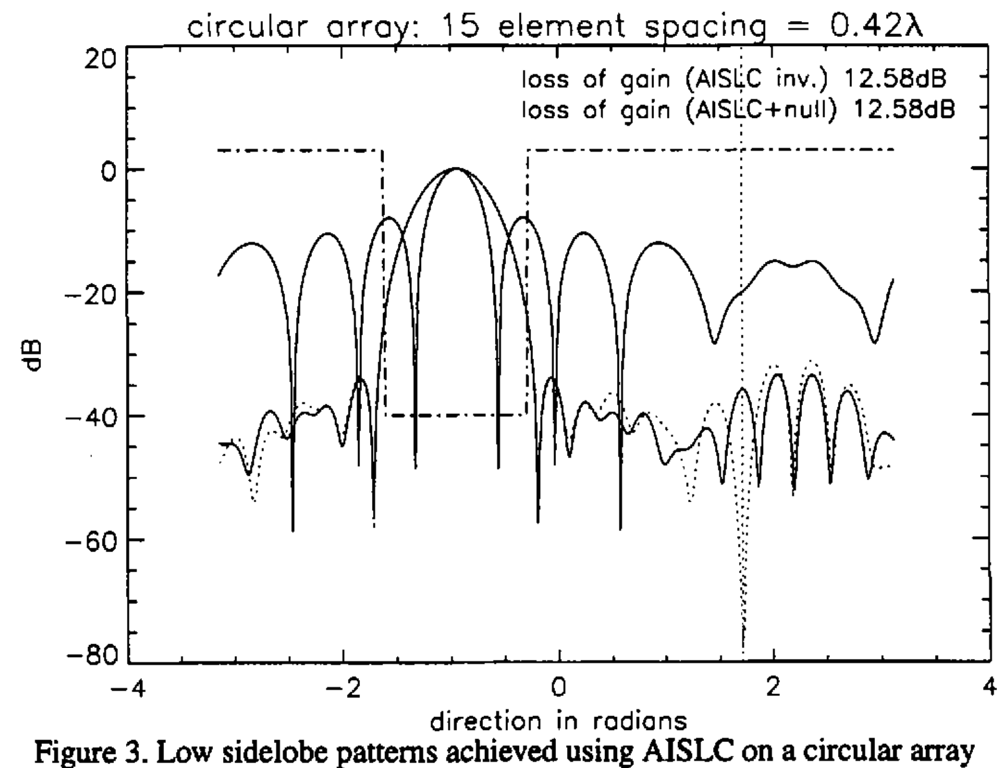

Figure 3. Low sidelobe patterns achieved using AISLC on a circular array of omni-directional sensors.

5.3 Circular array (cardiod sensors)

Figure 4 shows plots for a similar array with outward-facing cardiod pattern sensors. Note the significantly better loss of gain. This occurs because the sidelobe leakage is already lower than for the omni-directional case. Individual sensors should also exhibit higher gain arising from the (half-sinusoid) directional patterns and there is scope for using AISLC to optimise these.

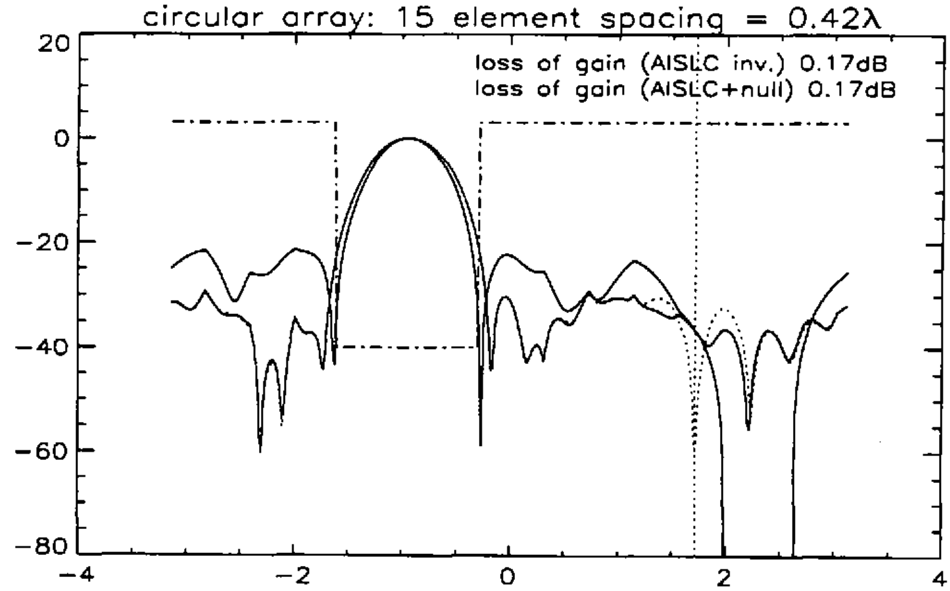

Figure 4. Low sidelobe patterns achieved using AISLC on a circular array of outward facing cardiod pattern sensors.

5.4 Stave array.

Figure 5 shows similar plots for a 5 stave array with 3 sensors per stave, again with a radius of one wavelength. Low sidelobes (with some asymmetry) are achieved despite the poor initial pattern. The stave directions and three nulls, steered using diagonal loading, are also shown.

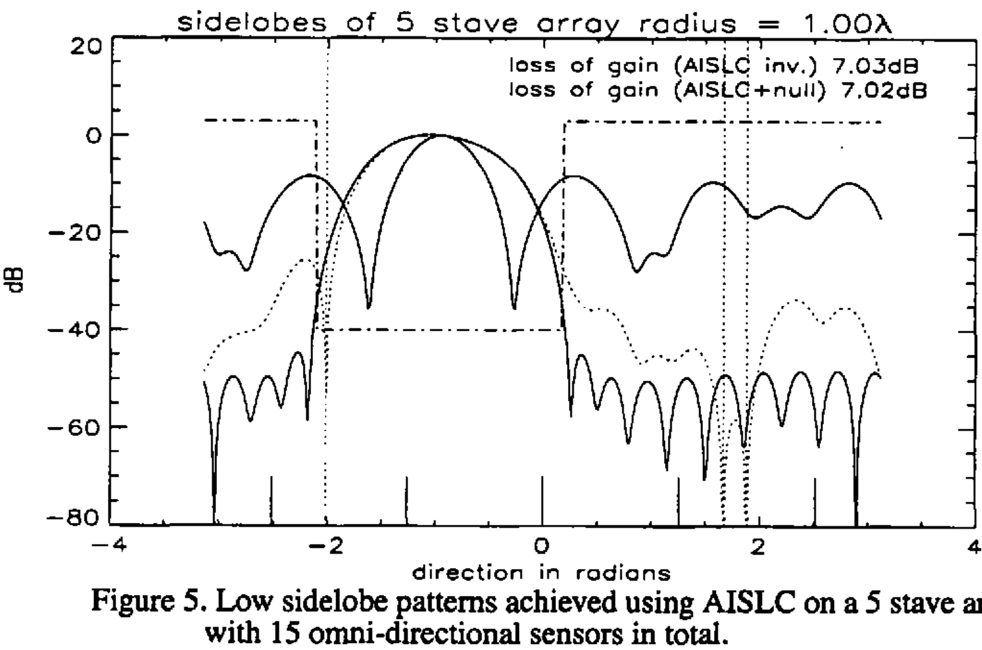

Figure 5. Low sidelobe patterns achieved using AISLC on a 5 stave array with 15 omni-directional sensors in total.

6. CONCLUSIONS

It has been demonstrated that fixed weight, low sidelobe beamformers can, in principle, be successfully computed for arrays of arbitrary geometry by applying the established principles of adaptive cancellation to an simulated field of fictitious interferers. This field could define a deemed potential threat from jammers, clutter or reverberation and is equivalent to the prior in a Bayes approach to the problem. It is shown that it is possible to control the target sidelobe level to within reasonable accuracy without iteration and that the detailed shape of the pattern can be subsequently adjusted by changing the weights of some of the artificial jammers. It is also concluded that it is possible to steer several sidelobe nulls using a subsequent computationally-efficient partially adaptive process without destroying the predefined low-sidelobe properties. Steering nulls into the mainlobe causes an upward scaling of the sidelobe levels.

To minimise the inevitable loss of gain, it is important not to lower sidelobes more than is necessary to met the specified threat. In the AISLC methods outlined here, control of the mean sidelobe level can be achieved by careful choice of the width and depth of mainbeam notch in the field of artificial jammers. Clearly, a trade-off between the different desirable features of fixed sidelobe patterns can be implemented by tailoring the relative weighting of artificial interferers in the field. To cater for a wider variety of operating conditions in a specific application, it is feasible to switch between several sets of pre-calculated low sidelobe weight vectors and augment these by partially-adaptive null steering as necessary in prevailing conditions.

It is noted that accurate calibration of the sensor system is a prerequisite and there are inherent limitations on the degree of sidelobe reduction that can be achieved for a specific geometry. Further investigation is desirable into understanding the fundamental principles of the array design. There is scope, at the design stage, for optimising both the array geometry and the directional response of individual sensors or sub-arrays to minimise loss of gain.

It is suggested that application-specific versions of AISLC, with or without null steering, can be developed from the basic approaches outlined here, in particular to suit conformal arrays, wideband signals and to compensate for defective sensors.

REFERENCES

1. PJ Wright and DH Brandwood. "Re-optimisation of linear and planar arrays with failed elements." *Antennas and Propagation, 4-7 April 1995*, pp81-84 Conference publication No. 407 IEE 1995.

2. M Bertero, P Brianzi, P Parker and RE Pike. "Resolution in diffraction-limited imaging, a singular value analysis." *Optica Acta*. 1984 vol. 31 no. 2 pp 181-202.

3. DT Hughes and IJ Clarke. "Jitter Reduction in Adaptive Sonar Signal Processing." See proceedings of this conference.

4. See for instance BD Van Veen and KM Buckley. "Beamforming: a versatile approach to spatial filtering." *IEEE ASSP Magazine*, pp4-24 April 1988.

5. Monzingo and Miller. Introduction to adaptive arrays. J Wiley (1980).

6. L J Griffiths. "New results in constrained beamforming: non-linear constraints and constraint modulus output." Eusipco 90, Signal Processing V, Vol. 1 Theories and Applications, p1-8. Elsevier (Sept.1990).