

ENERGY EFFICIENCY IN SOUND REINFORCEMENT

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1 INTRODUCTION

Anthropologically-driven climate change is showing no signs of abatement, research suggests that CO₂ emissions must be reduced to net zero by 2050 to avoid catastrophic warming. In addition to global lifestyle changes and transition to renewable energy sources, a huge factor in achieving this goal is simply increasing the energy efficiency of existing technology. Loudspeakers are extremely inefficient transducers, typically 1-5%. Improving the efficiency of loudspeakers has been an important area of research at Funktion One, since the 1970s and 80s when amplifier power was in short supply(!)

In this paper we look at the measured efficiency of Class D/PWM amplifiers, waveguide/horn loudspeakers, effects of reactive power and the total efficiency from mains to acoustic energy, with results from real events.

2 LOUDSPEAKER EFFICIENCY

2.1 EFFICIENCY

Efficiency is defined as:

$$\eta = \frac{P_{out}}{P_{in}}$$

In terms of a loudspeaker, P_{out} is the total sound power radiated by the loudspeaker and P_{in} is the total electrical input power.

To understand which parameters affect the efficiency of a loudspeaker, we can begin by looking at an approximation of the radiation impedance of a baffled plane piston *at low frequencies*:

$$\hat{Z}_r \approx \rho_0 c \left[\frac{(ka)^2}{2} + j \frac{8ka}{3\pi} \right]$$

Where ρ_0 is the density of air, c the speed of sound, k is wavenumber, a the radius of the piston. Introducing velocity (u) into the equation, and we have an approximation for the total sound power:

$$P \approx S_d \frac{(\hat{u}_d)^2}{2} \rho_0 c \frac{(ka)^2}{2} \approx \frac{\rho_0 c k^2 (\hat{q}_d)^2}{4\pi}$$

Where the volume velocity of the diaphragm, q_d is:

$$\hat{q}_d = \hat{u}_d S_d$$

The key parameters are therefore the velocity of the diaphragm, and the area of the diaphragm. The area of the diaphragm is of particular importance, when $ka \ll 1$ the radiated power becomes entirely reactive – performing no useful work.

Turning attention to real electrodynamic loudspeakers, with a given motive force, increasing the size of the diaphragm increases efficiency up to a point, where the gain is outweighed by the increase in mass (and also limited in practice by directivity and rigidity).

With a given diaphragm size, increasing velocity requires an increased motive force. The force on a current-carrying wire is:

$$F = Bli$$

Where F is the generated force, B the magnetic field strength, l the length of wire and i the current in the wire. The velocity of a diaphragm attached to the coil of wire can approximately be described by considering the mechanical impedance Z_m of the diaphragm assembly and air load:

$$\hat{u} \approx \frac{Bli}{Z_m}$$

Again, there are practical limits to how much the electromotive force can be increased. Typically pure iron saturates at a magnetic flux density of around 2T, most magnetic steels saturate at a lower level than this. Increasing the length of wire in the voice coil increases mass and inductance, and the maximum current capacity is limited by the thermal dissipation of the voice coil and motor assembly. The "art" of loudspeaker design is balancing this highly complex and interconnected parametric problem.

The efficiency of real drive units is usually described by a reference efficiency η_0 , defined in terms of lumped parameters as:

$$\eta_0 = \left(\frac{\rho_0 B^2 l^2 S_d^2}{2\pi c M_{ms}^2 R_E} \right) \times 100\%$$

The relationship between motive force, cone area and moving mass (M_{ms}) and R_E is clearer here. A high efficiency loudspeaker must have a large radiating area, high field strength, many turns and low mass.

2.2 SENSITIVITY

Because power amplifiers generally behave as constant voltage sources (with low output impedance), a common parameter for comparing loudspeakers is sensitivity, defined as the sound pressure level (dB) produced at 1m from the source at 2.8V (IEC60268-5 and AES2-2012). Sensitivity can be converted from efficiency using the reference sound power level (one picowatt) over a hemisphere with area $2\pi \text{ m}^2$, although direct measurement is preferable.

$$10 \log \left[\frac{\left(\frac{1}{2\pi} \right)}{P_{ref}} \right] = 112.1 \text{ dB}$$

$$\text{Sensitivity (dB)} = 112 + 10 \log \left(\frac{P_{out}}{P_{in}} \right)$$

2.3 RESONANT SYSTEMS

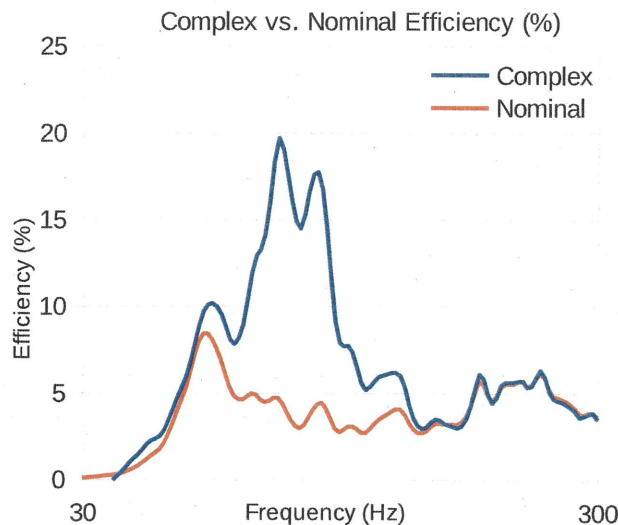


Figure 1: Comparison of complex and nominal derived efficiencies of a 15" bass reflex loudspeaker

While voltage sensitivity is often a useful parameter in comparing "efficiency" of loudspeakers, it becomes less related to "true efficiency" at low frequencies where loudspeakers are typically operating in their stiffness-controlled region into highly resonant loads (bass reflex, bandpass etc.).

Figure 1 demonstrates the magnitude of the error between the "true" efficiency and calculated nominal efficiency from voltage sensitivity at low frequencies.

The error is greatest around the tuning frequency of the bass reflex cabinet, as the resonant behaviour of the system presents a higher acoustic impedance to the driver, increasing the sound power output.

Highly resonant systems present a correspondingly reactive load to the amplifier, which is examined in more detail in section 4.

2.4 MOTOR EFFICIENCY FACTOR

Another useful parameter is the motor efficiency factor, Bl^2/R_E .

2.5 HIGH EFFICIENCY DRIVE UNIT EXAMPLES

Table 1 shows some measured parameters of two drivers designed according to the principles in section 2, with clear improvements in efficiency. Common examples of conventional units are included for comparison. Parameters were measured by the delta mass method.

	High Efficiency Midrange driver	Conventional Midrange driver	High Efficiency Bass driver	Conventional Bass driver
Nominal diameter	10"	6.5"	24"	18"
Nominal impedance	16Ω	8Ω	4Ω	8Ω
Operating band	200Hz – 4kHz	100Hz – 4kHz	20Hz – 100Hz	20Hz – 100Hz
Effective diaphragm area (Sd)	380cm ²	145cm ²	2376cm ²	1210cm ²
Force factor (Bl)	28Tm	12Tm	42Tm	30Tm
Motor Efficiency Factor (Bl^2/R_E)	52	26	534	163
Sensitivity (2.8V/1m)	100dB	95.5dB	101.5dB	97dB
Reference Efficiency η_0	4.9%	2.2%	8.6%	1.9%

3 HORNS AND WAVEGUIDES

The high efficiency drive units in Table 1 are efficient enough to be used in a direct radiating PA system, but further improvements can be made by coupling them to suitable horns.

Horn loudspeakers were the dominant technology in public address until advancements in materials and amplifier technology increased power availability and handling.

An acoustic horn behaves as an impedance matching transformer converting high pressure, low velocity at the throat, to high velocity low pressure at the mouth, maximising energy *transfer*.

Viewed from another perspective, horns and waveguides increase efficiency by increasing the acoustic load on the diaphragm (see equation 2) increasing radiation impedance (larger area), and restricting the radiation angle.

3.1 BASS HORNS

Following traditional design methods, a "true" exponential bass horn with a cutoff frequency of 40Hz would be nearly 3 metres long, with a 6m² mouth! This is clearly impractical for almost any application, and both the length and mouth size must be significantly compromised in practice in order to produce reasonably sized cabinets. Truncated and folded bass horns have existed at least since the 1940s. Klipsch's famous *Klipschorn* (1946) was designed to radiate into the corner of a room to extend the effective horn path and increase the low frequency efficiency.

A range of ingenious methods of extending the low frequency response of truncated bass horns have since been explored by engineers, including port assisted, tapped (re-entrant) and bandpass varieties.

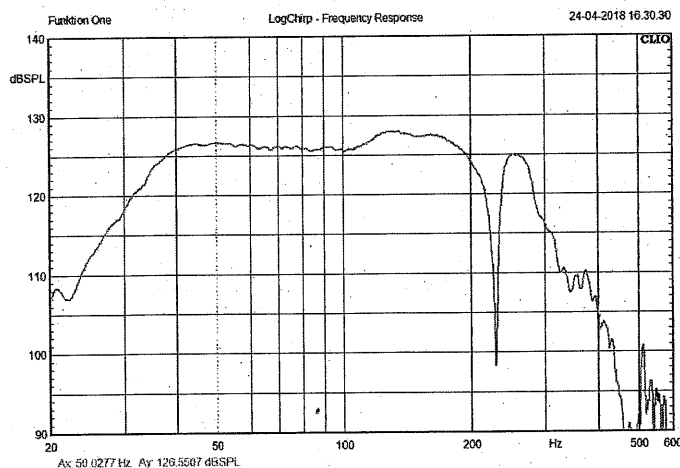


Figure 2: Array of eight 24" bass horns measured at 20m and corrected to 2.8V/1m. Two stereo amplifiers were used in a parallel configuration (4x2 ohm loads)

high sensitivity of 126dB/2.8V/1m.

The maximum continuous SPL *per cabinet* is in excess of 139dB with minimal power compression. This is not possible to achieve with a conventional dual 18" bass reflex subwoofer, with a sensitivity on the order of ~95dB/2.8V.

However, as discussed in section one, sensitivity is not the full story for subwoofers, and even a bass reflex subwoofer is more efficient than the sensitivity would suggest around its resonant frequency(ies), especially when powered by PWM/Class D amplifiers.

Once a necessity due to limitations in amplifier power, bass horns have fallen out of the mainstream in favour of dual 18" bass reflex subwoofers for most PA applications, but bass horns still have a number of attractive advantages.

The principal advantages of all bass horns, are the increase in efficiency and improved mutual coupling due to the entire front surface of the cabinet radiating in phase (at low frequencies).

With Neodymium magnets, the high efficiency 24" drive unit in Table 2 enable the construction of a bass horn with a mass of just over 100kg, and a volume of 735 litres. Figure 2 demonstrates an array of eight of these units reaching a very

4 PWM (CLASS D) AMPLIFIER EFFICIENCY

Pulse Width Modulation (PWM) (or "Class D") amplifiers are not a new idea, Clive Sinclair offered an early PWM amplifier DIY kit (the Sinclair X10) in the 1960s. However, it wasn't until the late 1990s/early 2000s that the technology matured to the point where the reliability and audio quality was competitive enough with Class AB and G/H topologies to find use in sound reinforcement.

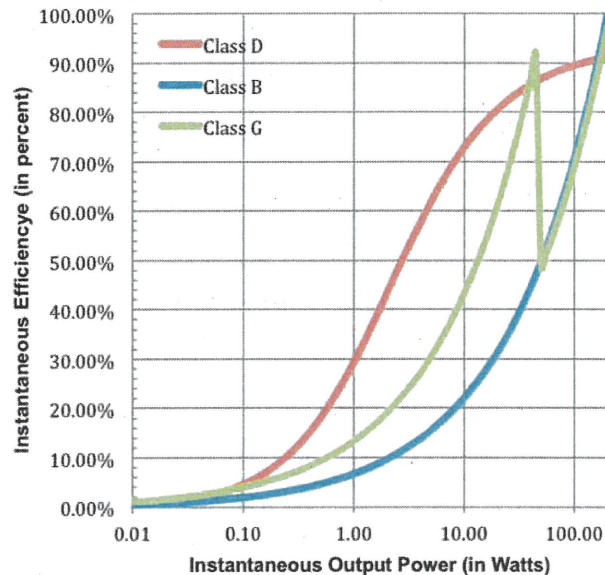


Figure 3: Efficiency vs. Output Power (Angus)

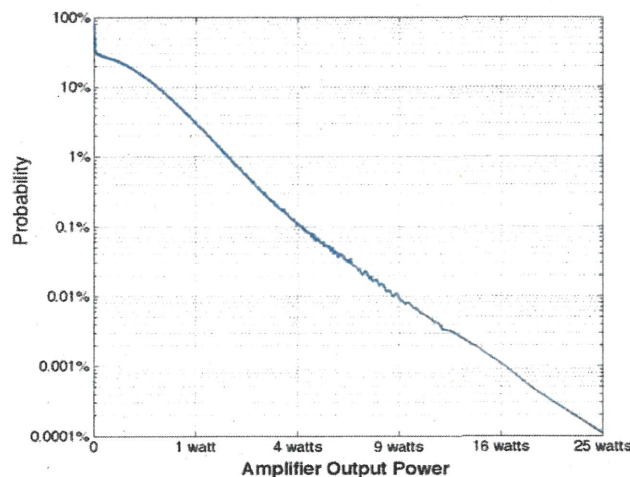


Figure 4: Example Probability Density Function of music (Angus)

Amplifier classes can be characterised by their conduction angle; the proportion of time in a cycle of signal where current is flowing in the output devices. The output devices of a Class A amplifier conduct 100% of the time (360°) giving a maximum theoretical efficiency of just 50% (in reality often much lower than this, limiting their applications to small scale i.e. microphone preamplifiers).

Depending on the bias, a Class AB amplifier has a conduction angle of $180\text{--}270^\circ$ with a marginally higher theoretical efficiency of 60%.

A Class D amplifier switches the output devices on and off very rapidly, spending very little time in the linear region, resulting in a conduction angle close to 0° , and a theoretical maximum efficiency of 100%. Real Class D amplifiers have been measured with efficiencies in the high 90s.

MOSFETs with very high switching speeds are required, which provide the additional advantage of bidirectional energy flow to and from the load.

Class D amplifiers appear to be the answer to sound reinforcement efficiency, but Angus points out that Class D amplifiers only reach their maximum efficiency at maximum power (Figure 3), and due to the average probability density function of music (Figure 4), this is an unusual case. Note the logarithmic axes.

The crest factor of bass signals is significantly lower, and combined with the ability to recycle back EMF (see next section) the biggest efficiency gains from Class D amplification are likely to be seen in use with sub arrays.

4.1 BIDIRECTIONAL ENERGY FLOW AND REACTIVE LOADS

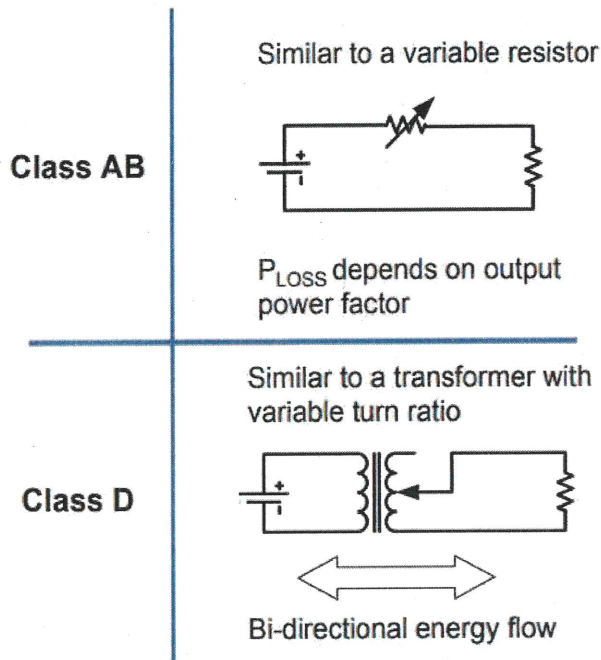


Figure 5: Comparison of AB and D (adapted from <http://www.irf.com/product-info/audio/classdtutorial2.pdf>)

Like other electric motors and generators, electrodynamic loudspeakers and microphones are reciprocal transducers. The current and voltage produced by the motion of the loudspeaker as a generator is often referred to as "back EMF". The amount of back EMF is related to the reactivity of the load (or the phase angle of the impedance).

In a Class AB amplifier, the back EMF is dissipated by the output devices as heat (although this isn't a total "waste" as the amplifier is doing a very important job of providing electrical damping for the loudspeaker).

In a Class D amplifier, the back EMF can flow back through the output devices and into the power supply, recharging the bulk storage capacitors and reducing the power required from the mains.

This effect is small, but useful in the context of large subwoofer arrays. Lastrucci (2018) showed that when driving real subwoofers, Class D output stages can have a 14.6% higher net electrical efficiency and 3.7% high total mains to acoustic efficiency compared to a Class AB output stage.

5 RESULTS

To quantify the relative and combined efficiency gains of high efficiency drivers, horns and PWM amplification, a range of electrical and acoustic measurements were made in both laboratory conditions and real-world applications.

5.1 SOUND POWER MEASUREMENTS OF A HORN LOUDSPEAKER

To investigate the effects of horn loading on midrange efficiency, the sound power level of a 10" drive unit was measured with and without a horn in a reverberation chamber.

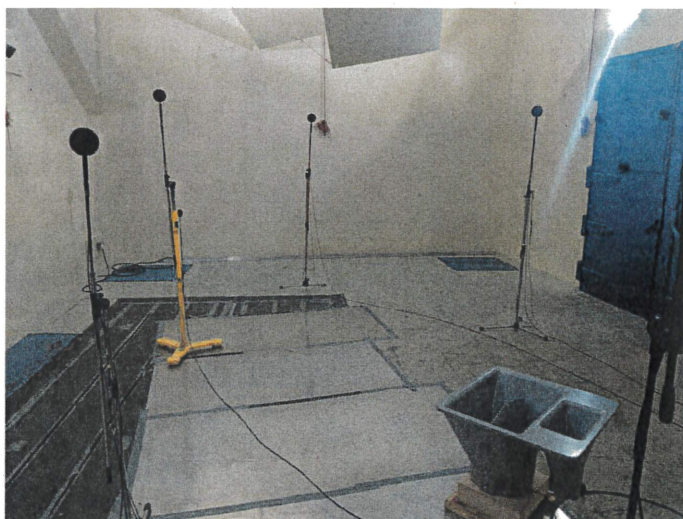


Figure 6: Sound power measurement in reverberation chamber

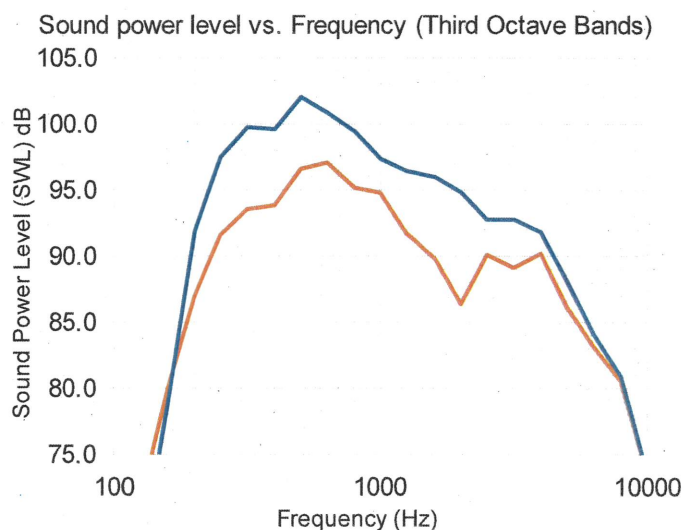


Figure 7: SWL of driver with waveguide (blue) and direct-radiating (orange)

Figure 4 shows the measured sound power level vs. frequency of the 10" drive unit in a back bowl (approximating an infinite baffle), and mounted to a 40"x20" horn.

At 1600Hz the SWL of the driver mounted to the horn is more than 6dB higher (four times the sound power).

The overall A-weighted SWL was 107.5dB with the horn and 103.2dB for the driver radiating into free air.

At the limits of the operating bandwidth, the efficiency gain of the waveguide reduces to zero. At low frequencies this is a similar case to the driver radiating into free air; as frequency decreases, the radiation impedance becomes entirely reactive and no power is transferred to the air. At high frequencies the roll-off is the result of the combined moving mass of the drive unit and the compliance of the air mass in the horn throat.

5.2 CLASS D AND CLASS G OUTPUT STAGES AND REACTIVE ENERGY RECYCLING

To compare the efficiency of Class G and Class D output topologies, power consumption and current was measured delivering a constant RMS noise voltage into a resistive "dummy" load and a real loudspeaker (24" bass horn). The nominal load impedance was 4Ω in both cases. Both amplifiers have the same voltage gain (32dB). The noise signal was pink noise with a crest factor of 4 (12dB) high pass filtered at 20Hz and low pass filtered at 80Hz (to emulate a subwoofer application). Line voltage was 244V.

Table 2: Results

	Class G (Resistive load)	Class G (Loudspeaker)	Class D (Resistive Load)	Class D (Loudspeaker)
Power into load (W)	250	250	250	250
Mains Current (A_{RMS})	7.6	5.3	3.9	2.9
Real Power (W)	1273	680	745	447
Apparent Power (VA)	1854	1293	951	707
Power factor	0.68	0.52	0.78	0.63
Efficiency (%)	19%	36%	33%	55%

The difference in efficiency between Class G and D is clear. However, the difference in efficiency is not as large as marketing material might suggest, especially when examining Figure 8. This supports Angus' ideas that Class G can compete with Class D in efficiency when driving real loudspeakers with music (and music-like signals). However, a significant factor in this could be that only one loudspeaker was connected. Connecting more loudspeakers in parallel to reduce the load impedance may result in significantly increased efficiency. The results in the following section show similarly low efficiency with a single 4Ω loudspeaker load.

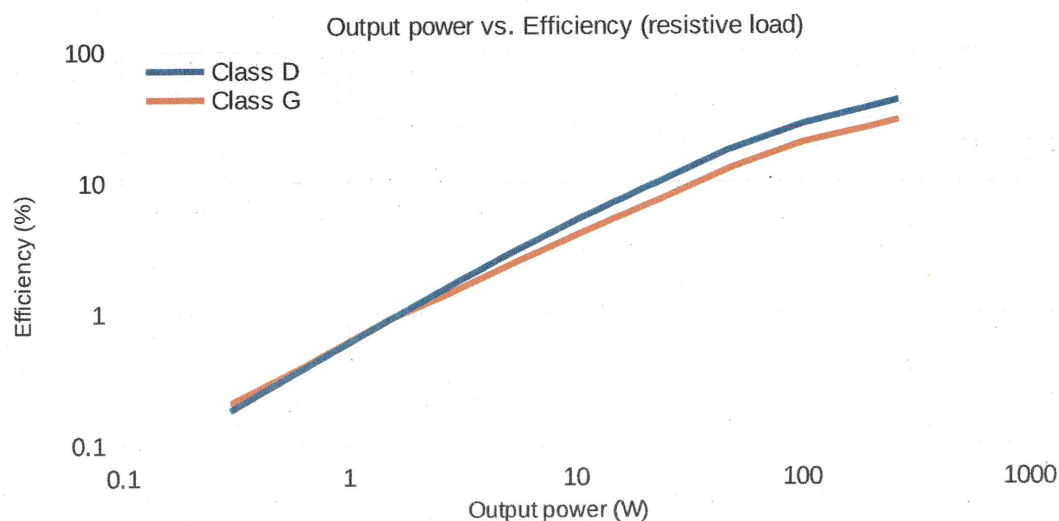


Figure 8: Comparison of Class D and G amplifiers operating into a resistive load

5.3 MEASURED TOTAL EFFICIENCY FROM MAINS TO ACOUSTIC POWER

One bass horn loudspeaker was connected to one channel of a Class D amplifier, presenting a nominal load impedance of 4Ω . The test stimulus was 12dB crest factor Pink Noise, all standard crossover filters were applied (as would be used in a real application) with a high pass filter at 20Hz and low pass at 80Hz.

The sound power was measured using the "engineering" free field method (ISO 3744) of spatially averaged sound pressure over a hemisphere (in a low reflection room), electrical power was calculated from the mains voltage, current and power factor.

For reference, the on-axis sound pressure level at 2m was 117dB (unweighted).

Averaged (unweighted) pressure over the measurement surface:

$$L_{p(ST)} = 10 \log \left[\frac{1}{N_M} \sum_{i=1}^{N_M} 10^{0.1 L_{p_i(ST)}} \right] \text{dB}$$

Where N_M is the number of microphone positions and $L_{p(ST)}$ is the time averaged sound pressure level at the i th measurement position. The total sound power level is found from:

$$L_W = (L_{p(ST)} - K_1 - K_2) + 10 \log \frac{S}{S_0} \text{dB}$$

Where K_1 and K_2 are background noise and environmental corrections respectively.
 S is the area of the measurement surface in m^2 and S_0 is the reference area, 1m^2 .

Table 3: Results

	Test 1	Test 2 (+6dB)
Sound power level L_W	125.9dB	131.9dB
Sound power output (W)	3.9	15.5
Mains Power input (W)	245	400
Quiescent power (W)	160	160
Total System Efficiency %	1.59%	3.89%

Despite the high electroacoustic efficiency of the loudspeaker, the measured total efficiency from mains to sound power is very low. This is likely due to high quiescent power and the single loudspeaker load. This highlights the importance of using Class D amplifiers with low load impedances (many speakers in parallel) so they are operating near to their maximum output, for maximum efficiency. In the second test, the signal level was increased by 6dB.

In future experiments, checking the accuracy of sound power measurement with a reference sound power source would be useful.

It was assumed that there would be some efficiency loss due to one channel of the amplifier remaining idle, but it was surprising to see that the quiescent power consumption was around 160W, and the loudspeaker had to be driven at a high level (~120dB) to see any significant power consumption above the quiescent power level.

The main losses in a Class D amplifier are directly related to the non-ideality of MOSFETs, including conduction, switching and gate charge losses.

5.4 FIELD MEASURED ENERGY USAGE OF SOUND REINFORCEMENT SYSTEMS

Modern generator sets are equipped with sophisticated load monitoring and logging systems. Combined with the schedule of noise propagation testing, this provided an interesting comparison of energy usage between similar sized sound reinforcement systems at real events. Sources of error are high, including variation in level between different stages during the noise propagation testing, different program material and also stage lighting and non-audio power consumption. However, all systems in this comparison used the same amplifier model.

	System A	System B	System C
Acoustic design	All horns	All horns	Direct-radiating
Amplifier type	Class AB with switched-mode power supply (SMPS)	Class AB with SMPS	Class AB with SMPS
Number of subwoofers	12	40	45
Number of main array elements	14	30	40
SPL limit at FOH (dBA)	98	102	102
SPL limit at FOH (dBC)	105	115	115
Average apparent power (KVA)	50*	55	207
Hours in use	51	36	36
Estimated CO₂ emissions** (kg)	2019	1416	4276

Unfortunately the power usage of system A was hidden under the “noise floor” of lighting and other loads on the generator. However, amplifier load monitoring revealed that the total mains current draw measured from the bass amplifiers was typically 9A, reaching peaks of 15A. This equates to an average power consumption of 2160-3600W for 12x bass horns, comfortably covering an audience of over 10,000 at peak times, with an SPL limit of 105dBC. To put this into perspective, the bass requirements for this stage could have been delivered by a domestic 13A socket (don't try this at home).

Systems B and C provide interesting results. Both systems have horn loaded midrange and high frequency units. The large difference in power consumption is most likely due to horn loaded vs direct radiating subwoofers. Additionally, system B used an entirely ground-stacked configuration, whereas the majority of subwoofers in System C were suspended. Ground-stacked subwoofers experience a 6dB gain from the floor reflection, which is reduced when they are suspended. Corteel (2018) presents some interesting simulations of flown subwoofer arrays, demonstrating that flown arrays are only marginally less efficient than ground stacks when considering far field SPL and distribution. The motivation for flown subwoofer arrays is improved level consistency from the front to the back of the audience area, some system designers may consider this to be worth sacrificing some system efficiency.

**Including non-audio loads*

***Assuming 2.64kg of CO₂ per litre of Diesel consumed*

5.5 BONUS – LOUDSPEAKER SENSITIVITY OVER TIME

Datasheets for 22 popular large format PA loudspeakers from 1975 to 2018 were collected and a linear regression performed to reveal any trends.

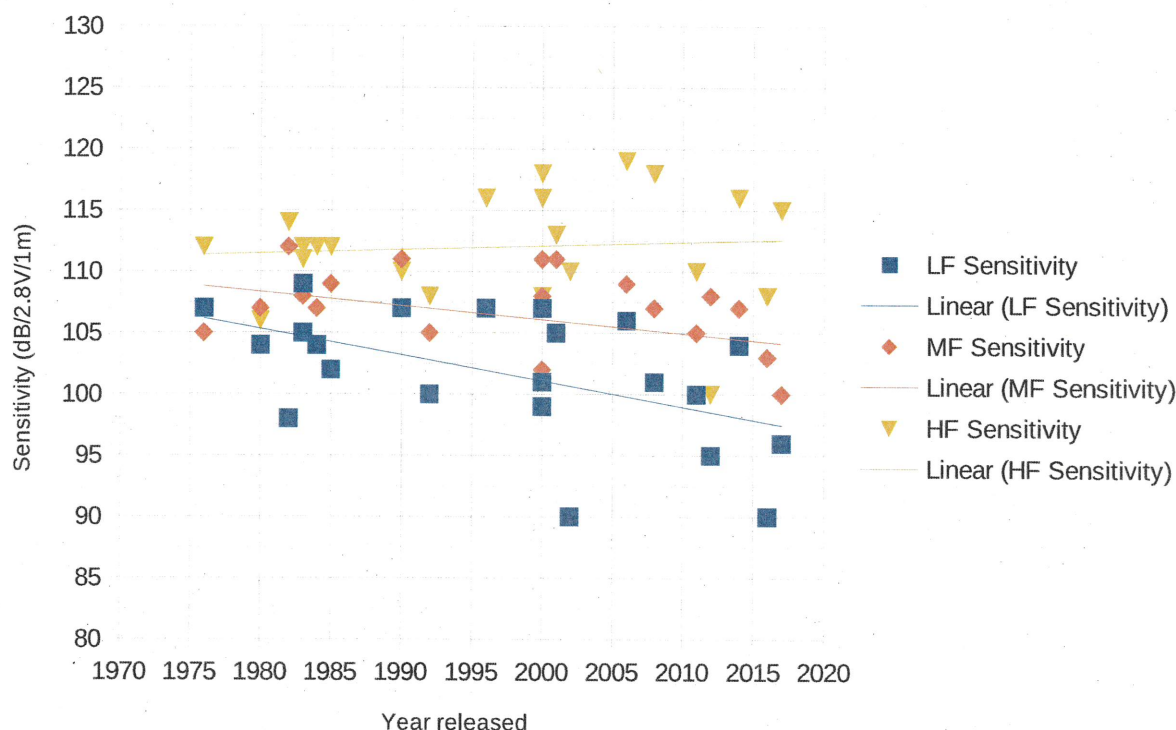


Figure 9: Sensitivity figures and trends for 22 large format 3-way PA speakers from 1975 to 2018

LF and MF sensitivity appears to have decreased since 1975, whilst HF sensitivity has marginally increased. This is likely due to improved materials technology and increased amplifier power reducing the necessity for high sensitivity at low frequencies. An additional factor is the dominance of Line Arrays in the marketplace; mutual coupling in arrays reduces the LF requirements of a single cabinet, but coupling at high frequencies is generally incoherent due to the short wavelengths.

One factor not accounted for, is that older cabinets usually had one large compression driver on a large horn, whereas modern line array cabinets often have at least two (even four) compression drivers in one cabinet. Driver for driver HF sensitivity has probably also decreased!

Comparing to averaged amplifier power available in the same years, a moderate correlation coefficient of -0.57 was found for LF sensitivity against amplifier power, -0.41 for MF sensitivity and no correlation, -0.17 for HF sensitivity. These results suggest that amplifier power is not the only factor affecting efficiency in loudspeaker design, a somewhat obvious and logical conclusion, but an interesting exercise nonetheless.

6 DISCUSSION

The comparison of sound power outputs of direct radiating and horn loaded drive units produced expected, but still interesting results. The increase in sound power output is very significant, and essentially "for free". An interesting extension would be to measure the change in electrical impedance and quantify the increase in efficiency, but unfortunately there wasn't enough time to perform these measurements in exactly the same environmental and acoustic conditions.

Plotting the efficiency vs. output power graphs was a quick and easy procedure. It would certainly be useful to system designers and installers for amplifier manufacturers to provide this kind of data, particularly with multiple curves at different load impedances. This would greatly assist in maximising system efficiency.

The original plan for the total efficiency experiment was to use four full-range array loudspeakers connected in parallel. In practice it was not possible to read any power consumption above the quiescent power level until the loudspeakers were running at over 120dB, potentially disturbing the neighbours and unsafe even with ear protection(!). An intermittent 120dB low frequency "rumble" proved to be much more tolerable.

One unexpected finding was the issue of quiescent power consumption. This made power measurements difficult in every configuration tested.

None of the amplifiers tested consumed less than 100W while idle, and one had a quiescent power consumption of over 200W. In the context of a large fixed installation that may be idle most of the time, this represents an enormous wastage of energy, that could well be larger than the losses when the system is in use. Should more amplifiers have automatic idle detection and low power states? How practical is it to reduce the quiescent power consumption?

In the field measured energy consumption data, systems B and C provided useful results as the measurements were from dedicated audio circuits. The results from system A were not useful, as despite the system in question being by far the largest audio load on the generator, the combined loads of lighting and miscellaneous areas were even larger still, preventing any meaningful comparison to data obtained from other areas.

7 CONCLUSIONS

Class D amplifiers have revolutionized sound reinforcement with unprecedented power density and efficiency, but in a similar fashion to the introduction of the first solid state hifi amplifiers in the 70s, loudspeaker efficiency has reduced.

It's good engineering practice and environmentally responsible to take advantage of every efficiency gain possible, including high efficiency transducer design and horns.

8 FURTHER WORK

Repeating the electrical efficiency experiments with many loudspeakers in parallel would be interesting, if impractical (due to the requirement of a very large reverberation or anechoic chamber). The total efficiency of real world systems is unquestionably much higher than the single loudspeaker results presented here.

As the generator load data was not as useful as anticipated, a future experiment would be to directly measure the total power consumption of large scale sound systems with various program material, over the duration of an event. This could be easily achieved with commercially available data logging equipment measuring current and line voltage on the audio circuits only, removing the extraneous (and more significant) lighting and miscellaneous loads.

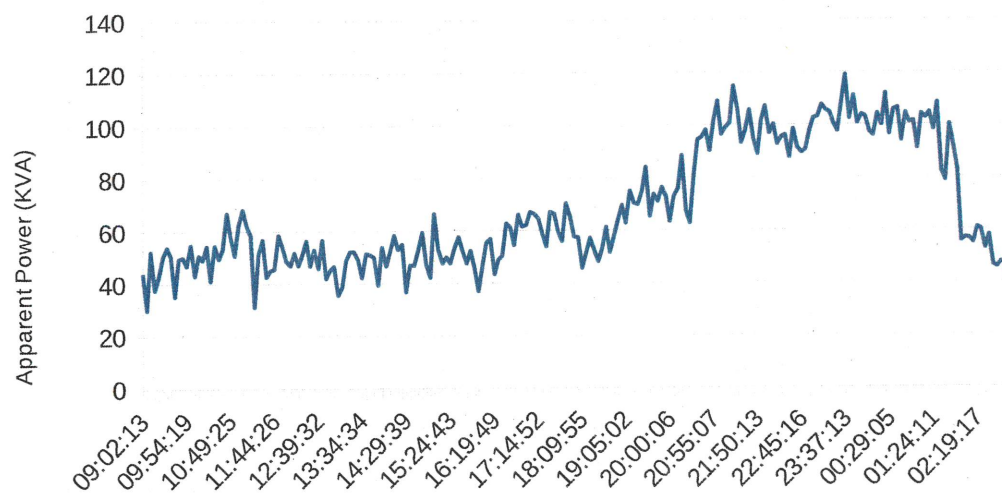
Further investigation into efficiency and mutual loading in bass horn arrays would also be useful, as there is little to be found in literature, and many misconceptions and unsubstantiated claims in common discussion of the topic.

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10 APPENDIX

10.1 GENERATOR LOAD PROFILE OVER ONE DAY (FESTIVAL)



11 ACKNOWLEDGEMENTS

Special thanks to Tom Devaney and Greg Shepherd from Aggreko plc for assistance with generator load data used in this paper.