NEW ANALYTICAL EXPRESSIONS FOR PREDICTING OUTDOOR SOUND PROPAGATION OVER A COMPLEX TERRAIN

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1. SUMMARY

The purpose of the present work is to develop an original simple method to predict traffic noise propagation. New analytical expressions have been found to evaluate both ground and diffraction effects at quite large distances. The ground effect expressions are approximations for soft and hard surfaces of the well-known Chien and Soroka formulation. The diffraction expression is based on the properties of the first Fresnel ellipsoid applied to single absorbing screen attenuation. Then it is extended by means of G.T.D. principles to more complex impedance barriers such as embankments, wedges and three-sided obstacles. If combined, these two approaches give a new simple method for calculating sound attenuation over most traffic terrain configurations.

2. GROUND EFFECT

The following ground attenuation formulas are derived from the expression of the reflected field of a spherical harmonic wave emitted by a point source S above an impedance plane ground in the rest atmosphere [1]. Here the specific admittance \( \beta \) is calculated using the Delany and Bazley's model [2] for a ground characterized by its flow resistivity \( \sigma \). Attenuations stand for \( 20 \log_{10} \left( \frac{\text{total field}}{\text{free field}} \right) \) throughout the paper.
Attenuation for rather soft surfaces \((\alpha < 10^7 \text{Pa.s.m}^2)\) and long range propagation. In traffic noise situations this is related to the receiver side ground effect. Attenuation is given by (see notations on Fig. 1):

\[
\text{Att}_g = 10 \log_{10} \left[ \frac{4k^2}{d^2} \left( z_S^2 - \sqrt{2} C z_S + C^2 \right) \left( z_R^2 - \sqrt{2} C z_R + C^2 \right) \right] \leq 6 \text{dB} \tag{1}
\]

with: \(C = \frac{d}{k} \times \frac{1 + 3W e^{-\frac{\sqrt{W}}{x}}}{1 + W} \), \(W = k \cdot \rho^2\), \(k\) being the wave number.

Attenuation for rather hard surfaces \((\alpha \geq 10^7 \text{Pa.s.m}^2)\) and middle range propagation. It is related to the source side ground effect. We have:

\[
\text{Att}_g = 10 \log_{10} \left[ \left( A + \cos \left( \frac{2kz_S z_R}{d} \right) \right)^2 + \left( B - \sin \left( \frac{2kz_S z_R}{d} \right) \right)^2 \right] \leq 6 \text{dB} \tag{2}
\]

with: \(A = \left( 1 - \frac{2kz_S z_R}{d^2} \right) \left( 1 + \frac{\sqrt{2} \rho d}{z_S + z_R} (F - 1) \right)\),

\[B = \left( 1 - \frac{2kz_S z_R}{d^2} \right) \frac{\sqrt{2} \rho d}{z_S + z_R} (F + 1), \quad F = \frac{1}{1 + \frac{\sqrt{2W} + W}{k(z_S + z_R)^2}}.
\]

\[W = k(z_S + z_R)^2\]

3. DIFFRACTION EFFECT

The Attenuation expressions are derived from Fresnel theory and G.T.D. [3] [4]. We consider the diffraction negligible when the screen is not penetrating the first Fresnel ellipsoid.

Attenuation by a screen. The Fresnel number \(N\) associated to the path \(S-Q-R\) (see notations Fig. 2, \(\lambda\) being the wavelength) is:

\[N = \text{sign}(\theta - \pi) \times 2(a + b - d) / \lambda, \quad \text{with: sign}(\alpha) = \begin{cases} 1 & \text{for } \alpha < 0 \end{cases}
\]

In a traffic terrain configuration, \(d \gg 1\) and \(\theta\) is close to \(\pi\). Deygout [4] gave a system of simple equations to evaluate attenuation \(\text{Att}_d\) versus \(N\):

\[
\begin{align*}
\text{Att}_d = 0 & \quad \text{for } N < -0.25 \\
\text{Att}_d = -6 + 12 \sqrt{-N} & \quad \text{for } -0.25 \leq N < 0 \\
\text{Att}_d = -6 - 12 \sqrt{N} & \quad \text{for } 0 \leq N < 0.25 \\
\text{Att}_d = -8 - 8 \sqrt{N} & \quad \text{for } 0.25 \leq N < 1 \\
\text{Att}_d = -16 - 10 \log(N) & \quad \text{for } N > 1
\end{align*}
\tag{4}
\]

Fig. 2, screen

Wedge. From [3], the diffracted field for a rigid wedge may be written as the sum of 4 elementary diffracted fields relative to the 4 paths \((i = 1-4)\) SQR, S'QR, SQ'R and S'QR'(see Fig. 3). Applying the simple screen approximations, one gets:

\[
\text{Att}_d = -20 \log_{10} \left[ \sum_{i=1}^{4} \frac{\text{Att}_d(N_i)}{20} \right] \tag{5}
\]
where: \[ \zeta = \frac{d}{a+b}, \quad \gamma_i = \frac{a + b + d_i}{2(a+b)}, \]
and \[ \xi = \frac{1}{2 + \frac{1}{4} \sqrt{\frac{d_i + a - b}{d_i - a + b}}} \]

\( d_i \) is the \( i \)th direct path appearing in the GTD formulation (that is SR, S'R, SR' and S'R').
Att\( _i(x) \) is calculated from eq. (4). In many traffic noise configurations, \( \zeta, \gamma \) and \( \xi \) are close to 1 as in the following examples.

**Double-wedge.** From [5], diffraction by a rigid double-wedge can be easily expressed as the product of two simple-wedge diffractions. Thus all the approximations made before remain valid and the attenuation takes the form:

\[
\text{Att}^{3w}_d = -20 \log_{10} \left( 2^\zeta \prod_{i=1}^{2} \left( \sum_{i=1}^{2} \xi_i^{10} \right)^{20} \right)
\]

where \( \zeta = d / (a + w + b), \) \( \xi_i, \gamma_i \) and \( N_i \) are related to the \( i \)th path diffracted on the \( j \)th edge, and calculated as previously (see Fig. 4). These paths defined in [5] are either SQ1Q2, S'Q1Q2, S''Q2R, S''Q2R', or RQ2Q1, R'Q2Q1, R''Q1S, R''Q1S', according to the position of Q1 and Q2.

### 4. APPLICATION TO REAL CASES. COMPARISON WITH B.E.M. [6]

The different studied cases are shown in Fig. 5. Comparison with B.E.M. calculations are shown in Fig. 6.

![Fig. 5. Description of the different terrain configurations [distances in m, \( \sigma \) in 10^3 Pa.s.m^−1]](image-url)
In a real traffic configuration, the impedance on one surface of the obstacle or ground is taken into account by multiplying the elementary diffracted field related to the \(i^{th}\) path by \((10^{\text{Att}^{(20)}_{i}} - 1)\) or/and \((10^{\text{Att}^{(20)}_{i}} - 1)\); however, this approximation fails when the grazing angle is not small enough.

![Graphs showing attenuation for each configuration](image)

**Fig. 6.** Attenuation for each configuration. B.E.M., ——Analytical method

### 5. CONCLUSIONS

Comparisons with B.E.M. calculations show good agreement. Thus this method may be used widely for traffic noise propagation problems, especially for engineering purpose for it is easy to compute. Further developments are in progress in order to include rounded obstacles as well as meteorological effects in this analytical model.

### REFERENCES


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