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LOCAL ACTIVE CONTROL IN PURE TONE DIFFRACTED DIFFUSE SOUND FIELDS

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1 - INTRODUCTION

A local system for the active control of sound uses a secondary source to cancel the pressure at a closely spaced error microphone. This creates a zone of quiet around the cancellation point which becomes larger as the error microphone is moved further away from the secondary source. Since in a practical application the local active control may be implemented close to a listener's head, the physical presence of the head may affect the zone of quiet produced. The acoustic performance of such a local control system will be mainly determined by the geometric arrangement of the secondary source, error microphone and the head. This means that a suitable theoretical model has to take into account the diffraction effects on the secondary and primary fields due to the secondary source and the head.

The original arrangement for this sort of local active control system was proposed by Olson and May in 1953 [1]. Their "electronic sound absorber" used an error microphone which was placed close to a loudspeaker acting as a secondary source to cancel the pressure at the microphone location. Their arrangement used feedback control which although able to control broadband random noise, suffered from the disadvantage of instabilities due to changes in the response of the acoustic path from the secondary source to the error microphone.

Ross [2] performed computer simulations to calculate the effect of a point monopole secondary source used to cancel the acoustic pressure at a point with a plane wave primary sound field. He found that at low frequencies the zone of quiet was a shell-like volume surrounding the secondary source and that as the frequency increased the form and size of the zone of quiet degraded rapidly. Joseph *et al* [3,4] have recently investigated the zone of quiet created when the total pressure is driven to zero on the axis of a finite size piston source. They found that the near field characteristics of the secondary source are very important in determining the resulting on-axis pressure distribution. Joseph derived an analytical expression for the size of the on-axis quiet zone as a function of the near field properties of the secondary source. By assuming a pure-tone sound field and a feedforward control arrangement, David [5] has performed computer simulations to estimate the spatial extend of the zone of quiet created by a local active control system in which the secondary source is modelled as a piston in an infinite baffle controlling both uniform and diffuse primary pressure fields. He showed that the zone of quiet becomes larger as the control microphone is moved further away from the secondary source until, for large separations, the 10 dB zone of quiet approaches the limiting case of a sphere of diameter one tenth of a wavelength, as predicted theoretically by Elliott [10].

In this paper we will describe a more realistic model for a secondary source radiating noise in a free diffuse field and in the proximity of a diffracting spherical head. The diffraction effect of such a spherical head on the primary and secondary acoustic fields will be studied. Finally, the influence of the secondary source configuration and the diffracting head on the average zones of quiet created in a diffuse field around an error microphone will be presented.

2 - THEORETICAL MODEL OF A SECONDARY SOURCE. EFFECT OF SELF-SCATTERING

In any practical implementation of a local active noise control system, the secondary source is normally a loudspeaker in an enclosure. Previous workers have modeled the secondary source as a monopole [2] or as a piston in an infinite baffle [5] since the acoustic field radiated by these two types of sources are relatively simple to represent mathematically. For values of ka less than about 1, however, the pressure several diameters from a piston source is very similar to that due to the equivalent monopole source. In terms of local active control, this means that the achievable zone of quiet using these two types of sources will be very similar provided that the error microphone is at least a diameter away from the centre of the sources [6].

The main limitation when using this simple models is that they do not take into account the diffraction caused by the secondary source enclosure. The distortion of the secondary field due to the presence of the enclosure must have some effect on the near field zone of quiet around an error microphone. The estimation of this effects deserves special attention, especially for those cases where the dimension of the active part of the source is small compared with the overall size of the secondary source enclosure.

Our model for a secondary source consists of a rigid sphere with a pulsating segment defined by a certain solid angle. This idealization of a loudspeaker in a realistic enclosure is based on the assumption that the diffraction effects caused by a sphere and a cube, for example, are much the same if the dimensions of the diffracting bodies are considerably smaller than the acoustic wavelength. The frequency range of interest and the dimensions of a practical secondary source suggest that this assumption will introduce an small error in the value of the calculated scattered pressure [7].

The sound field at any point (r,θ) outside the spherical source with a pulsating segment can be written in terms of an infinite sum of spherical harmonics as [8]:

$$p(r,\theta)=\sum_{m=0}^{\infty} A_m . P_m(\cos\theta) . h_m^{(2)}(kr) \qquad (1)$$

where $P_m(\cos\theta)$ is the Legendre polynomial of order m and $h_m^{(2)}(kr)$, the second order spherical Hankel function or order m. Since the radiation is axi-symmetric, there is no azimuthal angle dependence. A_m are pressure coefficients which have to be determined in order to calculate the value of $p(r,\theta)$. The values of the pressure coefficients A_m are found by calculating the normal acoustic velocity at any value of the polar angle θ on the surface of the sphere. These velocities can then be matched with the normal surface velocity defined as boundary conditions and A_m may be determined. Morse solves this problem analytically [8] but we have used a numerical method which is used for more complicated problems below, which involves solving a system of linear equations that can be expressed as [9]:

$$\sum_{m=0}^{M} A_m . D_m(r_p, \theta_p) = \dot{w}(r_p, \theta_p) \qquad (2)$$

where $D_m(r_p, \theta_p)$ is a function of Legendre polynomials and second order Hankel functions evaluated at those points on the surface of the secondary source defined by the coordinates r_p and θ_p and $\dot{w}(r_p, \theta_p)$ is the normal acoustic velocity associated to them. One can see that in order to solve this system of linear equations, we have to impose an upper limit m=M on the infinite series representing the pressure field in equation (1). This implies that the number of pressure coefficients A_m to be calculated is M+1. Using vector notation, equation (2) can be expressed as:

$$\mathbf{Z}\,\mathbf{c} = \mathbf{w} \qquad (3)$$

where $\mathbf{Z}$ is the matrix formed by the terms $D_m(r_p, \theta_p)$, $\mathbf{w}$ is the column vector of the outward surface velocities $\dot{w}(r_p, \theta_p)$ and $\mathbf{c}$ is the column vector of the pressure coefficients A_m. Watson [9] concluded that in order to minimise the error caused by the truncation of the infinite series (1), the number of points on the surface of the spheres at which the boundary conditions must be satisfied should be made greater than the number of coefficients required. In this way, the system of linear equations (3) becomes an overdetermined system which can be solved by minimising the cost function

$$J = \sum_{r=1}^{P} |\delta_r|^2 \qquad (4)$$

where P is the number of points chosen on the sphere and $\delta = \mathbf{w} - \mathbf{Z}\,\mathbf{c}$. Since equation (2) represents an overdetermined system, P has to be greater than M+1. It can be shown [9] that the cost function J is a quadratic function of the vector $\mathbf{c}$ having a minimum at

$$\mathbf{c}_0 = (\mathbf{Z}^H \mathbf{Z})^{-1} (\mathbf{Z}^H \mathbf{w}) \qquad (5)$$

where the superscript H denotes the hermitian transpose. Using the pressure coefficients A_m given by $\mathbf{c}_0$ in the series expansion (1) with m going from zero to M, the secondary acoustic field can be calculated at any location (r, θ).

3 - ZONES OF QUIET IN A FREE DIFFUSE PRIMARY FIELD

In many practical applications the primary pressure field will be due to a large number of acoustic modes excited in an enclosure. A suitable model for the average performance of a local control system under this conditions is provided by averaging the residual pressure field in front of the secondary source over an ensemble of examples of such a diffuse field. Each sample of diffuse field pressure on an arbitrary x-y plane was calculated using the following expression [5]:

$$p_p(x,y) = \sum_{k=1}^{k_{max}} \sum_{l=1}^{l_{max}} (a_{kl} + jb_{kl}) \sin\theta_k \exp(jk(x\sin\theta_k\cos\phi + y\sin\theta_k\sin\phi_k)) \quad (6)$$

which combines a total of k_{max} x l_{max} plane waves coming from all possible directions. The values of a_{kl} and b_{kl} in equation (6) are chosen from a random population with Gaussian distribution N(0,1) and the multiplicative factor $\sin\theta_k$ is included to ensure that, on average, the energy associated with the incident waves was uniform from all directions. The ensemble obtained with equation (6), taking k_{max}=6 and l_{max}=12, for a given frequency over a grid of 101x101 points in the x-y plane, was stored and used in subsequent calculations of the control sound field. The average of the squared modulus of the controlled fields from 20 such examples of diffuse primary fields were then divided by the average of the squared modulus of all the primary fields. The result of such a calculation yields the average zone of quiet for a diffuse primary field.

The average extent of the zones of quiet created by a monopole secondary source in a free diffuse field for different values of kL, where L is the distance between the cancellation position and the monopole source, are shown in *figure 1*. We observe that for low excitation frequencies (kL=0.2) there is a shell of quiet around the secondary source, whose thickness decreases with excitation frequency. At higher frequencies (kL=2), the extent of the zone of quiet is drastically reduced by the phase variation in the diffuse primary field. At higher values of kL, the zone of quiet is practically spherical with a diameter of approximately $\lambda/10$, as theoretical predicted by Elliott *et al* [10].

The results shown in *figure 1* reveal a striking similarity in the -10 and -20 dB contours to those obtained by David [5] for the case of a piston secondary source. This agreement is due to the fact that the secondary acoustic fields due to a piston and a monopole source are very similar for the range of ka (0.1-1), where a is the piston radius, and kL (0.2-2) covered by David. However if the cancellation point moves closer to the secondary source, that is, for a very small values of kL, the secondary field due to a monopole source changes much more rapidly with position than that originated by a piston, causing a considerable discrepancy in the shape and size of the zone of quiet created by both types of sources [6].

The theoretical model for the secondary source introduced in section 2, i.e.,a rigid sphere with a pulsating segment, is now used to calculate the effect of scattering of the secondary field by the secondary source itself on the average zone of quiet. *Figure 2* shows the progressive change in the average zone of quiet in a free diffuse field for a'=L/2, where a' is the spherical source radius, and kL=0.5 for four different angles of the active segment (360°,180°,45°and 10°). These results show that for a realistic secondary source the effect of the self-scattering on the secondary field and, thus, on the average zones of quiet is important and that as the relative size of the active segment is reduced, the average zone of quiet reduces in size and appears to be centred on a virtual source location which moves progressively towards the surface of the active segment.

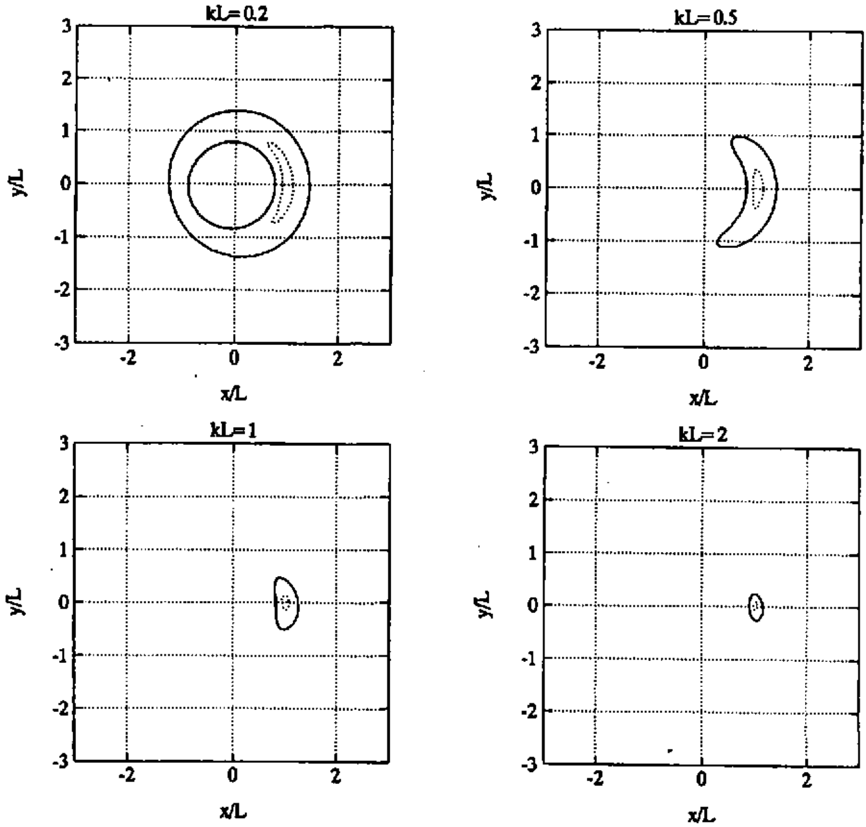

Figure 1: The average pressure field in the x-y plane due to the superposition of a free diffuse primary field and the field created by a secondary monopole source centred at (0,0).The monopole strength has been adjusted to cancel the primary pressure at the cancellation point x/L=1, where L is the distance between the cancellation point and the centre of the secondary source. The solid line represents reductions in the pressure of 10 dB; the dotted line, reductions of more than 20 dB.

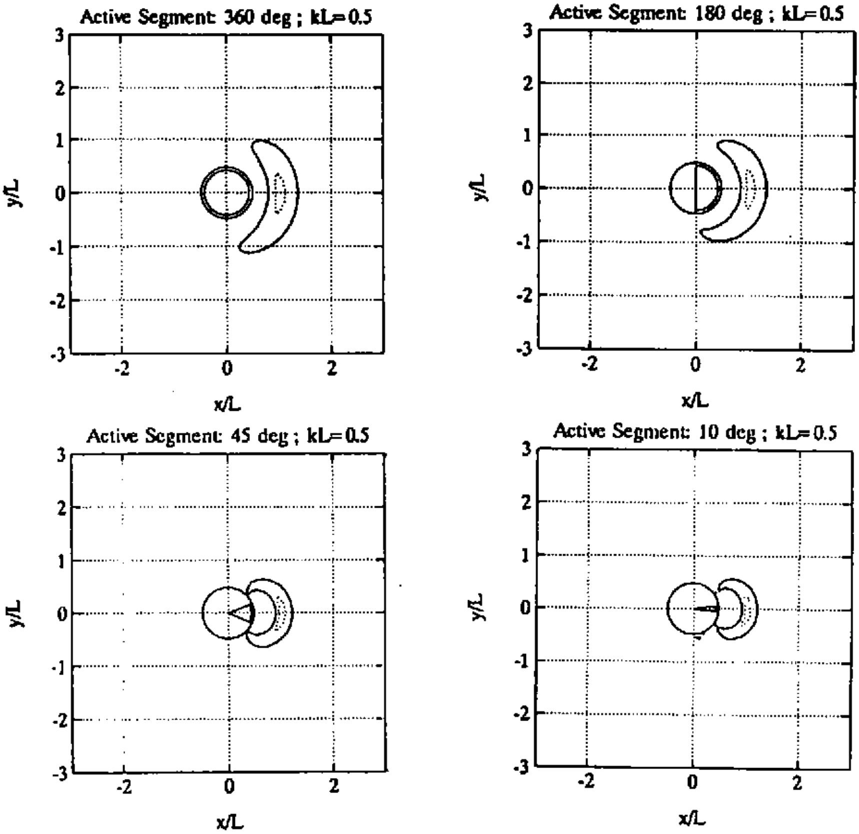

Figure 2: The average pressure field in the x-y plane due to the superposition of a free diffuse primary field and the field created by a secondary spherical source with an active segment of variable size (shown with double line). The secondary source has a radius equal to L/2 and its strength has been adjusted to cancel the primary pressure at the error microphone located at x/L=1. The solid line represents reductions in the primary pressure of 10 dB, the dotted line, reductions of more than 20 dB.

The samples of diffuse field used to generate the contour plots of *figure 2* where calculated with equation (6) which does not take into account the diffraction of the primary field by the secondary source. A complete simulation has been performed [6] which included the effect of diffraction on each of the components in the diffuse primary field. The results are barely distinguishable from those shown in *figure 2* which do not include the effect of diffraction on the primary field. This is perhaps to be expected for the small values of ka shown here.

4 - THEORETICAL MODEL OF HEAD INTERFERENCE

Since a practical local control system generally has to be implemented close to a listener's head, the effect of such a diffracting body on both the secondary and primary sound field is considered next in this section. The head will be modelled, in the first instance, as a rigid sphere to estimate the magnitude of the diffraction effects.

4.1 - Effect of a diffracting sphere on the secondary field.

The effect of a close rigid sphere on the acoustic field generated by a spherical secondary source with an active segment is considered next. Watson [9] has recently calculated the total acoustic field radiated by two closely located spheres of different radii and surface velocities. The sound field at any point outside the two spheres can be written as the superposition of the outgoing waves from each sphere at that point. The outgoing waves from each sphere can be expressed in terms of an infinite sum of spherical harmonics as:

$$p(r,\theta)=\sum_{m=0}^{\infty} A_m . P_m(\cos\theta)\, h_m^{(2)}(kr)+\sum_{n=0}^{\infty} A'_n . P_n(\cos\theta')\, h_n^{(2)}(kr') \qquad (7)$$

where the meaning of the terms of the infinite series for each sphere coincide with those of equation (1). Similarly to the procedure described in section 2, the method of finding the pressure coefficients A_m and A'_n is to derive and expression for the normal acoustic velocity at any point θ and θ' on the boundary surface of each sphere. By matching these velocities with the normal surface velocity defined as boundary condition, the pressure coefficients A_m and A'_n can be determined by solving the following system of linear equations

$$\sum_{m=0}^{M} A_m D_m(r_p,\theta_p) + \sum_{n=0}^{N} A'_n E_n(r'_p,\theta'_p) = \dot{w}(r_p,\theta_p)$$
$$\sum_{m=0}^{M} A_m F_m(r_q,\theta_q) + \sum_{n=0}^{N} A'_n G_n(r'_q,\theta'_q) = \dot{w}(r'_q,\theta'_q) \quad (8)$$

which is expressible in the same vector form as equation (3), with **Z** in this case being the matrix formed by the terms $D_m(r_p,\theta_p)$, $E_n(r'_p,\theta'_p)$, $F_m(r_q,\theta_q)$ and $G_n(r'_q,\theta'_q)$; **w**, the column vector of the normal acoustic velocities on both spheres, $\dot{w}(r_p,\theta_p)$ and $\dot{w}(r'_q,\theta'_q)$ and **c**, the column vector of pressure coefficients A_m and A'_n.

By using an identical approach to that presented in section 3, and applying equation (5), the M+N+2 pressure coefficients A_m and A'_n can be calculated and used, together with equation (7), to generate the acoustic field created by a spherical secondary source with a pulsating segment radiating close to a rigid spherical head.

4.2 - Effect of both a diffracting sphere and a spherical secondary source on the primary diffuse field.

In section 3 we discussed the theoretical model for a free diffuse field used in the calculation of the average zones of quiet without diffraction effects shown in *figure 1.* This model consists of a superposition of a finite number of plane waves ,with random phase and amplitude, coming from equidistant angular directions. The same idea can be applied to generate samples of a diffuse field diffracted by the presence of two close spheres, but, in this case, the individual diffraction suffered by each incoming plane wave has to be considered. The analysis of the diffraction effects on a plane wave coming from arbitrary direction due to two close spheres can be carried out using the same theory developed in section 4.1, but, because the problem is not axi-symmetric, the expansion of the acoustic field radiated by each sphere has to include one term to account for the azimuthal dependence. The three dimensional expansion for the total acoustic field radiated by two spheres after truncation is:

$$p(r,\theta,\phi) = \sum_{m_1=0,\, m_2=-m_1}^{m_1=M,\, m_2=m_1} A_{m_1 m_2} \, . \, e^{-jm_2\phi} \, . \, P_{m_1}^{m_2}(\cos\theta) \, . \, h_{m_1}^{(2)}(kr) \, + \\ + \sum_{n_1=0,\, n_2=-n_1}^{n_1=N,\, n_2=n_1} A_{n_1 n_2} \, . \, e^{-jn_2\phi'} \, . \, P_{n_1}^{n_2}(\cos\theta') \, . \, h_{n_1}^{(2)}(kr') \tag{9}$$

where P_v^μ is the associated Legendre Function of the first kind of order μ and degree v and the terms $e^{-jm2\phi}$ and $e^{-jm2\phi'}$ include the azimuthal dependence of the waves radiated by each sphere respectively. All the other terms have been similarly defined in equation (1). Following the same procedure described in section 4.1, the pressure coefficients A_{m1m2} and A_{n1n2} can be calculated by solving the following system of linear equations

$$\sum_{m_1=0,\, m_2=-m_1}^{m_1=M,\, m_2=m_1} A_{m_1 m_2} \, . \, D_{m_1 m_2}(r_p, \theta_p, \phi_p) + \sum_{n_1=0,\, n_2=-n_1}^{n_1=N,\, n_2=n_1} A_{n_1 n_2} \, . \, E_{n_1 n_2}(r'_p, \theta'_p, \phi'_p) = \dot{w}(r_p, \theta_p, \phi_p) \\ \sum_{m_1=0,\, m_2=-m_1}^{m_1=M,\, m_2=m_1} A_{m_1 m_2} \, . \, F_{m_1 m_2}(r_q, \theta_q, \phi_q) + \sum_{n_1=0,\, n_2=-n_1}^{n_1=N,\, n_2=n_1} A_{n_1 n_2} \, . \, G_{n_1 n_2}(r'_q, \theta'_q, \phi'_q) = \dot{w}(r'_q, \theta'_q, \phi'_q) \tag{10}$$

which is a three dimensional extension of equation (8). The total number of coefficients A_{m1m2} and A_{n1n2} is $(M+1)^2+(N+1)^2$ and they can be calculated using the approach described in section 3.

For the diffraction of a plane wave due to two close rigid spheres, the column vector of the normal acoustic velocities on both spheres was set equal and opposite to the particle velocities that the incident plane wave would cause in free space in a direction normal to the surface of the spheres. Each sample of diffracted diffuse field was calculated by adding the pressure of 72 diffracted plane waves with random phase and amplitude. For each contributing plane wave, the system of linear of equations given by equation (10) had to be solved with the terms $\dot{w}(r_p,\theta_p,\phi_p)$ and $\dot{w}(r'_q,\theta'_q,\phi'_q)$ defined by the direction of the incident plane wave.

5 - ZONES OF QUIET IN A DIFFRACTED DIFFUSE PRIMARY FIELD

The theory developed in section 4.1 and 4.2 allow us to calculate the effect of a diffracting sphere on the secondary and primary field respectively.

Applying the averaging procedure described in section 3, the average zone of quiet created by our secondary source in a diffracted diffuse field can be calculated. *Figure 3* shows the effect of a diffracting rigid sphere on the average zone of quiet for different values of kL. The secondary source has a radius equal to L/2 and the radius of the rigid sphere is 1.5 times that of the secondary source. For these simulations, the distance between the secondary source and the rigid sphere has been chosen equal to 2L. We observe that the distortion of the zone of quiet caused by the proximity of the rigid sphere is quite small and somehow beneficial since the overall size of the quiet zone tends to increase slightly at the same time that it extends towards the rigid sphere. In fact if these simulations are repeated without the effects of diffraction on the primary field, the results are very similar. The distortion in the zone of quiet shown in *figure 3* is thus predominantly due to the effects of the rigid sphere on the field due to the secondary source.

6 - CONCLUSIONS

The average extent of the zone of quiet created by a monopole secondary source in a free diffuse field has been calculated and compared with those produced by a piston in an infinite baffle [5]. The results are very similar provided that ka is less than approximately 1 and the cancellation point is at least one piston diameter from the secondary source. However, if the cancellation point moves closer to the secondary source, the secondary field due a monopole source changes much more rapidly with position than that originated by a piston, causing a considerably discrepancy in the shape and size of the zones of quiet [6].

The zones of quiet created by a rigid spherical secondary source with an active segment in a diffuse field have been presented.

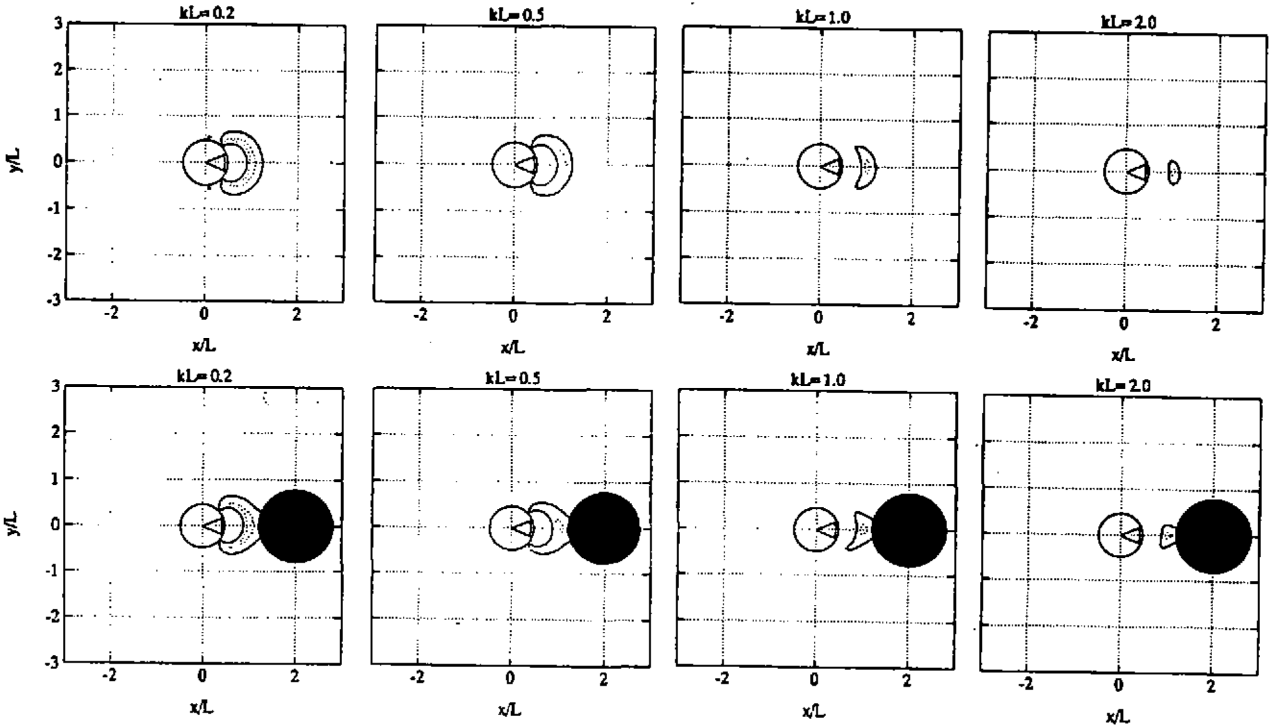

Figure 3: The average field in the x-y plane due to the superposition of a diffracted diffuse primary field and the field due to a secondary spherical source with a 45 degree active segment, to cancel the pressure at an error microphone located at x/L=1. The secondary source has a radius equal to L/2 and the radius of the rigid sphere is 1.5 times that of the secondary source. The upper contour plots show the zones of quiet without scattering effect for four different values of kL. The bottom plots depict the expected change in the zones of quiet due to the diffraction effect caused by the rigid sphere. The solid line represents reductions in the primary pressure of 10 dB; the dotted line, reductions of more than 20 dB.

It has been shown that the relative size of the active segment has an important effect on the size and form of the zone of quiet generated. One can observe that as the angle of the active segment decreases, the average zone of quiet reduces in size and appears to be centred on a virtual source location which moves progressively towards the surface of the active segment.
The effect of the diffraction from a rigid sphere close to our secondary spherical source on the zones of quiet has been considered. The results show that for values of ka', where a' is the radius of the spherical source, in the range from 0.1 to 1, the distortion of the zone of quiet caused by the proximity of the rigid sphere is quite small and predominantly due to the effect of the rigid sphere on the secondary acoustic field.

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