MODELLING GROUND ROUGHNESS EFFECTS IN A SUBSTITUTE SOURCES METHOD (SSM)

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ABSTRACT

In a previous implementation of the substitute sources method (SSM), a random medium is modelled as a transfer from a coherent field into an incoherent, random field along the propagation path [1]. Outdoor acoustics with focus on urban environments is the main topic of interest, for which the random medium is due to the turbulence in the atmospheric boundary layer. The aim of the present study is to incorporate a randomly rough surface in a similar way. In the model the calculation is made in steps along the propagation path and in each step one removes from the initially coherent field the random part, which can be seen as the diffuse ground reflection. The properties of the random field at each step are used together with the coherent field to calculate the sound pressure level at a receiver.

1 INTRODUCTION

Small local changes in the sound field due to a random medium or random boundary properties can greatly affect the sound reaching a listener. For outdoor acoustics shadow regions formed by barriers or refraction can experience largely changed sound pressure levels due to the random, turbulent medium. In the present paper the influence on the sound field of random roughness elements in the ground surface are at focus. The largest influence is expected to take place for situations with strong interference effects. An example thereof could be a reverberant courtyard or a city canyon with multiple reflections that interfere positively in the absence of roughness elements, giving strong resonances.

The models for the random roughness effects are here developed to be part of a substitute sources method (SSM) previously used for the modelling of turbulence effects [1]. The SSM is applicable to steep geometries, e.g. a high barrier close to the source [2]. In the SSM only forward propagation has been considered so far. Taking account of backscattering, as in two-way parabolic equation (PE) methods [3], gives the possibility to model e.g. a reverberant courtyard.

In the SSM, the sound field due to an original source is represented by a distribution of (substitute) sources on a plane surface. The surface is called the substitute surface and the substitute sources thereon can be seen as Huygens' secondary sources, which can give the sound field using the Rayleigh integral. Many substitute surfaces can be put between the source and the receiver, with separation distances large compared with the wavelength. In the modelling of turbulence effects in Ref. [1], the propagation is calculated in steps from one surface to the next, first for a non-turbulent atmosphere.

The effect of turbulence is that it causes a loss in coherence of the sound field. Within each step the unperturbed, coherent field loses power into a residual, random field. The coherent field is further propagated toward the receiver and at each substitute surface a residual, random part is taken out. The contributions from different surfaces are assumed to be uncorrelated, and the total power at the receiver is found by adding the power from the coherent field to the powers from the residual fields.

The Green function for the sound pressure contribution at the receiver due to each substitute source is found for a non-turbulent atmosphere. All the contributions are summed up to give the estimated power by taking into account their mutual coherence due to the turbulence. A special mutual coherence function for the residual field is used. Since the calculation of the source strengths and the Green functions do not involve turbulence, many methods could be used. For instance ray-methods would be efficient for a homogeneous atmosphere or a linear sound speed profile. A fast field program (FFP) was used in Ref. [1].

In the present paper, the randomness is given by the ground only, i.e. the atmosphere is here assumed to be free from turbulence. The effects of random ground roughness are planned to be incorporated in a similar way as the turbulence effects were modelled in Ref. [1]. This means that the separation into coherent and residual fields and the coherence of the residual field need to be estimated for a randomly rough ground. This is shown in the next Section and thereafter follow concluding remarks. The paper describes a work in progress and no results are shown from calculations using the SSM including ground roughness.

2 THEORY

2.1 The Rayleigh integral

The Rayleigh integral can be used to calculate the sound pressure level in a medium provided that the particle velocity in normal direction to a plane surface and the Green function for the medium are known. (Alternatively, a Rayleigh integral can be formulated for known pressure.) The propagation through a non-turbulent atmosphere, with for instance refraction, or through a snap-shot of a turbulent atmosphere could be described by a Green function. The plane surface is here one of the substitute surfaces S_i, $i=1...N$, and the normal component of the velocity, v_i, of the sound field at the surface S_i constitutes the strengths of the substitute sources. (See Figure 1.) In the two-dimensional (2-D) implementation used here, the surface is transformed into a line, but still referred to as a substitute surface. The resulting pressure amplitude at a receiver position, $p(x,z)$, from the velocity on surface S_i can be written

$$p(x,z)=\frac{\mathrm{j}\omega\rho}{2\pi}\int_l v_i(\sigma)G(r)\,\mathrm{d}\sigma \tag{1}$$

where l is the line of integration, ω is the angular frequency of a time-oscillation, $\mathrm{e}^{\mathrm{j}\omega t}$, with time t, and ρ the medium density. In equation (1) G is the Green function $\pi/\mathrm{j}H_0^{(2)}(kr)$, with $H_0^{(2)}$ the Hankel function of zero order and second kind, and r the distance to the receiver from the z co-ordinate σ on the surface.

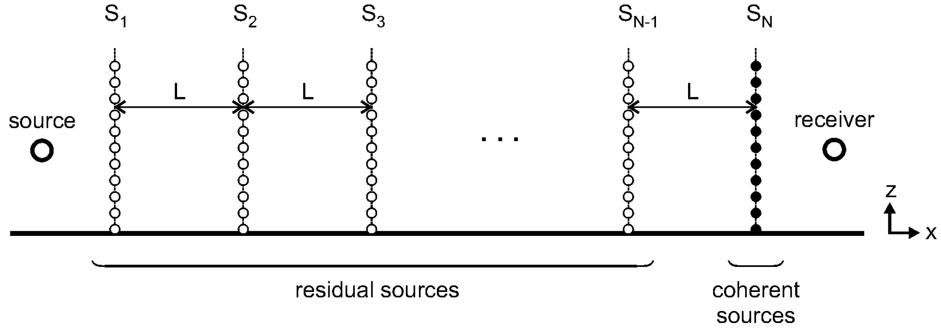

Figure 1. Substitute surfaces, S_i, with separation distances L.

2.2 Numerical estimation of ground reflections

The ground modelled here is hard except from small holes that are randomly distributed. That the holes are small here means that their openings, a, and depths, b, fulfil $ka, kb << 1$, where k is the wave number of the sound in the air. By furthermore assuming that the velocity, v, in normal direction upward from each hole, is constant over the opening, each hole can be treated as a point source whose amplitude is determined by fulfilling the boundary condition. The interaction between all holes together with the excitation from the original sound source then forms a system of equations. Solving the equation system gives the velocities of the hole openings, and the pressure at the receiver can be calculated.

Let $p_{\mathrm{inc},m}$ be the free field pressure from the original source at the centre position of hole m. (See Figure 2.) If there are M holes, the final pressure above each hole, $p_{\mathrm{tot},m}$, can be written as

$$p_{\mathrm{tot},m} = p_{\mathrm{inc},m} + \sum_{n=1}^{M} v_n g_{mn} \tag{2}$$

where g_{mn} is the Green function for the pressure contribution to hole m due to the velocity v_n at hole n. The final pressures, $p_{\mathrm{tot},m}$, and the velocities, v_m, must also fulfil the impedance at the opening m:

$$p_{\mathrm{tot},m} = Z_m v_m \tag{3}$$

where the impedance, Z_m, used for the small cavities can be written as

$$Z_m = -\frac{\rho c^2}{\mathrm{j}\omega b} \tag{4}$$

where $\omega = k/c$, c is the sound speed and b the depth of the hole.

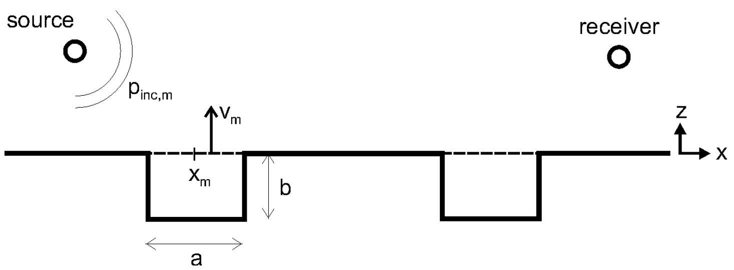

Figure 2. Geometry for the modelling of the ground roughness elements.

The equation system arising from equations (2) and (3) can be written in matrix form as

$$(\mathbf{G} - \mathbf{Z}) \times \mathbf{v} = -\mathbf{p}_{\mathrm{inc}} \tag{5}$$

where g_{mn} make up the elements (m,n) of the matrix $\mathbf{G}$ with size $M \times M$, $\mathbf{Z}$ is diagonal with elements Z_m given by equation (4), $\mathbf{v}$ is the vector with M elements v_m and $\mathbf{p}_{\mathrm{inc}}$ is the vector containing elements $p_{\mathrm{tot},m}$.

Since the velocities on the openings are assumed to be constant, the Green functions g_{mn} can be written as (cf. equation 1)

$$g_{mn} = \frac{\omega\rho}{2} \int_{x_n - a/2}^{x_n + a/2} H_0^{(2)}[k(x - x_m)]\,\mathrm{d}x \tag{6}$$

where the x axis is along ground and where x_m and x_n are at the centres of holes m and n respectively.

From the assumption that the opening is small ($ka << 1$), the approximation is made that the integrand is constant over the opening, giving

$$g_{mn} \approx a\frac{\omega\rho}{2}\ H_0^{(2)}[k(x_n - x_m)]. \tag{7}$$

The above approximation is only used for $m \neq n$ since for $m = n$ the integrand is singular. An analytical integration is made for the case $m = n$, resulting in Hankel and Struve functions, $S_{0,1}$, which thereafter are approximated by low order terms of series expansions. In detail, one gets, from Ref. [4, Eq. 11.1.7]

$$\int_0^t H_0^{(2)}(\tau)\,\mathrm{d}\tau = tH_0^{(2)}(t) + \frac{\pi}{2}t[S_0(t)H_1^{(2)}(t) - S_1(t)H_0^{(2)}(t)]. \tag{8}$$

Using the expansions in Ref. [4, Eq. 9.1.7–9.1.9], gives $S_0(t) \approx \frac{2}{\pi}t$, $S_1(t) \approx \frac{2}{\pi}\frac{t^2}{3}$, $H_0^{(2)}(t) \approx 1 - \mathrm{j}\frac{2}{\pi}\ln(t)$ and $H_1^{(2)}(t) \approx \frac{t}{2} + \mathrm{j}\frac{1}{\pi}\frac{2}{t}$. Neglecting second and higher order terms in t, and going back to the notation of equation (7) results in

$$g_{mn} \approx a\frac{\omega\rho}{2}\left\{1 + \frac{2\mathrm{j}}{\pi}\left[1 - \ln\left(\frac{ka}{2}\right)\right]\right\}. \tag{9}$$

The pressure at the receiver, p_{rec}, is the sum of the direct wave from the original source, p_{dir}, and the wave reflected in the ground surface, p_{refl}. A decomposition is made of p_{refl} into two parts. The first part is the reflection as from an infinitely hard ground, $p_{\mathrm{refl},\infty}$, which is calculated as from an image source with phase and amplitude equal to that of the original source. The second part consists of the sum of contributions from the hole openings with velocities v_m. Again using that $ka << 1$, as in equation 7, gives

$$p_{\mathrm{rec}} = p_{\mathrm{dir}} + p_{\mathrm{refl},\infty} + a\frac{\omega\rho}{2}\sum_{m=1}^{M} v_m H_0^{(2)}(kr_m) \tag{10}$$

where r_m is the distance from the centre of opening m to the receiver.

2.3 Coherence of ground reflections

In Figure 3 a geometry with two sources and two receivers at range L are shown. For the case where the pressure p_1 is only due to source 1 and pressure p_2 is only due to source 2, the average of the square of the combined pressure amplitudes after random realisations of the ground roughness can be written as

$$\left\langle |p_{\text{tot}}|^2 \right\rangle = \left\langle |p_1 + p_2|^2 \right\rangle = \left\langle |p_1|^2 \right\rangle + \left\langle |p_2|^2 \right\rangle + 2\Re \left\langle p_1 p_2^* \right\rangle . \qquad (11)$$

The quantity $\left\langle |p_{\text{tot}}|^2 \right\rangle$ is proportional to the power of the signal at the receiver, and in the following the quantity $W = \frac{1}{2}\left\langle |p_{\text{tot}}|^2 \right\rangle$ is referred to as the power. (The sound pressure level is then found as $L_p = 10\lg \frac{W}{2\cdot 10^{-5}}$ dB re $2\cdot 10^{-5}$ Pa.)

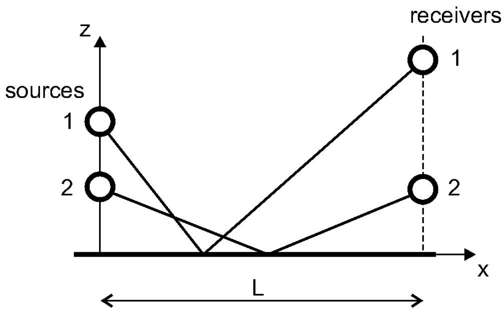

Figure 3. A pair of sound waves reflected in the ground surface.

Let $\hat{p}_i$ ($i = 1, 2$) have the phase of $\langle p_i \rangle$ (i.e. $\arg \hat{p}_i = \arg p_i$) and the amplitude $\left\langle |p_i|^2 \right\rangle^{1/2}$. If it is assumed that $\arg\left\langle p_1 p_2^* \right\rangle = \arg\left(\left\langle p_1 \right\rangle \left\langle p_2^* \right\rangle \right)$, one gets

$$W = \tfrac{1}{2}|\hat{p}_1|^2 + \tfrac{1}{2}|\hat{p}_2|^2 + |\hat{p}_1 \hat{p}_2| \cos\theta \, \Gamma^0 \qquad (12)$$

where $\theta = \arg\left(\left\langle p_1 \right\rangle \left\langle p_2^* \right\rangle \right) = \arg\left(\hat{p}_1 \hat{p}_2^* \right)$ and

$$\Gamma^0 = \frac{\left\langle p_1 p_2^* \right\rangle + \left\langle p_1^* p_2 \right\rangle}{\hat{p}_1 \hat{p}_2^* + \hat{p}_1^* \hat{p}_2} \qquad (13)$$

is the mutual coherence factor (MCF). The MCF Γ^0, will here depend on the heights of the sources (z_{S1} and z_{S2}) and receivers (z_{R1} and z_{R2}), as well as of the propagation range, L.

2.4 Coherence of the residual field

To separate the residual, random field from the coherent field, it is assumed that $\langle p_i \rangle = \hat{p}_i \exp(-\alpha)$, with α independent of source and receiver heights. Thus α is the same for p_1 and p_2, and equation (12) can be rewritten as

$$W = \mathrm{e}^{-2\alpha}\left(\tfrac{1}{2}|\hat{p}_1|^2 + \tfrac{1}{2}|\hat{p}_2|^2 + |\hat{p}_1\hat{p}_2|\cos\theta\right) + \left(1-\mathrm{e}^{-2\alpha}\right)\left(\tfrac{1}{2}|\hat{p}_1|^2 + \tfrac{1}{2}|\hat{p}_2|^2 + |\hat{p}_1\hat{p}_2|\cos\theta\,\frac{\Gamma^0 - \mathrm{e}^{-2\alpha}}{1-\mathrm{e}^{-2\alpha}}\right). \quad (14)$$

In the above equation the first term, being equal to $\left|\langle p_1 \rangle + \langle p_2 \rangle\right|^2$, is the contribution of the coherent field to the power. The second term is the contribution from the residual field, and the corresponding MCF is identified as

$$\tilde{\Gamma} = \frac{\Gamma^0 - \mathrm{e}^{-2\alpha}}{1-\mathrm{e}^{-2\alpha}}. \quad (15)$$

The factor $1-\mathrm{e}^{-2\alpha}$ in equation (14) can be seen as the fraction of diffuse field power due to the ground reflection.

2.5 Numerical examples

The effects of random roughness elements in an otherwise hard ground surface are studied by randomly choosing the positions of holes, with a uniform probability density. The hole depths, b, and opening widths, a, are kept constant. Here $a = b =$ 0.01 m has been used, with openings covering 10% of the whole surface. Source and receiver heights from 0.5 to 9.5 m and a propagation range, L, of 10 m has been modelled for the frequency 500 Hz. The length of the ground containing roughness elements is extended to twice the source–receiver range to model an infinite extent. For the results shown below, 500 realisations of the hole locations have been used to estimate the statistical properties.

In Figure 4, to the left, the mutual coherence factor (MCF) Γ^0 is shown for a single source height of 0.5 m and different receiver pair heights. To the right in Figure 4 the relative strength of the residual, diffuse field, $1-\exp(-2\alpha)$, is shown as a function of source and receiver heights. In the theoretical derivation it was assumed that the strength is independent of the heights of the source and the receiver, which here can be seen not to be true. The value of α could be approximated by for instance the value at the average of the source and receiver heights.

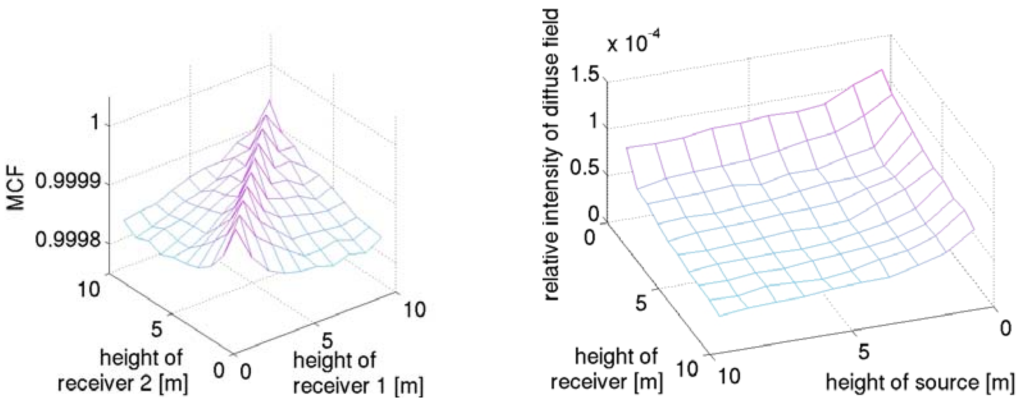

Figure 4. Calculated examples (see text for details). Left: Mutual coherence factor (MCF) for varying receiver positions. Right: relative intensity of the diffuse field as a function of source and receiver heights.

3 CONCLUDING REMARKS

The separation of the field reflected from a randomly rough ground into a coherent and a residual, diffuse part has been formulated, including a derivation of the mutual coherence factor (MCF) for the residual field, given the MCF for the total field and the relative power of the diffuse field. Numerical examples have been presented of the MCF for the total field. To conclude, the theoretical and numerical building blocks in this paper can be used to include random ground roughness in the substitute sources method (SSM) when calculating outdoor sound propagation, which is the planned continuation of the work presented here.

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