Proceedings of the Institute of Acoustics

PARTIALLY-BLIND MATCHED FILTERING

I J Clarke.

DRA Malvern, Worcs. WR14 3PS. UK.

Abstract: The adaptive signal separation techniques proposed in this paper are aimed at improving detection and estimation robustness in non-ideal propagation conditions such as caused by multipath. The basic idea is to relax the often excessively demanding ideal assumptions made in existing algorithms used for analysing antenna array data. For example, if the wavefronts arriving at the sensor system suffer distortion due to inhomogeneous propagation and these distortions are on a scale somewhat larger than the sensor separations then the longer-range correlations between widely separated sensors must be regarded as unpredictable and therefore noise-like. However local correlations between neighbouring sensors can be used for direction finding purposes.

We propose estimating degrees of confidence in the predicted cross-correlation between pairs of sensors and applying these as individual weights in a 'correlation shading' function. We then require a steering *matrix* to model the second order correlation statistics of a signal from a given direction rather than a conventional steering *vector*. Our approach is analogous to the maximum likelihood concept of prewhitening additive sensor noise but, in this case, it is the anticipated wavefront distortions that are regarded as the cause of estimation error or 'noise'.

We also propose a novel blind pre-processing stage for partially suppressing large signal components or interference. This technique, unlike the somewhat similar Wiener filter approach, does not fully cancel interference using a matrix inverse but is instead based on re-weighting Eigencomponents. Subsequent model-fitting artifacts are attenuated because the partial suppression is independent both of the shape of wavefront and of sensor geometry. Weak signals are passed essentially unchanged by the blind preprocessor while strong signals are attenuated to a predefined level chosen such the residuals due to calibration error become unimportant. Estimation and detection robustness achievable in the presence of inaccurate signal models are significantly improved.

1. INTRODUCTION

Sensors arrays are used in many different types of sonar application. Such arrays are generally required for the detection of an unknown number of separate emissions arriving from unknown directions in the presence of unknown noise, multipath and other interference. Existing data analysis algorithms often include means for beamforming or spatially filtering to separate possible emissions. For accurate beamforming, knowledge of the precise geometry of the array and calibration of the responses of the individual digitised sound transducers must be acquired. A further assumption is usually essential that propagating signal wavefronts are exactly spherical (or plane). Clearly such ideal assumptions are not always fully satisfied in a typical sonar envi-

2. CALIBRATION

A major calibration task in beamforming is that of finding the best complex weights and compensating delays to provide the optimum spatially matched filter for any given direction-of-arrival (DOA). In the ideal case, the optimum beamformer in a given direction is defined either directly by prior experiment using a real emitter at a known location or, indirectly, by analytic computation. The latter requires prior knowledge of the physical sensor geometry, propagation constants and transducer characteristics. In second order processing, expected cross-correlations between the two waveforms from all pairs of sensors are recorded or predicted for a selected DOA and stored either in a correlation matrix or, more economically, as a steering vector. It is normally sufficient for such calibrations to be provided for discrete DOA using a grid of closely-spaced values rather than a true analytic parametric model with continuous variables.

3. CORRELATION SHADING

For simplicity of notation, the basic principle of correlation shading is described here in terms of a *narrowband* spatial analysis using complex-valued weighting, a wideband system would require that the relative propagation delays also be compensated by the beamformer.

In a classical beamformer, unit response can be specified in a chosen mainbeam direction θ using a inner product of two vectors:

$$w^H(\theta)c = 1 \qquad \textit{equation 1}$$

where $w(\theta)$ denotes the complex-valued beamforming weight vector and c is termed the constraint vector. c is derived from the calibration and can be regarded as describing the relative phase delay and magnitude responses of the uncompensated array of sensors for a DOA of θ. Otherwise termed a direction or steering *vector*, c defines the responses to the wavefront from a unit magnitude, zero phase emitter impinging on the array from the required direction. The sensor elements of the array need not be identical, coplanar or regularly spaced. In the case of spatially uncorrelated white noise, the optimum beamformer that satisfies equation 1 is given by:

$$w_c^H(\theta) = \frac{c^H}{c^H c} \qquad \textit{equation 2}$$

where $w_c(\theta)$ denotes the unweighted beamforming vector. The vector inner product $\hat{s} = w_c^H(\theta)d$ performs the required scaling, phase shifting and summation of the individual sensor signals. The latter are denoted by data vectors d (snapshots) and $\hat{s}$ denotes the beamformed estimates of the phase and magnitude of the emitter temporal samples for a given value of θ.

Reconstructed versions of the data vectors are obtained by scaling the constraint vector c by $\hat{s}$:

$$\begin{aligned} \hat{d} &= c\hat{s} \\ &= c\frac{c^H}{c^H c}d \end{aligned} \qquad \textit{equation 3}$$

The vector outer product cc^H in the numerator defines the predicted correlation *matrix* (unweighted) for a unit magnitude emitter. The inner product $c^H c$ in the denominator normalises this matrix such that the combination acts as a projection matrix or spatial filter with unit gain.

It is a common practice to taper or shade the elements of the vector c^H with a vector t such that:

$$\hat{d} = c\frac{[t^H \bullet c^H]}{[t^H \bullet c^H]c}d \qquad \textit{equation 4}$$

where [•] denotes an element by element multiply, in this case between two between vectors. The usual objective of shading is that of reducing the sidelobe leakage levels of the spatial filter. The penalty for shading is a reduced suppression of noise due to an increase in the width of the spatial filter. In this case, the effective signal correlation model is defined by the weighted columns of the outer vector product $c[t^H \bullet c^H]$ normalised to unit response by the scalar inner product term in the denominator.

A primary objective of this paper is to show that, rather than weighting the columns of cc^H, the elements can be individually weighted using $W \bullet cc^H$ and performance degradation due to certain types of calibration error can be minimised.

We note from equation 5 that each element of the estimated vector $\hat{d}$ is evaluated using one of the rows of a shaded correlation matrix $[W \bullet cc^H]$. In principle, the norms of the rows of this matrix can be scaled arbitrarily and it becomes necessary to separately re-normalise each element of the estimated data vector:

$$\begin{aligned}\hat{d} &= \bullet\!\!-\frac{[W \bullet cc^H]d}{[W \bullet cc^H]c} \\ &= [C_W(\theta)]d\end{aligned} \qquad \textit{equation 5}$$

where •— indicates that an element by element divide is required, in this case of the vector in the numerator by the vector in the denominator. For simplicity of notation, $[C_W(\theta)]$ denotes a shaded correlation matrix where the rows are appropriately normalised.

We see from beamformer defined by equation 5 that the phases and magnitudes of $\hat{d}$ at a given sample in time are individually estimated for each sensor using a row vector from $[C_W(\theta)]$. It is possible, by suitable choice of shading *matrix* W, to select which input sensor waveforms in d are averaged to evaluate each element of $\hat{d}$. For example, W can be of diagonally banded form with values well away from the leading diagonal are set to zero. In the case of a linear array, the outer diagonal widely-spaced elements correspond to correlations between outer sensors where poor correlation might be expected. Only local correlations between near-neighbour sensors are utilised by this form of shading. We realise that the detrimental effects of an unknown smooth distortion of an incident wavefront are then significantly reduced and there is an improved potential capability for fully cancelling a large signal in a Least Mean Squares manner. However, because fewer sensors are averaged, it must be expected that the improvement of SNR (beamforming gain) will be reduced. This loss is inevitable in cases where longer range correlations are unpredictable through wavefront distortion or where array curvature is not known accurately.

Furthermore, since the time series waveforms can be evaluated individually for each sensor, it is possible to extract the shape of the wavefront for each signal or, in some cases, to deduce the shape of the array. In the latter case, it is then possible to adaptively correct the constraint vectors c and to reduce the degree of shading applied. As a consequence, the loss of beamformer gain would then be reduced and weaker signals would become easier to detect and estimate.

Clearly, as in any non-adaptive filtering process, if more than one signal is present simultaneously then, in general, sidelobe leakage occurs, detections may be missed and estimates of DOA may be biased. This effect can be reduced by using Eigen-based high resolution algorithms such as MUSIC or, more effectively, by using a modified form of an iterative technique such as IMP.

In the case of MUSIC, for each DOA, each of the selected Eigenvectors is projected in turn onto the normalised shaded steering *matrix* and a sum-of-squares measure of likelihood is evaluated:

$$S(\theta) = \sum_{i}^{\text{subspace}} \left\| [C_W(\theta)] u_i \right\|^2 \qquad \textit{equation 6}$$

u_i denotes one of either the set of signal or set of noise subspace eigenvectors of the data covariance matrix. The relevant DOA are estimated from the maxima or minima respectively of the likelihood estimator $S(\theta)$.

In the case of iterative algorithms, the largest signal can be first estimated by finding the DOA at the global maximum of:

$$S(\theta) = \sum_{\text{time}} \left\| [C_W(\theta)] d \right\|^2 \qquad \textit{equation 7}$$

The temporal summation provides non-coherent integration of several data 'snapshots' equivalent to that of a spatial covariance estimator. Subsequently, any already detected and estimated signals are first subtracted from the data and an estimate of DOA of a possible additional signal is made using for example:

$$S(\theta) = \sum_{\text{time}} \left\| \frac{[C_W(\theta)](d - \Sigma\hat{d})}{[C_W(\theta)](c - \Sigma\hat{c})} \right\|^2 \qquad \textit{equation 8}$$

The subtracted terms in equation 8 represent predictions of the effects of leakage, using the currently estimated values of DOA for detected signals. As in IMP, nulls are steered in the denominator to compensate for the similar nulls steered in the numerator. It is also necessary, immediately after each successful additional detection, to iteratively re-estimate all of the values of DOA to remove possible bias caused by leakage of the previously undetected component. This re-estimation process can also be used to track non-stationary conditions.

The weighting *matrix* W can be defined directly by measuring or, indirectly, by analytically predicting the consistency of correlation between each pair of sensor waveforms across many different environmental conditions. Predictable terms are given unit weight, other terms lower weight in a manner that ensures that the unpredictable residuals are suppressed at least to the level of other noise. The technique represents a powerful generalisation both of the concepts of pre-whitening and of the sub-aperture or sliding window approaches already in widespread use. The idea can also be applied to the analysis of time series data.

4. BLIND PARTIAL CANCELLATION

The Wiener filter method of beamforming is established for pre-whitening non-white noise and for adaptively cancelling interference. It takes the form, with suitable re-normalisation to provide unit gain as required in equation 1:

$$w_w^H(\theta) = \frac{c^H R^{-1}}{c^H R^{-1} c} \qquad \textit{equation 9}$$

where $\boldsymbol{R}$ denotes the expectation of the covariance of the noise and interference. In practice, it is only possible to estimate this matrix and imperfect prewhitening results. A particular problem is that of ensuring that the estimate of $\boldsymbol{R}$ does not comprise leakage from signals because then these are also cancelled to some degree. We propose to modify the Wiener filter formulation in such a way that strong signals are partially cancelled but still easily detectable while weak signals are not affected and thus also detectable if they exceed the detection threshold:

$$w_w^H(\theta) = \frac{c^H U_R \lambda_{\text{mod}} U_R^H}{c^H U_R \lambda_{\text{mod}} U_R^H c} \qquad \textit{equation 10}$$

where U_R is a column matrix comprising the Eigenvectors of $\boldsymbol{R}$ and λ_{mod} denotes a diagonal matrix comprising modified Eigenvalues. If the modification simply involves inverting the Eigenvalues then equation 10 reverts to the case of the Wiener filter. Alternatively, if it is arranged that the values of the products $\lambda_{\text{mod}}\lambda$ do not exceed some suitable value (about 15-25dB above the mean noise subspace Eigenvalues), then Eigenvalues below this limit are not affected and conventional beamforming occurs. However, larger Eigencomponents are attenuated such that, in general, the corresponding larger signals are reduced to the limit level. This effect is independent of the constraint vectors and hence of the accuracy of calibration.

A similar effect is achieved without the expense of Eigen decomposition by a technique resembling diagonal loading. Here a matrix comprising a scaled estimate of the noise covariance matrix N is added to $\boldsymbol{R}$ prior to evaluating the inverse:

$$\begin{aligned} w_w^H(\theta) &= \frac{c^H (R + kN)^{-1}}{c^H (R + kN)^{-1} c} \\ &= \frac{c^H (R/k + N)^{-1}}{c^H (R/k + N)^{-1} c} \end{aligned} \qquad \textit{equation 11}$$

We see from the equivalent second equation that, if N is approximately an identity matrix then the effect of the loading is equivalent to 'diluting' the contribution from the signals matrix R by a factor k such that weak signals are again unaffected. If N is non-white then prewhitening can, in principle, be achieved simultaneously without additional computation.

Either form of partial cancellation can be used to improve the appearance of a display showing beamformed data. The masking of weaker signals by large components is small and the effective beamwidth of strong localised interferers is much reduced.

Where necessary, it is also possible to apply high resolution methods to estimate multiple closely-spaced emitters using partial cancellation to reduce the sensitivity to calibration errors. In the case of the signal subspace version of the MUSIC algorithm, equation 10 allows a partial cancellation to be implemented as follows:

$$S(\theta) = \frac{c^H U_{R\mathrm{sig}} \lambda_{\mathrm{modsig}} U^H_{R\mathrm{sig}} c}{c^H U_R \lambda_{\mathrm{mod}} U^H_R c} \qquad \textit{equation 12}$$

where the subscript $_{\mathrm{sig}}$ identifies signal subspace Eigencomponents. For iterative methods, equation 11 permits better economy in the computations.

6. CONCLUSIONS

We have presented two ideas for addressing the difficulty caused by poor accuracy of calibration. Such errors limit the degree of cancellation of an emitter from the data. Valid prior calibrations are clearly not feasible if propagation conditions are variable or, for example, if the shape of a long towed array varies according to unknown currents or tow-ship track. However, if it can be established that wavefront distortions are on a scale somewhat larger than the sensor separations, then local correlation estimates can still be predicted accurately. Clearly, attempting to exploit longer-range correlations between widely separated pairs of sensors is potentially detrimental to the beamforming summation because incorrectly weighted sensor waveforms may have arbitrary relative phase thus preventing coherent spatial integration. Since cancellation residuals are proportional both to signal magnitude and the magnitude of calibration errors, stronger signals tend to cause the greatest problems.

The first solution that we have proposed involves applying a 'correlation shading' function to the correlation *matrix* used to specify the weights of a beamformer. Although the resulting spatial filter achieves a lower gain than an ideal coherent beamformer (without error), the algorithm promises to be more robust to errors in the signal models. The beamforming gain achieved is greater than can be obtained by non-coherent non-beamforming methods.

The second idea takes the form of a 'blind' pre-processing stage that relies on re-weighting Eigenvalues of the data covariance matrix such that larger signal components are partially suppressed. This suppression is independent both of the array calibration and shape of wavefront. Localised signals remain detectable after partial cancellation and can be subsequently fully cancelled using an iterative high discrimination technique such as IMP to provide an estimated residual that is primarily non-localised noise. It is therefore not essential to provide an independent signal-free interference matrix and this advantage is potentially important to many active and passive sonar designs. It is somewhat of a bonus that this method can be used to suppress sidelobe leakage of interferers, allowing a cleaned up display presentation of beamformed data with or without calibration errors.

Since the respective waveforms of the emitters (time series) can be estimated more accurately by using either or both of the proposed methods, it is in principle, possible to estimate valid mutually orthogonal time domain filters. In principle, by re-applying these to the data matrix, the relevant spatial response vectors of the array can be estimated adaptively. In turn, these might be used adaptively re-estimate either selected parameters of the transducers, array geometry or relatively difficult calibration variables such as mutual coupling.

The combinations of blind pre-processor and (model-fitting) beamformers with modified weights can be regarded as 'partially blind'.

REFERENCES

1. I J Clarke. "High discrimination target detection algorithms and estimation of parameters." Proc. NATO ASI on Underwater Acoustic Data Processing July1988, *ed Y T Chan,* Kluwer Academic pp 273-278 1989.

2. DT Hughes and IJ Clarke. "Jitter Reduction in Adaptive Sonar Signal Processing." Proc. IOA Vol 17 Pt 8 1995 Sonar Signal Processing Conference, Loughborough University of Technology pp 305-314 Dec. 1995.

3. I J Clarke. "Low sidelobes for arbitrary arrays." Proc. IOA Vol 17 Pt 8 1995 Sonar Signal Processing Conference, Loughborough University of Technology pp 233-242 Dec. 1995.

4. See for instance BD Van Veen and KM Buckley. "Beamforming: a versatile approach to spatial filtering." *IEEE ASSP Magazine*, pp4-24 April 1988.

5. Monzingo and Miller. Introduction to adaptive arrays. J Wiley (1980).

6. L J Griffiths. "New results in constrained beamforming: non-linear constraints and constraint modulus output." Eusipco 90, Signal Processing V, Vol. 1 Theories and Applications, p1-8. Elsevier (Sept.1990).