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A FORCED SOLUTION FOR MULTICHANNEL FEEDFORWARD ADAPTIVE SYSTEMS

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1. INTRODUCTION

As is well known, compensation in the reference-signal path in conventional feedforward systems must be identical to the forward transfer-function between the secondary sources and the detection points, in which case the adaptive filter ideally converges to the Wiener solution. It is shown however that by appropriately altering the compensation from the conventional choice, an adaptive system can be forced to converge to any predetermined solution of interest. In particular, with this approach it is shown that systems of relatively small dimensionality (numbers of error channels) can be made to converge to solutions that could otherwise be achieved only by systems of much larger dimensionality, whose implementation could be prohibitive. Results of computer experiments are presented validating the theory that is developed.

2. ANALYSIS

The general configuration of multichannel filtered-X feedforward systems is shown in Fig. 1. The analysis will be carried out in frequency space with X(m) and D(m) the discrete Fourier transforms of the discrete-time-series reference and disturbance signals x(n) and d(n), and $W_k(m)$ the transfer function of the kth iteration of the adaptive-filter impulse response $w_k(n)$, given by

$$X(m) = \sum_{n=0}^{N-1} x(n)\, e^{\frac{-i2\pi nm}{N}}, \qquad D(m) = \sum_{n=0}^{N-1} d(n)\, e^{\frac{-i2\pi nm}{N}},$$

$$W_k(m) = \sum_{n=0}^{N-1} w_k(n)\, e^{\frac{-i2\pi nm}{N}}$$

* Patent application in process

With this formulation all results that follow are to be understood to refer to specific frequency pickets.

In a general multidimensional case we consider K reference signals, L secondary sources and M disturbances, in which case we have:

$\vec{X}$ is a $K \times 1$ vector of reference signals

$\vec{D}$ is an $M \times 1$ vector of disturbances

P is an $M \times L$ matrix of transfer functions between the secondary sources and the M detection points

Q is the $M \times L$ compensation matrix

W is an $L \times K$ matrix of adaptive filter transfer functions between the reference signals and the secondary sources.

$\mathbf{T} = \overline{\vec{D}\vec{X}^{\dagger}}$ is an $M \times K$ matrix of cross spectral densities between the reference signals and the disturbances

$\mathbf{S} = \overline{\vec{X}\vec{X}^{\dagger}}$ is a $K \times K$ matrix of cross spectral densities between the reference signals.

Suppressing the discrete-frequency index m, the filter output $\mathbf{W_k X}$ drives a secondary source producing a response $\mathbf{PW_k X}$ at the detection point which yields the squared error $|\mathbf{D} - \mathbf{P W_k X}|^2$. In the conventional filtered-X LMS algorithm the transfer function **P** is compensated in the reference-signal path prior to the weight iteration stage ΔW by **P**. Here we denote the compensation operation by **Q** since, as noted, we wish to consider an alternative non-conventional compensation. The weight-iteration equation in frequency space then takes the form[1] , where for convenience, we omit the overbar on **W**.

$$\mathbf{W}(k+1) = \mathbf{W}(k) + 2\mu \mathbf{Q}^{\dagger} \left[\mathbf{T} - \mathbf{P}\,\mathbf{W}(k)\,\mathbf{S}\right] \tag{1}$$

With conventional perfect compensation $\mathbf{Q} = \mathbf{P}$, and $\mathbf{Q}^{\dagger}\mathbf{P} = \mathbf{P}^{\dagger}\mathbf{P}$, which is Hermitian and positive definite, a necessary requirement for convergence.

We now consider how **Q** may be chosen to force an adaptive system to converge to any desired ideal solution $\mathbf{W_D}$. This situation is diagrammed in Fig. 1. The desired or ideal system is shown in 1a with K reference signals, L secondary sources and N disturbances and detection points. The ideal desired system is conventional, with adaptive-filter transfer function $\mathbf{W_D}$ given by [1]

$$\mathbf{W_D} = \left[\mathbf{P_N^{\dagger} P_N}\right]^{-1} \mathbf{P_N^{\dagger}}\, \mathbf{T_D}\, \mathbf{S}^{-1} \tag{2}$$

where $\mathbf{P_N}$ is the $N \times L$ matrix of transfer functions from the L secondary sources to the N detection points and

[1] Minkoff, J., "Performance of Multichannel Feedforward Adaptive Systems," Proceedings of Inter-Noise 96.

$$T_D = \overline{\vec{D}_N \vec{X}^T} \quad , \qquad S = \overline{\vec{X}\vec{X}^T} \tag{3}$$

where $\vec{D}_N$ is the N × 1 vector of disturbances and $\vec{X}$ is the K × 1 vector of reference signals.

The physical system is shown in 1b with the same numbers K and L of reference signals and secondary sources but M disturbances, where M < N, and an M × L forward transfer-function matrix **P**. The problem is to choose **Q** such that **W(k)** will converge to $\mathbf{W_D}$. Now, referring to (1), we observe that in a conventional system with perfect compensation, convergence takes place when

$$\Delta \mathbf{W} = \mathbf{P}^{\dagger} (\mathbf{T} - \mathbf{P}\,\mathbf{W}(k)\,\mathbf{S}) = 0 \tag{4}$$

which requires

$$\mathbf{W}(k) \text{ -> } \mathbf{W_I} = \left(\mathbf{P}^{\dagger}\mathbf{P}\right)^{-1} \mathbf{P}^{\dagger}\,\mathbf{T}\,\mathbf{S}^{-1} \tag{5}$$

where $\mathbf{W_I}$ is the ideal Wiener solution. If compensation is implemented by some $\mathbf{Q}^{\dagger} \neq \mathbf{P}^{\dagger}$, the weight iteration equation is

$$\mathbf{W}(k+1) = \mathbf{W}(k) + 2\mu\,\mathbf{Q}^{\dagger}\,(\mathbf{T} - \mathbf{P}\,\mathbf{W}(k)\,\mathbf{S}) \tag{6}$$

and therefore, subject to certain conditions and constraints to be specified shortly, the system can be forced to converge to any arbitrary desired solution $\mathbf{W_D}$ if the **Q** that is selected satisfies

$$\mathbf{Q}^{\dagger} \left[\mathbf{T} - \mathbf{P}\,\mathbf{W_D}\,\mathbf{S}\right] = 0 \tag{7}$$

This will occur because the quadratic error surface has only a single minimum and if (7) is satisfied $\mathbf{W_D}$ is the only stable point of convergence.

The matrix **Q** will consist of L complex column vectors $\mathbf{q}_i$, and (7) is therefore equivalent to the sets of equations:

$$\mathbf{B}\mathbf{q}_i = 0, \; i = 1, 2, .. L \tag{8}$$

where

$$\text{where } \mathbf{B} = \left[\mathbf{T} - \mathbf{P}\,\mathbf{W_D}\,\mathbf{S}\right]^{\dagger} \tag{9}$$

is K × M, must be full rank, and of course cannot be square otherwise there is only the trivial solution $\mathbf{q}_i = 0$ for all i. The system (8) therefore consists of LK equations in LM unknowns, with LM > LK, and the $\mathbf{q}_i$ are therefore underdetermined. There are however certain necessary constraints that provide additional equations for the $\mathbf{q}_i$. Specifically, referring to (1), **Q** must be chosen so that $\mathbf{Q}^{\dagger}\mathbf{P}$ is Hermitian and positive definite. A particularly simple choice is to let **Q** satisfy

$$\mathbf{Q}^{\dagger}\mathbf{P} = \mathbf{P}^{\dagger}\mathbf{P} \tag{10}$$

where, since **P** must be full rank, $\mathbf{P}^{\dagger}\mathbf{P}$ is positive definite as well as Hermitian;

P must also be rectangular; if it were square, (10) could be satisfied only by **P**=**Q** which would not serve our purposes. Since $\mathbf{P}^{\dagger}\,\mathbf{P}$ is L × L, (10) provides L^2 constraint equations for the q_i as well as satisfying the necessary conditions for convergence.

This solution is optimal in the sense that the rate of convergence is identical to what would be achieved if conventional compensation were employed. It also provides for causality. Suppose M = 5, K = 2, and L = 2, and write

$$\mathbf{Q} = [\mathbf{q}_1, \mathbf{q}_2] \qquad \mathbf{P} = [\mathbf{p}_1, \mathbf{p}_2] \tag{11}$$

and (10) becomes

$$\begin{bmatrix} \mathbf{q}_1^{\dagger}\,\mathbf{p}_1 & \mathbf{q}_1^{\dagger}\,\mathbf{p}_2 \\ \mathbf{q}_2^{\dagger}\,\mathbf{p}_1 & \mathbf{q}_2^{\dagger}\,\mathbf{p}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{p}_1^{\dagger}\,\mathbf{p}_1 & \mathbf{p}_1^{\dagger}\,\mathbf{p}_2 \\ \mathbf{p}_2^{\dagger}\,\mathbf{p}_1 & \mathbf{p}_2^{\dagger}\,\mathbf{p}_2 \end{bmatrix} \tag{12}$$

where

$$\mathbf{q}_i = \begin{bmatrix} q_{i1} \\ q_{i2} \\ q_{i3} \\ q_{i4} \\ q_{i5} \end{bmatrix} \qquad \mathbf{p}_i = \begin{bmatrix} p_{i1} \\ p_{i2} \\ p_{i3} \\ p_{i4} \\ p_{i5} \end{bmatrix} \tag{13}$$

Hence, considering for the moment La Place transforms, the individual elements of the matrices in (12) satisfy relationships, such as for the (1, 1) terms,

$$\begin{aligned} &q_{11}(-s)\,p_{11}(s) + q_{12}(-s)\,p_{12}(s) + q_{13}(-s)\,p_{13}(s) + q_{14}(-s)\,p_{14}(s) + \\ &q_{15}(-s)\,p_{15}(s) = p_{11}(-s)\,p_{11}(s) + p_{12}(-s)\,p_{12}(s) + p_{13}(-s)\,p_{12}(s) + \\ &p_{14}(-s)\,p_{14} + p_{15}(-s)\,p_{15}(s) \end{aligned} \tag{14}$$

because all the relevant time functions are real and we therefore have $p_{11}(s) = p_{11}^{*}(-s)$, $q_{11}(s) = q_{11}^{*}(-s)$, etc. where, as usual, $s = \sigma + i2\pi f$. Now, since the transfer functions p_{ij} represent physical measurements, the associated impulse responses are causal and the poles of the $p_{ij}(s)$ are all in the left-half of the s plane and those of $p_{ij}(-s)$ are therefore all in the right-half plane. Therefore, since the locations of all poles and zeros on the left-hand side of (14) must be identical to those of the right-hand side, then, with one possible exception, all the poles of $q_{ij}(-s)$ must be in the right-half plane and the $q_{ij}(s)$ therefore all represent causal impulse responses. The only exception to this that could occur would be if the $p_{ij}(s)$ happened to have zeros in the left-half plane in *exactly* the same locations as poles of $q_{ij}(-s)$ in the left-half plane, which would violate casuality for $q_{ij}(s)$. In the event of this extremely unfortuitous situation it would be possible to relocate the error sensors to obtain suitable $p_{ij}(s)$.

Now, continuing with this example, and denoting $(\mathbf{B})_{ij} = b_{ij}$

we have the equations:

$$b_{11} q_{11} + b_{12} q_{12} + b_{13} q_{13} + b_{14} q_{14} + b_{15} q_{15} = 0 \quad (15)$$

$$b_{21} q_{11} + b_{22} q_{12} + b_{23} q_{13} + b_{24} q_{14} + b_{25} q_{15} = 0$$

$$p_{11}^{*} q_{11} + p_{12}^{*} q_{12} + p_{13}^{*} q_{13} + p_{14}^{*} q_{14} + p_{15}^{*} q_{15} = | \mathbf{p}_1 |^2$$

$$p_{21}^{*} q_{11} + p_{22}^{*} q_{12} + p_{23}^{*} q_{13} + p_{24}^{*} q_{14} + p_{25}^{*} q_{15} = \mathbf{p}_1^{\dagger} \mathbf{p}_2$$

and a similar set of equations for the q_{2j} s. Thus (15) and the equivalent set for $\mathbf{q}_2$ satisfy (8) as well as all the necessary conditions for convergence. We also note that in this example the number of unknowns, $2 \times 5 = 10$, exceeds the number of equations, 8, by 2, and therefore the values of one of the q_{1j} and one of the q_{2j} can be assigned an arbitrary value. There does not appear to be any reason why this is not perfectly satisfactory, and, indeed, could possibly be advantageous. However a unique solution for the q_{ij}s can also be obtained by, for example, increasing either K or L by 1, yielding in the latter case the constraint equations

$$\begin{matrix} \mathbf{q}_1^{\dagger} \mathbf{p}_1 & \mathbf{q}_1^{\dagger} \mathbf{p}_2 & \mathbf{q}_1^{\dagger} \mathbf{p}_3 \\ \mathbf{q}_2^{\dagger} \mathbf{p}_1 & \mathbf{q}_2^{\dagger} \mathbf{p}_2 & \mathbf{q}_2^{\dagger} \mathbf{p}_3 \\ \mathbf{q}_3^{\dagger} \mathbf{p}_1 & \mathbf{q}_3^{\dagger} \mathbf{p}_3^{\dagger} & \mathbf{q}_3^{\dagger} \mathbf{p}_3 \end{matrix} = \begin{matrix} \mathbf{p}_1^{\dagger} \mathbf{p}_1 & \mathbf{p}_1^{\dagger} \mathbf{p}_2 & \mathbf{p}_1^{\dagger} \mathbf{p}_3 \\ \mathbf{p}_2^{\dagger} \mathbf{p}_1 & \mathbf{p}_2^{\dagger} \mathbf{p}_2 & \mathbf{p}_2^{\dagger} \mathbf{p}_3 \\ \text{psub3}^{\dagger} \mathbf{p}_1 & \mathbf{p}_3^{\dagger} \mathbf{p}_2 & \mathbf{p}_3^{\dagger} \mathbf{p}_3 \end{matrix} \quad (16)$$

We now have 15 unknowns, 6 equations of the form $\mathbf{B}\mathbf{q}_{ij} = 0$ and 9 constraints in (16). In general, for a solution to exist we must always have

$$M \geq K + L \quad (17)$$

and for a unique solution for **Q** we require $LM - LK = L^2$ or

$$M = K + L \quad (18)$$

3. RESULTS OF COMPUTER EXPERIMENTS

In order to verify the foregoing theoretical results a series of computer experiments was carried out. Using (2) (conventional compensation) a causal solution for the ideal adaptive filter transfer function for a multidimensional system consisting of three reference signals, three secondary sources and 21 detection points was calculated.[2] Using this solution for $\mathbf{W}_D$, the **Q** compensation for six disturbances was calculated using (7) and (10), yielding a unique solution (18). Using this **Q**, the set of nine weights for six disturbances was calculated using (6) and compared with the ideal solution $\mathbf{W}_D$ for 21 error

[2] Novak, S., "Analytic Model for Broadband Feedforward Control I: Theory AT&T/Bell Laboratories Technical Memorandum, 31 October 1994.

channels. The results for three of the nine (to save space) are presented in Fig. 2. The ideal adaptive-filter transfer functions for 21 errors and conventional compensation are essentially identical to those for six error signals and Q-compensation. The six error signals in this case were a subset of the 21. This is unnecessary. Fig. 3 presents results for six error signals not included in the set of 21, with essentially identical agreement as in Fig. 2.

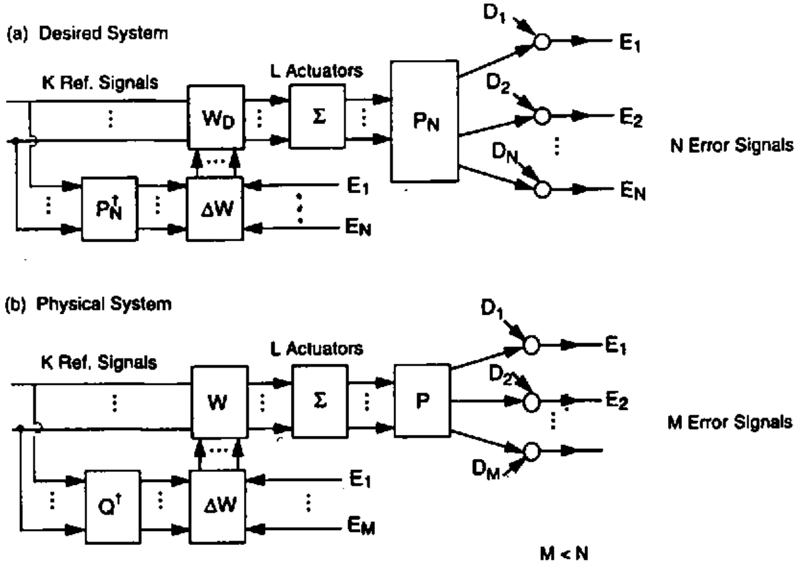
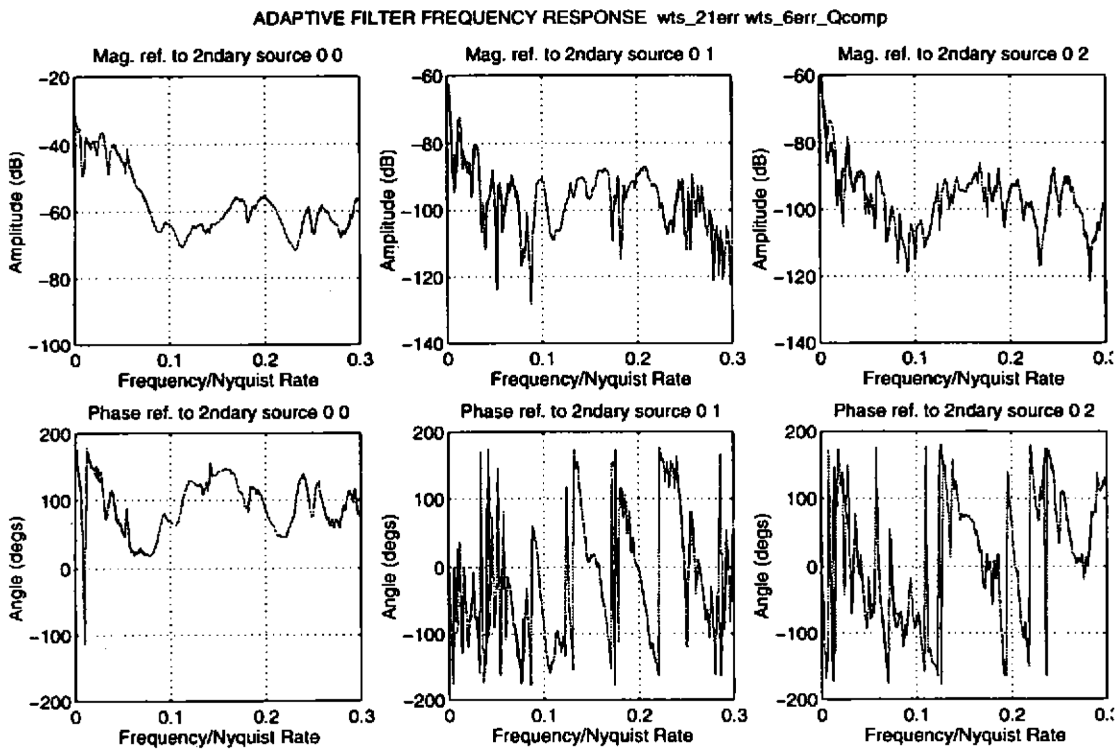
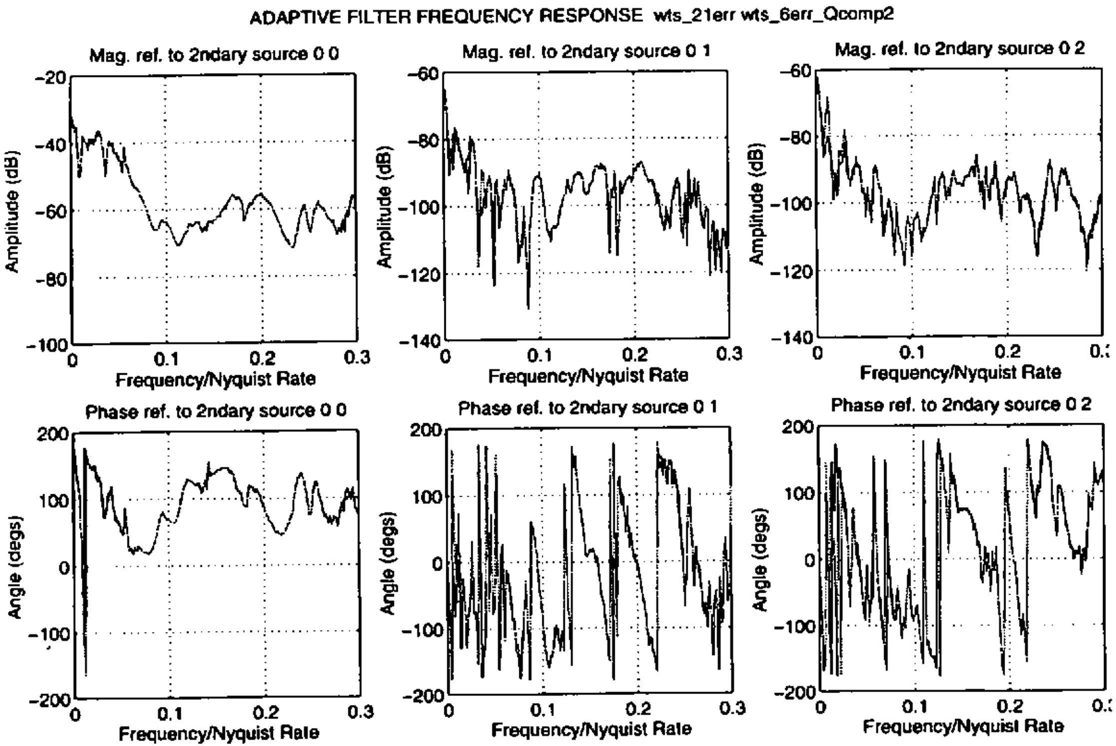


Fig. 1 Desired System and Physical System



File 1 : wts_21err
File 2 : wts_6err_Qcomp

Fig. 2



File 1 : wts_21err
File 2 : wts_6err_Qcomp2

Fig. 3