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GENERIC LIMITATIONS OF VIBRO-ACOUSTIC PREDICTION METHODS FOR PRODUCT NOISE

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1 BACKGROUND

Variations in the order of ± 5-8 dB at medium and higher frequencies for narrow-band vibro-acoustic transfer functions are often between nominally identical serial products such as road vehicles, aircraft, ships, appliances etc..

Such variation has been published for road vehicles [1]-[3]. Recently Kompella et. al. [1] showed measured scatter of frequency response functions for identical vehicles, see Figure 1. The FRFs show random behaviour and larger scatter as frequency increases. This scatter is usually taken as evidence for insufficient quality of supplied components or assembly, and QA-programs are used to try to reduce the variability. Use of highly detailed dynamic FE-models are also justified by this consideration of the variability to be due to "insufficient manufacturing quality". This paper will not address variability caused by actual quality or out-of-tolerance problems. Instead the fundamental scatter of dynamic response for multi-modal systems due to input parameter uncertainty is demonstrated, which limits deterministic predictability

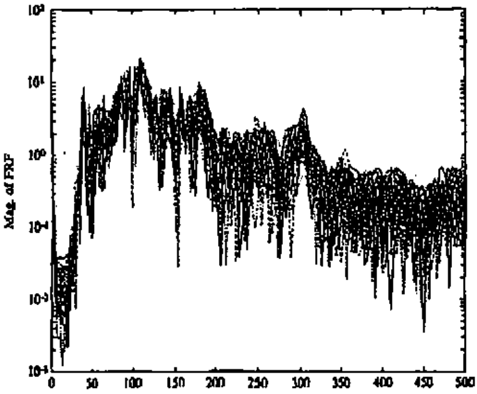

Fig. 1: Magnitudes of the 99 structure-borne FRFs for the RODEOs for the driver microphone [1].

A summary of fundamental limiting factors for deterministic modelling and analysis was presented in a recent SEA review paper by Fahy [4]. In addition to FE- or BE-models becoming very large at higher frequency due to the finer meshing requirements and a substantial increase in modelling effort due to more attention to geometrical detail, the fundamental limits for prediction of the response in detail are:

- *Uncertainty about precise dynamic properties.* Sensitivity of eigenfrequencies and phase to boundary conditions, thickness- and damping distribution etc. increases with mode order.

• *Modal summation.* Contributions from an increasing number of modes are added at each frequency as frequency and/or damping increases.

• *Uncertain dynamic properties of joints.* The dynamic force transmission properties of joints are not well defined and dynamic properties of joints are especially uncertain at higher frequencies.

• *Uncertain material properties.* Properties for alloys, polymers and composite materials are considerably harder to predict and they will vary much more due to temperature, static loads etc.

• *Uncertain modal damping estimation.* For detailed deterministic prediction, the correct spatial damping distribution or the correct estimates for individual modal damping has to be applied.

The response prediction of multi-modal systems is therefore a probabilistic problem. High-frequency response of a population of nominally similar products of which the individual members differ in many unpredictable details shall be described by an ensemble-average behaviour, together with statistical estimation of the distribution of responses around this average. One may randomise parameters and properties using some assumed distributions and make deterministic FE-computations (Monte-carlo simulation). This is impractical due to cost and time and probably not even feasible because of problems to model multi-dimensional joint probability distributions for a large number of parameters. An alternative is to use statistical energy methods (SEA) [4], [5].

2 THEORY

The response statistics of multi-modal systems was first derived for room acoustics [7]-[9] and has also been studied during the SEA development [10]. Schröder derived some basic relations already 1954 [7]. One considers a system where the response at each point and frequency is determined by the sum of a sufficient number of modes with random phase, and where no individual mode is dominating the sum. The real and imaginary parts of the complex response function are assumed to have a gaussian distribution [7]

$$W(\mathrm{Re}(x)) = \frac{1}{\sqrt{2\pi\overline{\mathrm{Re}(x)^2}}} e^{-\mathrm{Re}(x)^2/2\overline{\mathrm{Re}(x)^2}} \quad \text{and} \quad W(\mathrm{Im}(x)) = \frac{1}{\sqrt{2\pi\overline{\mathrm{Im}(x)^2}}} e^{-\mathrm{Im}(x)^2/2\overline{\mathrm{Im}(x)^2}} \qquad (1)$$

The gaussian distribution is valid when the response is given as the sum of several complex non-dominating independent (modal) vectors, see Figure 2a. The standard deviation of the frequency response function will be $\sigma = 5.57$ dB [7] in this case. Figure 2b shows a frequency response function with a dominating component, e.g., a dominating mode or the direct wave field of a damped system. In this case the gaussian distribution of real and imaginary parts does not apply.

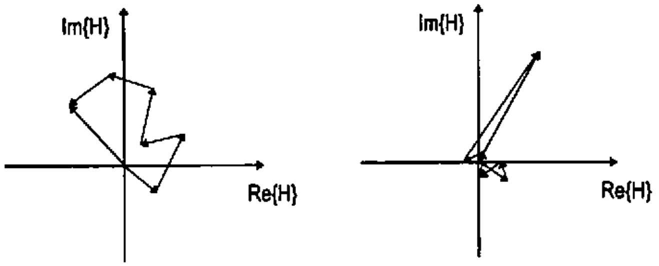

Fig. 2: The complex FRF and its modal components, for one frequency. a) No mode dominates. b) One component is dominating.

3 GENERIC VIBRO-ACOUSTIC MODELS

The variance given above was derived from a general summation of complex vectors. It should be valid for any system for which the frequency response function can be expressed as such a sum. For a system with N vibrational modes, the FRF between two points can be expressed as

$$H(x,x_e,\omega)=\frac{\bar{v}(x,\omega)}{F(x_e,\omega)}=\frac{4\,j\,\omega}{M}\sum_{i=1}^{N}\frac{\phi_i(x)\cdot\phi_i(x_e)}{\omega_i^2(1+j2\zeta_i)-\omega^2} \tag{2}$$

where x is the spatial vector, x = [x,y,z], index e refers to excitation point, M is total mass, ω is excitation frequency and ϕ_i is the eigenfunction of mode i.

Consider a generic dynamic system represented by a sum of "modes" according to Equ. (2) with eigenfrequencies distributed as $\omega_i = 200\pi\cdot\log(i+1)$ and the eigenfunctions at excitation and response positions being random numbers between -1 and 1. Eigenfrequencies for individual modes are randomized to simulate the effect of material and geometric parameter variations for real structures. The eigenfrequencies are shifted as:

$$\omega_{ij}=\omega_{ij0}\cdot(1+\varepsilon U)\quad\text{or}\quad\omega_{ij}=\omega_{ij0}\cdot(1+\varepsilon U_{ij}) \tag{3a, b}$$

where ω_{ij0} is the unshifted eigenfrequency, ε is the amplitude of the random variation and U and U_{ij} have normal distribution (m=0, σ=1). *Global* parameter variations (e.g. average plate density or modulus) correspond to all eigenfrequencies being shifted with the same εU (Equ. 3a). Examples are given in [6]. *Local* variations of thickness, mass, boundary conditions etc. gives individual shifts in eigenfrequency for each mode (Equ. 3b), where each ω_{ij} is shifted by εU_{ij}. Monte-Carlo simulation of an ensemble of plates is performed by using different sets of samples U_{ij}.

The same nominal modal damping ζ_{ij0}, is applied for all modes. The damping uncertainty is modelled as equation (4). U_{ij} is normally distributed with 0 mean value and a standard deviation of 1.This distribution is chosen as it provides a reasonably realistic modal damping distribution.

$$\xi_{ij}=\xi_{ij0}\cdot 10^{\varepsilon U_{ij}} \tag{4}$$

Figure 3 shows the difference in FRF that is obtained for two samples of this generic model.

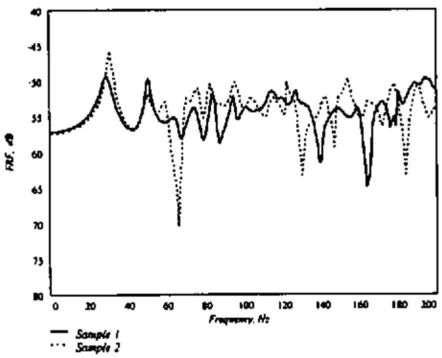

Fig. 3: FRFs for the generic modal expansion model with η = 5%. Standard deviations: 3% for eigenfrequencies, 30% for logarithm of the modal damping

The modal overlap factor, defined as

$$MOF = n(f)\eta f \tag{5}$$

where $n(f)$ is the average modal density (modes/Hz) and η the loss factor at frequency *f*, is larger than 1 for $f > 100$ Hz in this case. Schröder's formulation is applicable for modal overlap factor > 2-3. No single mode dominates the response in that case and the response is determined by several modes with different phase and amplitude.

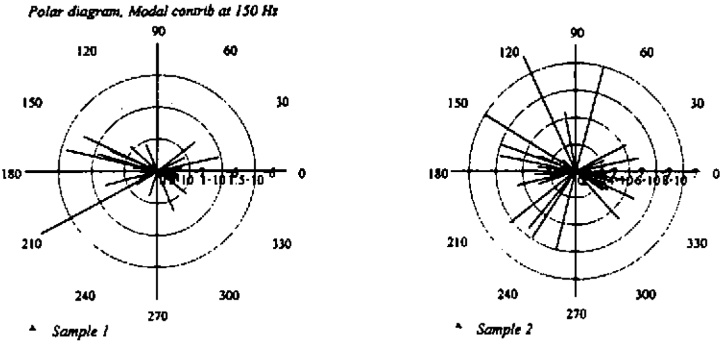

Fig. 4: Modal contributions at 150 Hz for the two samples from the generic modal expansion model.

The contributions from individual modes are shown for 150 Hz in Figure 4 a and b for the two samples. The large differences in both phase and amplitude of modal contribution vectors are obvious.

A thin rectangular, simply supported plate can be used to exemplify a real multi-modal component [6]. The analytic model of the plate was used to calculate the variations of FRFs resulting from localized parameter variations [6]. The excitation and response positions on the plate are arbitrarily chosen but the same for all plate samples. The same point-point frequency response function is plotted for all plate samples.

Local scatter in geometry, thickness, pre-stresses and boundary will introduce individual shifts of eigenfrequencies depending on how the mode shapes correlate with the localised variations of the structure. Also variations in individual modal damping factors, due to scatter in boundary conditions, material and joint damping distribution, sound radiation, etc. can be quite large. This is modelled as randomly distributed damping between the individual modes. The sensitivity to rather small eigenfrequency shifts is explained by the large FRF phase angle shift around the natural frequency of the single-degree-of-freedom system that represents each mode.

The necessary variation of the input parameters of the rectangular plate to get randomly varying FRFs for modal overlap larger than 2-3 is, e.g., a 2% eigenfrequency variation combined with a 20% variation in the logarithm of damping for the studied plate, see Figure 5a.

Energy methods (SEA) estimate spatial average responses for the subsystems [10]. They do not calculate the response at specific points on the system. However, as shown above, detailed FRF estimation is of limited practical value, since quite small input parameter uncertainties lead to low precision in the prediction anyway for significant modal overlap.

On the other hand, the spatial average energy response should fluctuate much less than the FRFs for corresponding variations in eigenfrequency and modal damping. The energy response of the same plate as before was calculated. For the same variation of eigenfrequencies and damping we obtain Figure 5b. The energy response levels show much less variation than the FRFs. energy model at frequencies where modal overlap is significant.

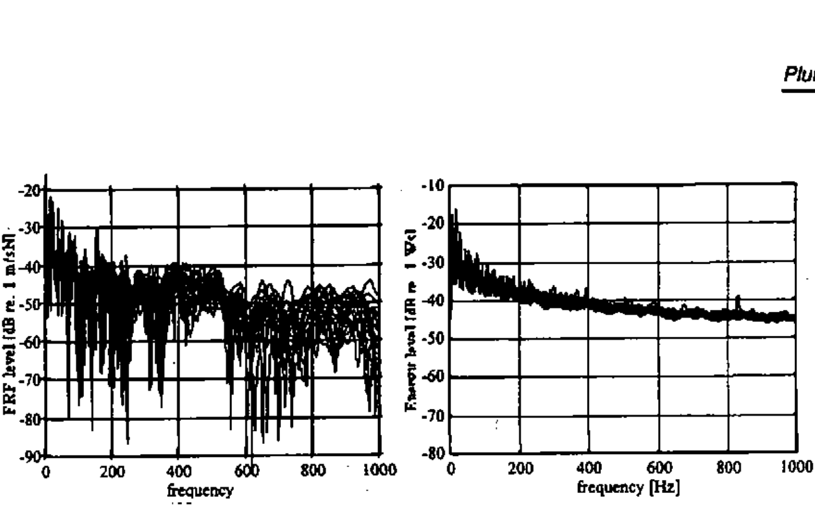

Fig. 5: a) FRFs for random variations of eigen-frequencies and modal damping. b) Variation of spatial average velocity level (proportional to the kinetic energy) between different samples of the plate. Standard deviations: 2% for eigenfrequencies and 20% for the logarithm of damping factor.

4 SCATTER IN BUILT-UP STRUCTURES

The energy flow model developed by Fredö [13] for two connected plates was used to demonstrate variation of FRFs for built-up structures. The result of combined variations of individual modal eigenfrequencies and damping, corresponding to those used for the simple plate, is shown in Figure 6a. The FRFs between an arbitrary point on plate 1 and a point on plate 2 is shown. The calculated response energy level in plate 2 with excitation in plate 1, corresponding to the result shown in Figure 5b for the single plate, is given in Figure 6b.

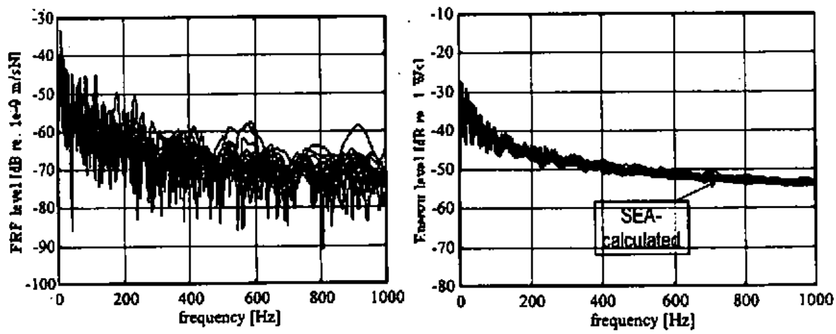

Figure 6:a) FRFs for random variations of eigen-frequencies and modal damping and b) Variation of spatial average velocity level in the receiving plate of the L-plate. Standard deviations: 2% for eigenfrequencies and 20% for logarithm of damping.

The response energy level calculated with Statistical Energy Analysis (SEA) has been included in Figure 6b for comparison. The plates are represented as bending wave subsystems only. The agreement with the exact analytical result is quite good.

5 CONCLUSIONS

Frequency response functions for simple as well as built-up vibro-acoustic systems with ovelapping modes are quite sensitive to small variations in local input parameters. Numerical simulation using realistic parameter tolerances show that the FRFs may become well randomised already for modal overlap factors of 2-3. Experimentally obtained variability by the author and published by other workers for nominally identical products like cars compare well with the simulations presented here and in [6].The standard deviation for FRFs between specific points and at a specific frequency is considerable. The variance is substantially smaller for spatial RMS-averages as well as for frequency band averages of the FRFs, which correspond to subsystem energy quantities, as they are used in existing statistical energy analysis (SEA) prediction.

The reliability of deterministic response prediction in road vehicles, aircraft, spacecraft etc. at medium and high frequency is not primarily limited by the size or the geometrical detail of a FE-model or even the modelling skill of the analyst. The limit is set by input parameter accuracy requirements as small variations in eigenfrequency and damping of individual modes will result in large FRF scatter. Updating of the FE/BE-model will not reduce this random error.

Statistical energy methods (e.g. SEA) for prediction are serious alternatives to deterministic modelling at medium and high frequency for many products. The modelling effort and computing resources are much less. The results also represent the average response for a random ensemble of structures. Result from a deterministic model may erroneously be interpreted as accurately representing the entire ensemble of products, especially if the model has been carefully updated.

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