

# EQUIVALENT ACOUSTIC ABSORPTION MODEL OF FLAT HETEROGENEOUS WALLS

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## 1 INTRODUCTION

This study concerns the acoustic characterization of flat heterogeneous walls in industrial rooms. The National Institute of Research and Security (INRS) employs the RAY+ software<sup>1</sup> to predict the sound levels in industrial rooms. This uses acoustic absorption coefficients associated to each wall (or obstacle) of the rooms. A wall in an industrial room can be constituted of different materials (brick, windows, doors,...) and in such a case the problem is to replace this heterogeneous wall by an equivalent homogeneous wall to which an average absorption coefficient is associated, in order to simplify the discretisation process needed by the software. The equivalent average absorption coefficient can be determined from the arithmetic average of the elementary absorption coefficients  $\alpha_i$  of every elementary area  $S_i$  constituting the wall (Sabine's technique). The work presented here concerns the study and the improvement of techniques for determining an equivalent acoustic absorption of flat heterogeneous facings.

The different theoretical reverberation time models, which depend on the average acoustic absorption of the room estimated by Sabine's method, were compared with measurements. The variations in reverberation time observed experimentally led us to modify the calculation method of the average acoustic absorption. The variations in the reverberation times obtained with the new expression of the average acoustic absorption were compared with the experimental results measured in a industrial room to study the validity of the new expression of the average acoustic absorption.

## 2 AVERAGE ACOUSTIC ABSORPTION FORMULAE

This paragraph reviews the models of the average acoustic absorption  $\bar{\alpha}$  used in each reverberation time formula respectively. For each formula, air absorption is ignored. The reverberation time expression depending on mean free path  $l_m$  is :

$$T_r = \frac{0,04.l_m}{\bar{\alpha}} \quad \text{where} \quad l_m = \frac{4V}{S} \quad (1)$$

V and S are the volume of the room and the total wall area respectively.

### 2.1 Sabine's formula

In diffuse sound field conditions, for a reverberant room with walls of a homogeneous geometrical and acoustic nature, Sabine<sup>2</sup> defines the average absorption coefficient of the walls  $\bar{\alpha}$  as the arithmetic average of the area elements  $S_i$  associated with the absorption coefficient  $\alpha_i$  :

$$\bar{\alpha} = \frac{1}{S} \sum_i S_i \alpha_i \quad (2)$$

$\alpha_i$  is the absorption coefficient used by professionals. It is not rare to find  $\alpha_i$  higher than 1 for certain absorbent materials. The average acoustic absorption expression doesn't take into account the materials locations on the walls.

## 2.2 Eyring's formula

The Eyring formula<sup>3,4</sup> takes into account relatively high absorption coefficients :

$$\alpha_{Ey} = -\ln(1-\bar{\alpha}) \quad (3)$$

The limited development shows that for low absorptions, the Eyring absorption coefficient is similar to that of Sabine :

$$\alpha_{Ey} = -\ln(1-\bar{\alpha}) = \bar{\alpha} + \frac{\bar{\alpha}^2}{2} + \dots + \frac{\bar{\alpha}^n}{n} \quad (4)$$

if  $\bar{\alpha}$  is low then  $\alpha_{Ey} \approx \bar{\alpha}$

## 2.3 Millington's formula

To treat the problem of  $\alpha_i$  higher than 1, Millington<sup>5</sup> suggests replacing  $\bar{\alpha}$  by :

$$\alpha_{Mil} = -\frac{1}{S} \sum_i S_i \ln(1-\alpha_i) \quad (5)$$

This model presents a drawback when one of the areas is very absorbent because, in this case, the reverberation time is close to 0. To allow the Millington formula to be used, Dance and Shield propose a conversion graph decreasing the high values of the Sabine absorption coefficient<sup>6,7</sup>.

## 2.4 Kuttruff's formula

Kuttruff<sup>8,9</sup> established a reverberation time formula for rooms with diffuse walls and for non-uniformly distributed absorption. He assumes that, for diffuse reflections, the walls of the room reflect the sound according to Lambert's law. He suggests correcting the Eyring formula by introducing the variance  $\gamma^2$  of the mean free path :

$$\alpha_{Kut} = \alpha_{Ey} \left( 1 - \frac{\gamma^2}{2} \alpha_{Ey} \right) + \frac{\sum_i (1-\alpha_i) \cdot (\bar{\alpha} - \alpha_i) \cdot S_i^2}{S^2 \cdot (1-\bar{\alpha})^2} \quad (6)$$

$\gamma^2 \approx 0.4$  for rectangular rooms.

## 2.5 Arau Puchades's formula

For rectangular rooms, Arau-Puchades<sup>10</sup> defines an average acoustic absorption based on the Eyring's model for every wall parallel to every direction of the space :

$$\alpha_{ArP} = \left[ -\ln(1-\alpha_x) \right]^{\frac{S_x}{S}} \times \left[ -\ln(1-\alpha_y) \right]^{\frac{S_y}{S}} \times \left[ -\ln(1-\alpha_z) \right]^{\frac{S_z}{S}} \quad (7)$$

where  $S_x, S_y, S_z$  are respectively the areas of walls normal to the coordinate system x, y, and z.  $\alpha_x, \alpha_y, \alpha_z$  are the average acoustic absorption coefficients of these walls.

## 2.6 Modified Fitzroy's formula

The acoustic absorption model used in this formula is the Eyring's model, as for the Arau-Puchades's formula. Recently, R.O. Neubauer<sup>11</sup> proposed a modified Fitzroy formula that takes into

account the non-uniformity of the absorption of parallel walls. The absorption model used in this new formula is based on Kuttruff's model :

$$\alpha^* \approx \alpha_{Ey} + \frac{\sum_i (1-\alpha_i) \cdot (\bar{\alpha} - \alpha_i) \cdot S_i^2}{S^2 \cdot (1-\bar{\alpha})^2} \quad (8)$$

## 2.7 Pujolle's formula

Pujolle<sup>12</sup> uses the equivalent absorption of Eyring in the reverberation time formula but he introduced a new formula for the mean free path  $l_m$  that takes into account the dimensions of the rooms. Pujolle firstly proposes :

$$l_m = \frac{1}{6} \times (\sqrt{L^2 + l^2} + \sqrt{L^2 + h^2} + \sqrt{h^2 + l^2}) \quad (9)$$

and then,

$$l_m = \frac{1}{\sqrt{\pi}} \times (L^2 \cdot l^2 + L^2 \cdot h^2 + h^2 \cdot l^2)^{1/4} \quad (10)$$

where L, h and l are respectively the length, the height and the width of the rooms. The relation (10) seems appropriate because it results directly from the guided propagation in rectangular ducts.

## 3 STUDY OF THE REVERBERATION TIME IN REVERBERANT ROOM FOR TWO FACINGS

To study the influence of heterogeneous areas on the average acoustic absorption, the reverberation times were measured in a reverberant room for two heterogeneous areas. The experimental results were compared with the theoretical reverberation time calculated using the average absorption models deccribed in the previous section.

### 3.1 Geometrical description of both facings

Both facings studied were made up of 19 glass wool panels, located flat on the floor around the walls (facing n°1, see figure 1 a) or up in periphery on the four walls of the reverberant room (facing n°2, see figure1 b). These panels have absorption coefficients relatively constant and close to 0.9 in the frequency range 500 Hz - 4000 Hz. In the reverberant room, the concrete walls were very reflective, the difference between both absorption coefficients creating the heterogeneity.

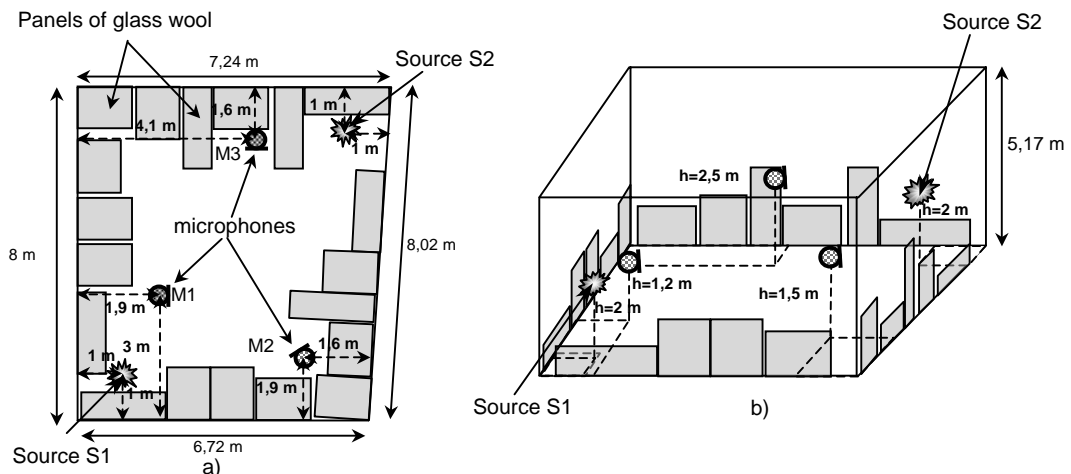


Figure n°1 Glass wool panels arranged in the reverberant room : a) Facing n°1 - b) Facing n°2

Sources S1 and S2 are explosions emitted successively by a 9 mm blank pistol. Reverberation time was estimated from the mean decay of the acoustic energy obtained for every measurement point (the three omnidirectional microphones M1, M2 and M3). It was accurately determined from the linear decay included 10 dB below the maximal sound level and 10 dB above background noise. The dynamic of the linear decay was about 30 dB.

### 3.2 Experimental results

The experimental results obtained for the two source positions and the two facing configurations are presented in third octave bands in figure 2.

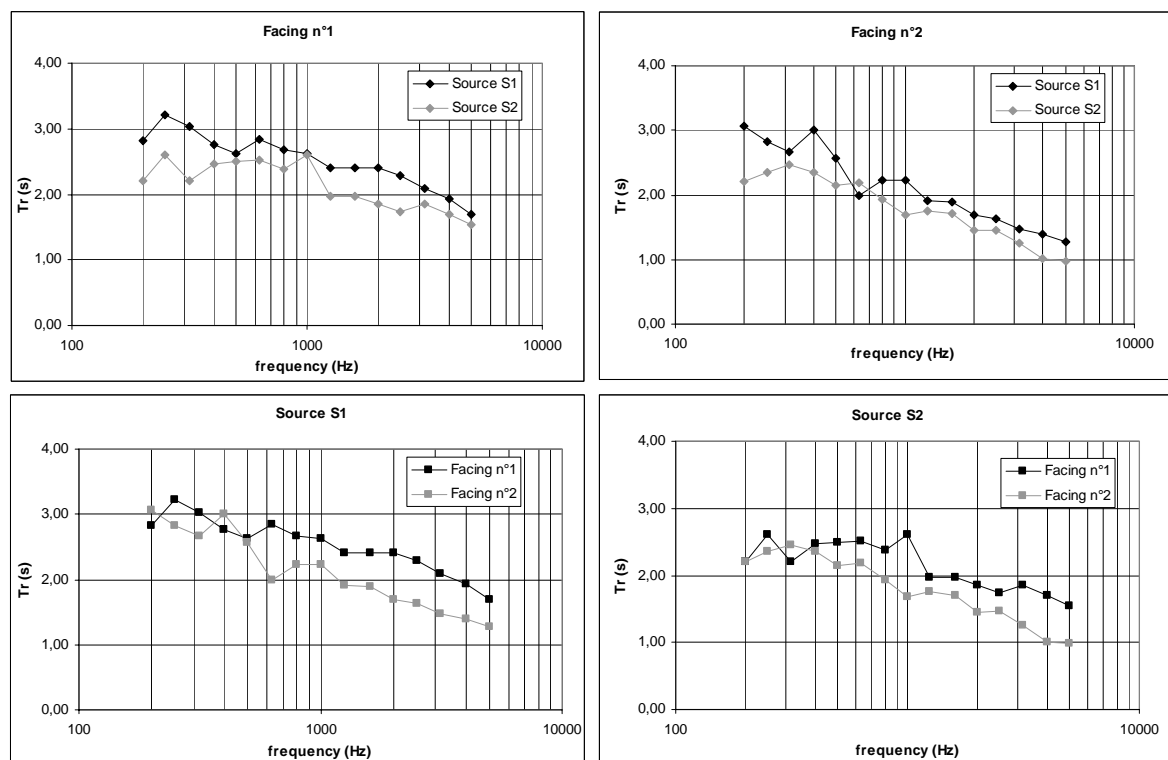


Figure 2 : Experimental reverberation time for the facings nos. 1 and 2 and for each source

Figure 2 shows that the position of both the source and the panels has an impact on reverberation time. The relative position of the sound source and absorbent panels must therefore be taken into account in the theoretical  $T_r$  model.

In another publication<sup>13</sup>, the reverberation times determined by the theoretical models were compared with those obtained experimentally. It resulted from this comparison that the  $T_r$  theoretical values were different from the experimental ones (maximal relative error was between 40 % and 60 %). The theoretical  $T_r$  models depend on the average acoustic absorption coefficient of the room which does not take into account the distribution of the absorbent panels on the walls and the source positions S1 and S2. In our experiment, the acoustic absorption area is constant for the facings n°1 and n°2. So, the theoretical  $T_r$  models don't predict the reverberation time variation observed between the two different facings and the two source positions.

## 4 MODIFIED FORMULA OF THE AVERAGE ACOUSTIC ABSORPTION

A geometric parameter which can take into account the position of every absorbent panel referred to the source position, is the solid angle  $\Omega$ . It is defined at the point source and contains all the sound beams coming from the source and directly absorbed by the panel. It represents the equivalent area of absorbent panels covered by the sound source.

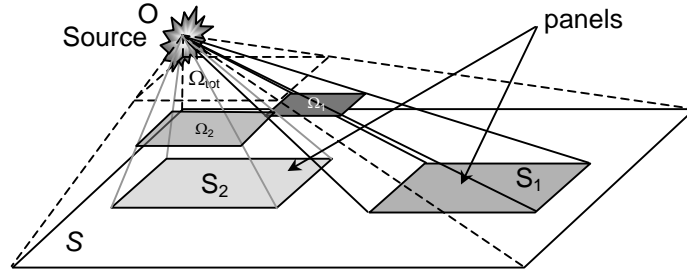


Figure 4 : Illustration of the solid angles ratio

We replace the area ratio by the solid angle ratio in the Sabine's theoretical equivalent acoustic absorption formula :

$$\bar{\alpha}' = \frac{1}{\Omega_{\text{tot}}} \sum_i \Omega_i \alpha_i \quad (11)$$

The models of Eyring, Millington, Kuttruff, Pujolle, Arau-Puchades and modified Fitzroy then become :

- Eyring :  $\bar{\alpha}'_{\text{Ey}} = -\ln(1 - \bar{\alpha}')$  (12)

- Millington :  $\bar{\alpha}'_{\text{Mil}} = -\frac{1}{\Omega_{\text{tot}}} \sum_i \Omega_i \ln(1 - \alpha_i)$  (13)

- Kuttruff :  $\bar{\alpha}'_{\text{Kut}} = \bar{\alpha}'_{\text{Ey}} \left( 1 - \frac{\gamma^2}{2} \bar{\alpha}'_{\text{Ey}} \right) + \frac{\sum_i (1 - \alpha_i) \cdot (\bar{\alpha}' - \alpha_i) \Omega_i^2}{\Omega_{\text{tot}}^2 (1 - \bar{\alpha}')^2}$  (14)

- Arau-Puchades :  $\bar{\alpha}'_{\text{ArP}} = \left[ -\ln(1 - \bar{\alpha}'_x) \right]^{\frac{S_x}{S_{\text{tot}}}} \times \left[ -\ln(1 - \bar{\alpha}'_y) \right]^{\frac{S_y}{S_{\text{tot}}}} \times \left[ -\ln(1 - \bar{\alpha}'_z) \right]^{\frac{S_z}{S_{\text{tot}}}}$  (15)
  - with  $\bar{\alpha}'_x = \frac{1}{\Omega_x} \sum_i \Omega_i \alpha_i$  ,  $\bar{\alpha}'_y = \frac{1}{\Omega_y} \sum_i \Omega_i \alpha_i$  ,  $\bar{\alpha}'_z = \frac{1}{\Omega_z} \sum_i \Omega_i \alpha_i$

- Modified Fitzroy :  $\bar{\alpha}'^* \approx \bar{\alpha}'_{\text{Ey}} + \frac{\sum_i (1 - \alpha_i) \cdot (\bar{\alpha}' - \alpha_i) \cdot \Omega_i^2}{\Omega_{\text{tot}}^2 (1 - \bar{\alpha}')^2}$  (16)

- Pujolle :  $\bar{\alpha}'_{\text{Puj}} = \bar{\alpha}'_{\text{Ey}} = -\ln(1 - \bar{\alpha}')$  (17)

## 5 CHANGE IN REVERBERATION TIME WITH THE SOLID ANGLE

To show the influence of the solid angle in the calculation of the average absorption, it was decided to study four different facings of a wall in an industrial room. 8 absorbent perforated panels (thickness 50 mm) and 16 absorbent fibralith panels (thickness 50 mm) were used to modify the facing of the wall. The dimensions of these panels were  $1.25 \times 2 \text{ m}^2$  in the case of the perforated panels and  $0.5 \times 2 \text{ m}^2$  for the fibralith panels. For the four different configurations the solid angle of every panel vary in relation to the source, which remained in the same position (see figure 5). The treated surface remained constant for the four configurations ( $36 \text{ m}^2$ ).

- Configuration (a) : Eight assemblies, each constituted of a perforated panel and two fibralith panels, are distributed along the entire length of the wall facade. They are 1,2 m apart.
- Configuration (b) : The panels are juxtaposed in the bottom of the room on the wall and form a long absorbent partition.
- Configuration (c) : The same arrangement as configuration (b) but the panels are distributed in relation to the center of the wall.
- Configuration (d) : The same arrangement as configuration (b) but the panels are arranged near the entrance of the room close to the source.

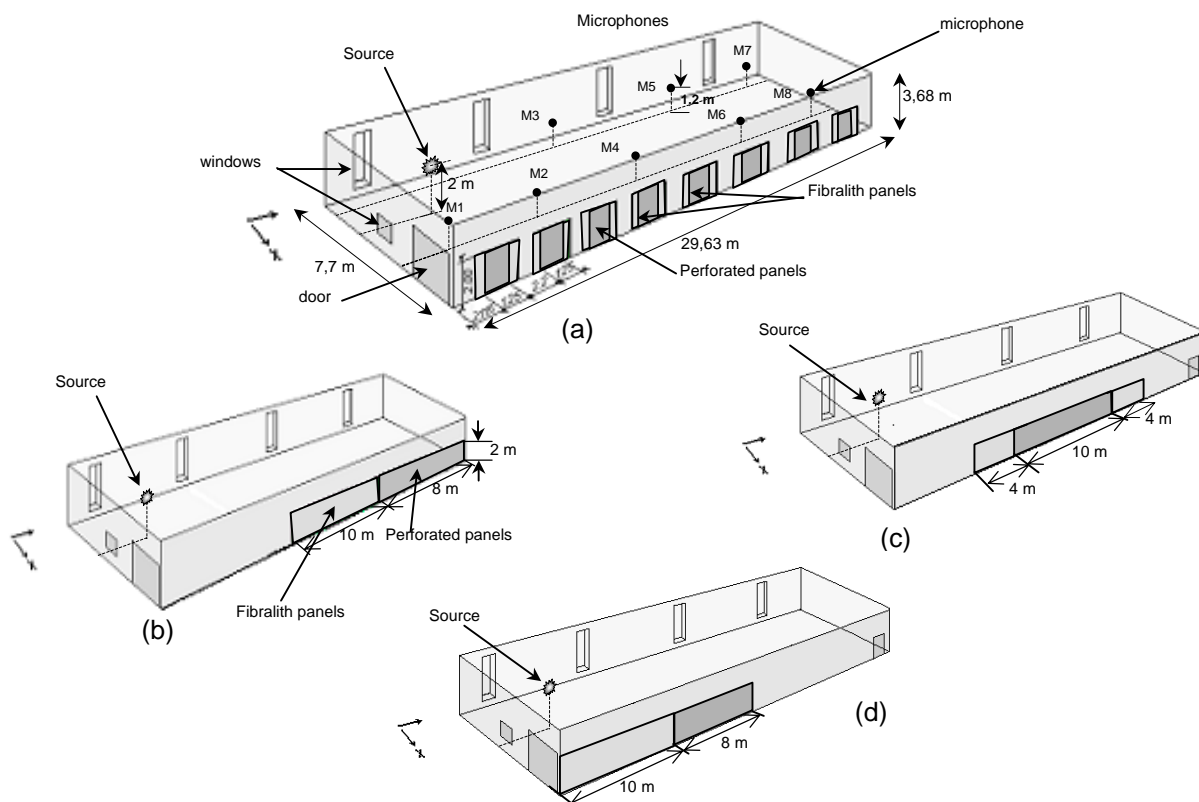


Figure 5 : Distribution of the absorbent panels on the wall facade of the industrial room

The reverberation time was measured for the four configurations using an explosion emitted by a blank pistol at a height of 2 m. Eight microphones placed along the entire length of the room were located 1,2 m high.

Taking the  $T_r$  of the configuration (a) as reference for the others configurations, we compared the change in the experimental and theoretical reverberation time. The theoretical models are those modified by taking into consideration the solid angles. As we noticed before, the classical models don't vary with the wall configuration since the panels area is the same. Figure 6 shows the theoretical and experimental differences in reverberation times between the reference configuration (a) and respectively configurations (b), (c), (d).

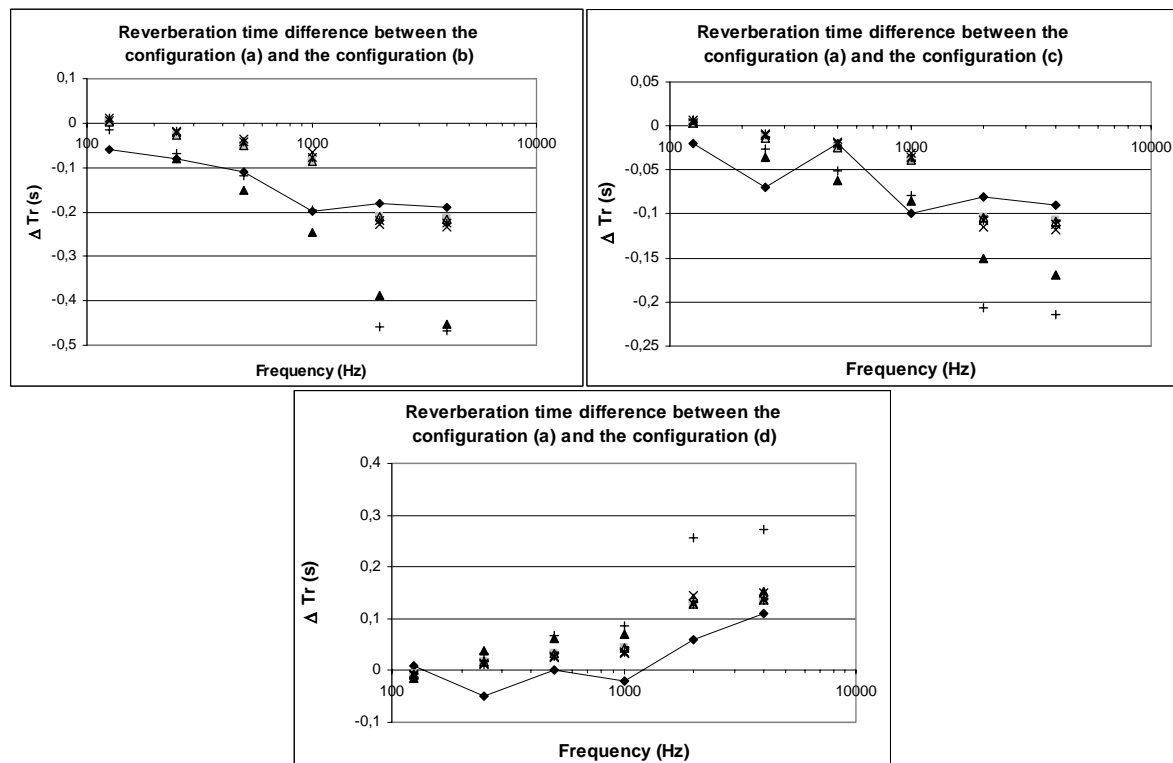


Figure 6 : Comparison of the change in the experimental and theoretical reverberation time  
 ■ Sabine's model ;  $\triangle$  Eyring's model ;  $\times$  Millington's model ; + Pujolle's model ; \* Kuttruff's model ;  
 ▲ Arau Puchades's model ; ▣ Modified Fitzroy's model ; —◆— Experiment

First, an evolution of the measured  $T_r$  between the different configurations exists. Then, between every configuration, the evolution in theoretical reverberation time agree with experiment. Taking into account solid angles in average absorption models is a sensitive way to determine the variations in  $T_r$  between every configuration. At low frequencies, the difference in the reverberation times is small ( $<0.05$  s) because the panels are not very absorbent. This can reach 0.2 s at high frequencies, particularly if the panels are located close to the source. The influence of the relative position of the absorbent panels and sound sources on the sound field has therefore been proved, and taking into consideration the solid angle in the absorption models allows this influence to be quantified.

The study showed that the sound field is always influenced by the relative positions of the sound source and materials. As the reverberation time is conditioned by the first reflections on the walls, if the absorbant material is close to the source, the standard Sabine's model can no longer be used, and it must be modified by taking into account the solid angles. During a study carried out by INRS in 2001, it was demonstrated that when the material is placed far from the source, models including the solid angle are no longer appropriate : the surface ratio must be again envisaged<sup>13</sup>.

## 6 CONCLUSION

This work has allowed the study of the influence on reverberation time of spatial non-uniformity of acoustic absorption in an industrial room. Many measurements of the reverberation time in a reverberant room were carried out with two facings. A difference in experimental reverberation time due to the spatial non-uniformity of absorption, and to the relative position of the sound source and materials, was observed. Hence, the source position in relation to the position of the absorbent panels in the calculation of the average absorption of the rooms was considered in the solid angle. We thus replaced the surface ratio by the solid angle ratio in all the standard average acoustic absorption formula. The change in reverberation time obtained for the analytical reverberation time formula compared to the experiment was clearly respected for all the models.

The possibility of replacing a heterogeneous facing by an equivalent homogeneous one was also studied at INRS<sup>13</sup>. The solid angles were therefore taken into consideration in the equivalent acoustic absorption model. We showed that the acoustic pressure field simulated with the RAY+ software for an industrial room in the case where one of the walls was heterogeneous, then in the case where this was replaced by an equivalent homogeneous wall, did not vary by more than 1 dB. But the method to calculate the solid angles requires precise knowledge of the location of the source and of all the dimensions of every homogeneous cover constituting the heterogeneous wall in relation to the position of the source. Geometric discretisation of the heterogeneous walls of the room therefore remains necessary to determine the equivalent wall.

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