Proceedings of the Institute of Acoustics

FAR-FIELD RADIATION FROM A SOURCE IN A FLAT RIGID BAFFLE OF FINITE SIZE

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ABSTRACT

The far-field radiation of a source in a flat rigid baffle of finite size is derived. The velocity distribution of the source is arbitrary as well as the shape of the source and the baffle. The derivation covers the two sided as well as the singled sided radiating case. The approach makes use of the Kirchhoff-Helmholtz equation as well as of spatial Fourier transforms of the source and the baffle shape.

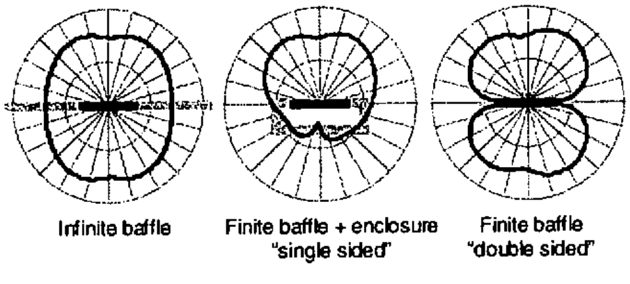
BACKGROUND AND APPLICATION

The radiation model is used in the simulation of the new Distributed Mode Loudspeaker (DML) [1]. The DML is usually a flat vibrating plate in bending, which is dimensioned in such a way that a smooth sound pressure response is achieved. The DML operates in the sub- as well in the super-sonic frequency range. Due to the bending-wave modulation a nearly omni-directional directivity is typical for the DML.

The DML might be built in an infinite baffle, a finite baffle, or it can be built into an enclosure, which is usually shallow in order to maintain the flat character of the device.

However, the presented diffraction calculation can also be used for the radiation of any plates, beams, membranes or flat apertures. The model can also be used for the simulation of any conventional loudspeaker systems.

In most DML applications there are three principle radiation conditions:



Three typical radiation conditions

The source, such as a vibrating plate, is built into an acoustically rigid baffle. The baffle can be of infinite size. This boundary condition model is applied if the source is mounted into a wall, such as the ceiling tile loudspeaker. If the rear radiation of the plate or diaphragm is sealed with the help of an enclosure then the applied boundary condition model is called "single sided" here. The so-called "double sided" case occurs if both sides of a plate or a diaphragm vibrate exactly in anti-phase and there is either no baffle present, or the baffle is of finite extent. In the presented model the baffle can be of arbitrary shape as long as it is flat and in the same plane as the source.

THEORETICAL MODEL

General boundary conditions

The theoretical model solves for the velocity potential in the far-field.

$$\varphi \rightarrow \varphi_f$$

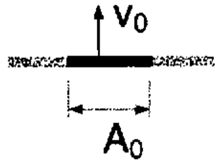
The potential-field obeys the Helmholtz equation

$$\nabla^2\varphi + k^2 \cdot \varphi = 0$$

and satisfies the boundary conditions at infinity and in the flat plane where the source is embedded.

$$\frac{\partial \varphi}{\partial n_0} = -v_0 \cdot A_0$$

The source has normal velocity, v_0. The baffle is acoustically rigid.

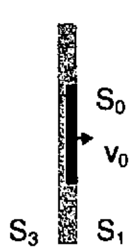


Source with normal velocity v_0, aperture function A_0 and finite baffle.

These inhomogeneous boundary conditions can be described with the help of the aperture function, A_0. $A_0 = 1$ for points, which belong to the source and $A_0 = 0$ for points on the baffle.

"SINGLE SIDED" CASE

The source with vibrating area, S_0, is embedded on one side of a finite baffle. For example, a plate in a shallow enclosure.

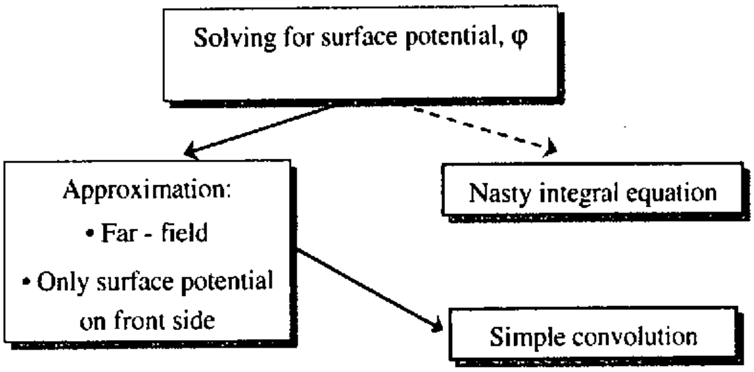


Source on one side only with normal velocity v_0 and finite baffle.

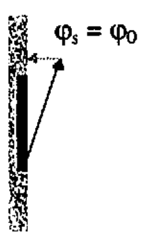
The Helmholtz integral gives the potential everywhere in the domain of radiation [2]:

$$\alpha \cdot \varphi = \int_{S_0} v \cdot G \, dS - \int_{S_1} \varphi \cdot G_n \, dS - \int_{S_3} \varphi \cdot G_n \, dS$$

The Helmholtz integral has terms involving surface velocities, v and terms involving surface potentials, φ. The free field Green function, G weights velocities. Potentials are weighted by the normal gradient of the Green function, Gn. The integration is performed over the total surface. However, the baffle has zero thickness and therefore the contribution from the sides can be neglected. If the baffle is assumed to be rigid then the velocity needs to be integrated only over S_0. The coefficient $\alpha = 1$ for points in the domain, $\alpha = 1/2$ on the boundary and $\alpha = 0$ otherwise.



Whereas the surface velocity, v_0 is given, the surface potential, φ is unknown. Solving for the surface potential usually yields to the problem of solving an integral equation. But, it is possible to derive a simple convolution-formula, which directly yields a good approximation for the aimed applications. For this only far-field solutions are taken into account and the approximation has been made of taking into account only the surface potential on the front-side of the baffle.



Moving test point onto surface

The procedure starts with moving the test-point down onto the surface. Because the baffle-source-plane is flat, the normal gradient of the Green function is zero here (Gn = 0). $\alpha = 0.5$ because of the pressure jump across the surface.

$$\frac{1}{2} \cdot \varphi_s = \int_{S_0} v \cdot G \, dS - \int_{S_1} \cancel{\varphi \cdot G_n \, dS} - \int_{S_3} \cancel{\varphi \cdot G_n \, dS}$$

Thus the surface potential is given by the velocity term only. This term is formally identical to the Rayleigh integral, which describes the radiation of the same source but from an infinite baffle.

$$\varphi_s = 2 \cdot \int_{S_0} v \cdot G \, dS$$

Because of this φ_s can be replaced by the inverse of the spatial Fourier transform at the surface, i.e. with z = 0 [3]:

$$\varphi_s = \frac{j}{4 \cdot \pi^2} \cdot \int_{\pm\infty} \frac{\breve{Q}_{(k_x, k_y)}}{\sqrt{k - k_x - k_y}} \cdot e^{-j \cdot k_x \cdot x - j \cdot k_y \cdot y} \, dk_x \, dk_y$$

$\breve{Q}$ is the spatial Fourier spectrum of the source, which is explained further below.

In the next step the Helmholtz integral is written down for points off the baffle plane and with the surface potential under the integral replaced by the inverse Fourier transform:

Replace by inverse Fourier transform, φ_s

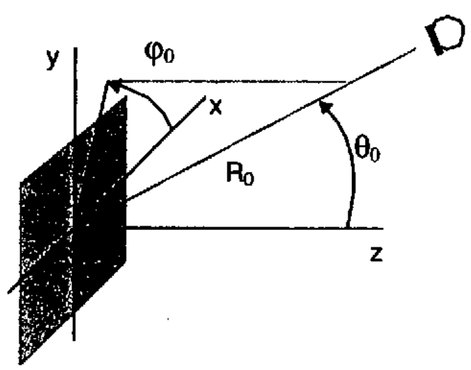
$$\varphi = \int_{S_0} v_0 \cdot G \, dS - \int_{S_1} \varphi_1 \cdot G_n \, dS$$

Note, that the contribution of the integral, which is over the rear of the finite baffle, is ignored ($\varphi_3 = 0$).

In the next step, all Green functions of the Helmholtz Integral are replaced by their far-field expression [3].

$$G_{(r', r)} = \frac{e^{-j k \cdot d_{(r', r)}}}{4\pi \cdot d_{(r', r)}} \quad \longrightarrow \quad g_{(x, y)} = G_0 \cdot e^{j k_{xo} \cdot x + j k_{yo} \cdot y}$$

$$G_{n(r', r)} = -G_{(r', r)} \cdot \left(j \cdot k + d^{-1}_{(r', r)} \right) \cdot \frac{\partial}{\partial n'} d_{(r', r)} \quad \longrightarrow \quad g_{n(x, y)} = -j \cdot k_{zo} \cdot G_0 \cdot e^{j k_{xo} \cdot x + j k_{yo} \cdot y}$$



Far-field co-ordinates of receiver

The arguments of the far-field version of the Green-function are wave-numbers which can be mapped to spherical co-ordinates of the receiver, where the angles are angular positions of the receiver.

$$k_{xo} = k \cdot \sin(\theta_0) \cdot \cos(\varphi_0)$$

$$k_{yo} = k \cdot \sin(\theta_0) \cdot \sin(\varphi_0)$$

$$k_{zo} = k \cdot \cos(\theta_0)$$

The far-field radial term becomes:

$$G_0 = \frac{e^{-j k \cdot R_0}}{4\pi \cdot R_0}$$

The radius, R_0 is the distance of the receiver to the center of the source.

After some algebraic manipulation the final expression for the far-field potential is obtained.

$$\varphi_f = G_0 \cdot \left(\breve{Q}_{(k_{xo},\, k_{yo})} + \breve{D}_{(k_{xo},\, k_{yo})} \right)$$

It is composed of the constant G_0, the above mentioned Fourier transform of the source term, $\breve{Q}$, and the diffraction term $\breve{D}$, which is a wave-number convolution:

$$\breve{D}_{(k_{xo}, k_{yo}, k_{zo})} = \frac{k_{zo}}{4 \cdot \pi^2} \cdot \int_{\pm\infty} \frac{\breve{Q}_{(k_x, k_y)}}{\sqrt{k - k_x - k_y}} \cdot \breve{A}_{B\,(k_{xo}-k_x,\, k_{yo}-k_y)}\, dk_x\, dk_y$$

or in short term notation:

$$\breve{D} = \frac{k_{zo}}{4 \cdot \pi^2} \cdot \left(\frac{\breve{Q}}{k_z} * \breve{A}_B \right)$$

The diffraction term involves the Fourier transform of the source, $\breve{Q}$, as well as of the baffle-shape, $\breve{A}_B$:

$$\breve{Q} = \breve{v}_0 * \breve{A}_0$$

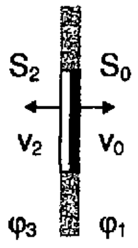
$$\breve{A}_{0\,(k_x, k_y)} = \int_{\pm\infty} A_{0\,(x, y)} \cdot e^{j k_x \cdot x + j k_y \cdot y} dx\, dy$$

$$\breve{A}_{B\,(k_x, k_y)} = \int_{\pm\infty} A_{B\,(x, y)} \cdot e^{j k_x \cdot x + j k_y \cdot y} dx\, dy$$

$\breve{v}_0$ is the Fourier transform of the source velocity function, which does not include the size of the source. $\breve{A}_0$ is the spatial Fourier transform of the shape of the source. It is a sinc-type function and, in most cases can easily be calculated analytically. $\breve{A}_B$ is the Fourier transform of the shape of the baffle and its character is identical to $\breve{A}_0$.

"DOUBLE SIDED" CASE

The Double Sided case has the same boundary condition as before but there are now two vibrating areas:

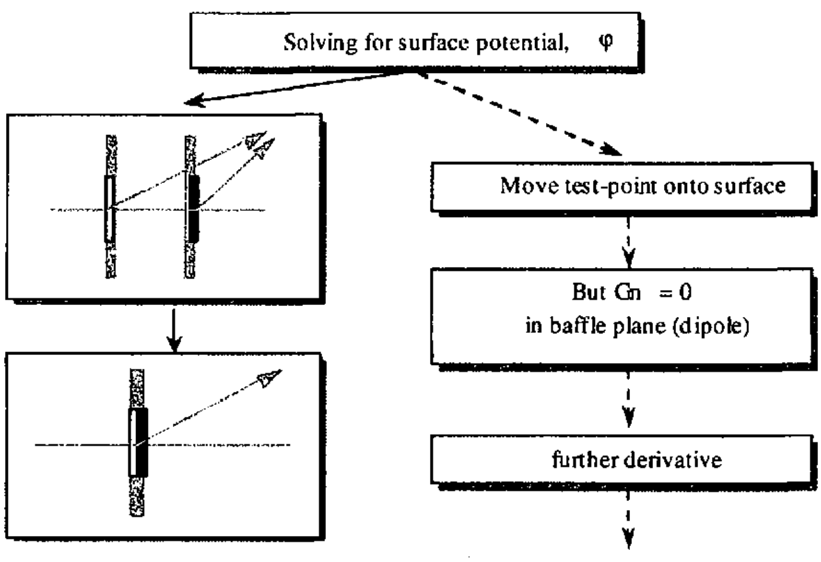


Source on both sides with normal velocity $v_2 = -v_0$ and finite baffle.

The second area, S_2, is at mirror position to S_0 on the rear of the baffle and vibrates exactly in anti-phase. Thus, a diaphragm or plate in a baffle is simulated. It can be shown that in this case the velocity integrals cancel and only potentials are part of the Helmholtz integral [4]:

$$\alpha \cdot \varphi \;=\; -\int_{S_1} \varphi \cdot G_n \, dS - \int_{S_3} \varphi \cdot G_n \, dS \;=\; -2 \cdot \int_{S_1} \varphi \cdot G_n \, dS$$

In order to solve for the surface potential under the integral, the test-point normally would be moved onto the surface. However, in this case the integral equation collapses because of the dipole character of the normal gradient of the Green function. Often, this so-called Thin-Shape Breakdown is solved with the help of a further derivation operating on the Helmholtz integral [5].



However, here a different route is followed. If the Double Sided case is constructed from the model of two Single Sided radiators, which are brought close together, back to back, so to say, then the diffraction formula for the far-field is obtained automatically:

$$\varphi_f = 2 \cdot G_0 \cdot \breve{D}_{(k_{xo},\, k_{yo})}$$

FORMULATION FOR ALL THREE CASES

The resulting far-field formula can be written down in compact form for all three radiation cases.

$$\varphi_{f(k_{xo},k_{yo},k_{zo})} = G_0 \cdot \begin{cases} 2\cdot\breve{Q}_{(k_{xo},k_{yo})} & \text{Infinite baffle} \\ \breve{Q}_{(k_{xo},k_{yo})} + \breve{D}_{(k_{xo},k_{yo},k_{zo})} & \text{Single sided} \\ 2\cdot\breve{D}_{(k_{xo},k_{yo},k_{zo})} & \text{Double sided} \end{cases}$$

For the Infinite Baffle case the far-field potential is proportional to the wave-number spectrum of the source term, $\breve{Q}$. For the Single Sided case it is the sum of $\breve{Q}$ and the diffraction term, $\breve{D}$. For the Double Sided case the far-field involves only the diffraction term $\breve{D}$.

The diffraction term, $\breve{D}$ is the previous shown wave-number convolution involving the source term and the Fourier transform of the baffle shape, $\breve{A}_B$.

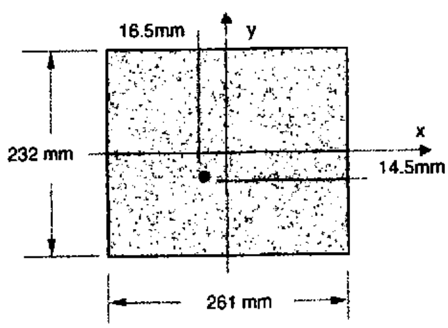
The advantage of this formula is that it is straightforward to use for directivity and radiation impedance calculations. In many cases the spatial Fourier transforms can be solved analytically.

The wave-number convolution must be carried out numerically in angle or wave-number space. The numerical integration can be performed directly or with the help of discrete Fourier techniques.

COMPARISON OF RESULTS

The results are compared with those obtained from a solver using the Boundary Element Method.

The wave-number convolution has been performed in angle space, which is slow in calculation but gives smooth results because all singularities are removed.

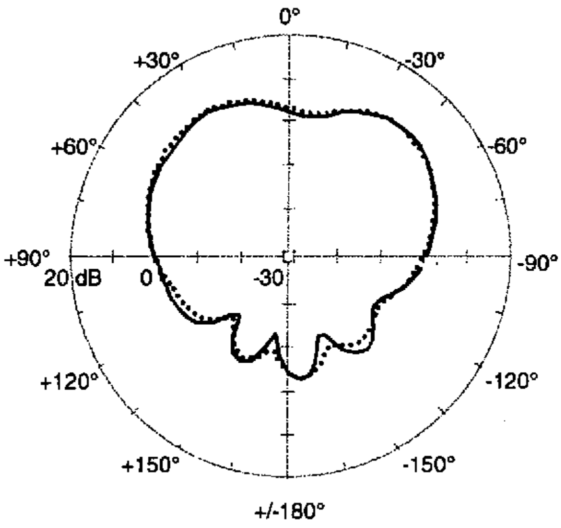


Point source on baffle

In order to test the diffraction caused by the finite size of the baffle a point source has been placed on the surface of the baffle, slightly offset from the center. For a point source $\breve{Q} = 1$.

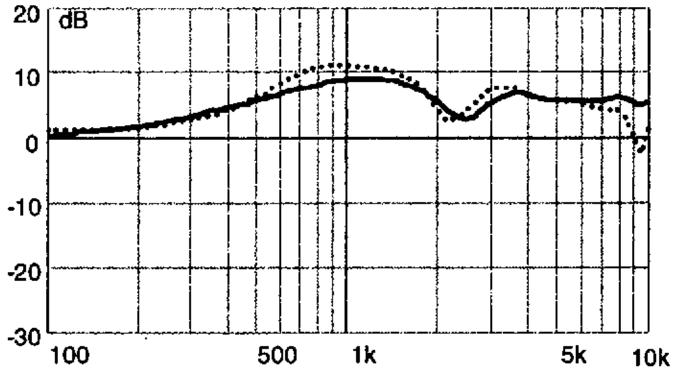
The BEM uses a direct approach and because of this the structure has to have a finite thickness (10 mm) in order to avoid Thin-Shape-Breakdown. In the used BEM package the source had to be at least 10 mm above the surface and the far-field response was simulated by calculating the field in R = 2 m distance.

"Single sided" case



Directivity at 2500 Hz, single sided, solid: Convolution, dotted: BEM
Abscissa: Horizontal receiver angle in degree
Ordinate: Level of potential in dB, range: 50 dB

The directivity is displayed in a polar plot, where the top is aligned with the normal of the baffle area. The frequency is 2500 Hz.



Frequency response of the far-field potential, single sided, solid: Convolution, dotted: BEM
Abscissa: Frequency log-scaled
Ordinate: Level of potential in dB, range: 50 dB

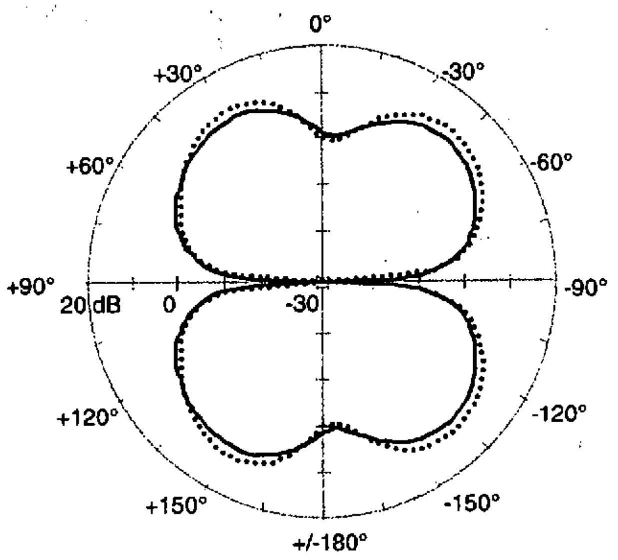
The level of frequency response of the far-field potential is displayed vs. a log-scaled frequency axis. The test-point is aligned with the normal of the baffle area.

The solid curve is the result of the convolution method, which is demonstrated here. The dotted curve is obtained from the BEM for comparison.

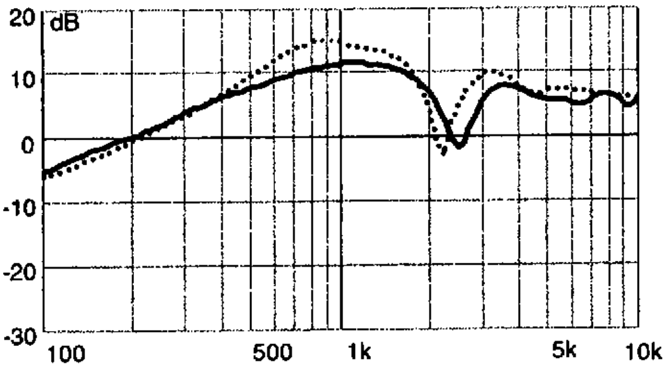
The curves display the radiation from a point source placed on a rigid finite baffle. Clearly visible are the effects of interference caused by diffraction. The dip in the BEM-curve just below 10 kHz is caused by interference between the source and the scattered field. The dip is not present in the convolution curve because the source is embedded in the surface of the baffle.

"Double sided" case

Following curves display the directivity and the frequency response of the Double Sided case.



Directivity at 2500 Hz, double sided, solid: Convolution, dotted: BEM
Abscissa: Horizontal receiver angle in degree
Ordinate: Level of potential in dB, range: 50 dB



Frequency response of the far-field potential, double sided, solid: Convolution, dotted: BEM
Abscissa: Frequency log-scaled
Ordinate: Level of potential in dB, range: 50 dB

Obvious is the dipole type radiation pattern and the increase of the interference effect due to double sided radiation. The interference dip around 2.5 kHz of the BEM response occurs at lower frequencies because of the finite thickness of the baffle, the displacement of the sources and the finite distance of the receiver point.

SUMMARY

A diffraction model using a wave-number convolution has been laid out. It can be used to calculate the directivity of flat structures such as plates and membranes in a finite baffle. The advantages are the fast calculation and the reasonable good approximation. The research focuses on the radiation of the Distributed Mode Loudspeaker. Current work investigates the coupling of this diffraction model to the Boundary Element Method.

Acknowledgements

The author would like to thank Dr Neil Harris and Dr Sean Kavanagh for valuable discussions and Dr Graham Bank for supplying the BEM simulations.

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