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Resolving uncertainty in mechanical vibration of thin structures: A novel Bayesian approach

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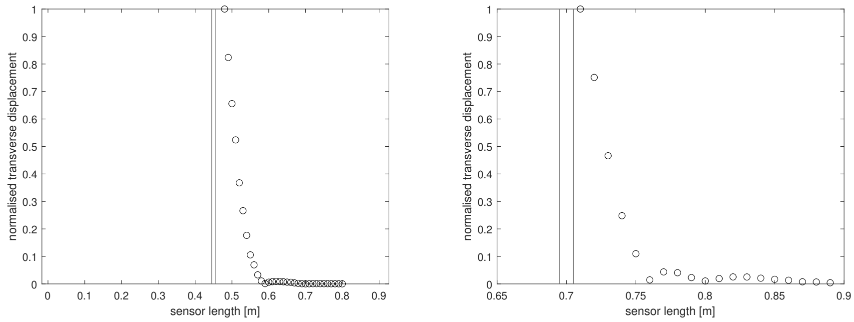
Abstract

Over the recent decades, the powerful framework of Bayesian inference has found applications in many areas of science and engineering, rooting itself as an invaluable tool to solve problems where classical deterministic methods fall short. Utilising measured data in conjunction with prior knowledge about the system in question in this Bayesian setting has birthed many robust techniques capable of performing uncertainty quantification, parameter estimation, model selection and more. In this talk, a novel adaptation of the conventional approximate Bayesian computation-sequential Monte Carlo (ABC-SMC) algorithm will be presented. Methodology of this type is used as a means for parameter estimation when there does not exist a tractable form of the associated likelihood function, which is replaced by a simulating model capable of producing artificial data. The aforementioned methodology has been utilised by the authors in the context of resolving uncertainty in measured data obtained from the mechanical vibration of thin structures, and demonstrates the value of this methodology when considering inference problems of this type. By attending this talk, participants will enhance their understanding of how Bayesian methods provide sound mechanisms for parametric inference in an engineering context, encouraging the use of this compelling machinery in their own research.

1 Introduction

Consider a thin rectangular plate of length l cm and width w cm, where $w << l$, fixed along its length, and free along its width. Consider the problem of experimentally recovering the lengthwise transverse displacement profile of this plate when subjected to known harmonic excitation over a small area at various points along its length. In this experiment, $l = 90$, and $w = 5$. Such experiments were carried out by the authors at the Laboratory for Verification and Validation (LVV) at The University of Sheffield. For clarity, the sides of the plate length were lined with 18 clamps either side, each 5 cm in length, designed to fix the plate along its entire length. The transverse displacement of the plate was recovered by use of a scanning laser-Doppler vibrometer (LDV) in centimetre increments at the centre of the plate width throughout the experiment. For verification, a model of the plate experiment was constructed in the FEM (Finite-Element Modelling) software COMSOL [1], with equivalent numerical data produced as a tool to ensure all assumptions made by the authors in the experimental setup of the problem were physically meaningful. Two examples of the absolute maximum displacement profiles of said plate are displayed in Figure 1. Due to the large difference between experimental and FEM data, it was realised the assumption of the plate being fixed along its length may be untrue.

This issue presented uncertainty in how to numerically model the plate response, and it was realised there may be uneven coupling between the clamps and the plate. It was decided to model each clamping section as either on or off, resulting in each 5cm along the boundary being fixed or free, respectively, and to apply the methodology presented in Section 2 to infer the true coupling between the clamps and the plate. This problem can be stated as follows: Given experimentally is a data set Y (Normalised maximum transverse displacement profiles for the plate), dependent on parameters $\theta \in \Theta$ (each θ being a particular configuration of fixed/free for all 36 clamping sections). Which members of Θ may be responsible for the observed Y?



(a) (b)

Figure 1: Normalised maximum transverse displacement profiles for the plate subjected to mechanical excitation of 0.041-0.044 N operating at 5Hz. Impact located at 45 cm along the 90 cm plate length in Figure 1(a). Impact located at 70cm along the 90cm plate length in Figure 1(b).

2 Problem formulation

If one were in possession of a mathematical model $f : \theta \to Y$ that could map different configurations of clamping on the plate (for a given harmonic excitation) to a maximum transverse displacement profile for the plate, then an optimal θ could be found that minimises the error between experimental data in Figure 1 and data produced through f. In a probabalistic setting, a probability density function $p\,(\,Y\,|\,\theta\,)$ expressing how likely the observed Y is given the choice of θ could be constructed through use of the forward model f. This expression will be termed the *likelihood* of the model. In Bayesian statistics, the *posterior distribution* is related to the likelihood via the relation

$$p\,(\,\theta\,|\,Y\,) \propto p\,(\,Y\,|\,\theta\,)\,p\,(\,\theta\,). \tag{1}$$

Where the prior belief about the distribution of the parameters $\theta \in \Theta$ is contained in $p\,(\,\theta\,)$ [2]. It is the posterior $p\,(\,\theta\,|\,Y\,)$, which if obtained correctly, provides inference on the parameters θ responsible for experimental data Y. However, there currently is no known model capable of mapping from θ to Y in the given context, and so a closed form of the function f is not available to be used as a forward model for the dynamic response of the plate, or as part of a likelihood function. Without a closed form of f, inference must be achieved using *likelihood-free* methods to compute an approximate form of the posterior in Equation (1) when taking a probabalistic approach.

3 Methodology

The likelihood-free method forming the basis of the methodology used in this work is the sampling scheme approximate Bayesian computation (ABC) [3]. ABC involves sampling θ^* from $p\,(\,\theta\,)$, and using these as inputs to a generative model f^* capable of modelling the system in question, simulating a likelihood function. Artificial data $Y^* = f^*(\theta^*)$ can then produced. Combining this with a suitable metric D capable of measuring similarity between experimental and artificial data Y, Y^*, it can be assessed whether given inputs $\theta^* \in \Theta$ produce outputs sufficiently similar enough to Y [4]. For a positive real number labelled the threshold α, defined by the user, θ^* is accepted if $D(Y, Y^*) \leq \alpha$. If this is case, it can be assumed that the element θ^* producing Y^* is contained in a distribution that approximates the posterior. If not, the proposed θ^* is rejected and another possible θ is sampled from $p\,(\,\theta\,)$. This process is repeated until the desired number of samples are accepted.

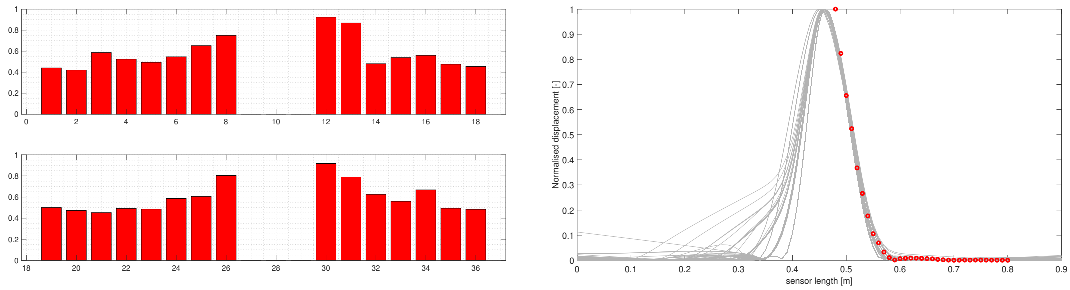
When considering sample spaces Θ that may be computationally expensive to explore fully, or spaces that contain only a very small subspace of appropriate samples, the traditional ABC sampling scheme may fall short due to inefficient or slow searching of the sample space, thus reducing the acceptance rate. For improved efficiency, augmented versions, such as ABC-SMC (Approximate Bayesian Computation-Sequential Monte Carlo) can be utilised [5]. This scheme projects accepted samples through a predefined number of intermediate distributions n, each with their own decreasing threshold $\alpha_n \leq, ..., \leq \alpha_1$. The idea is that the distributions formed from the accepted samples on each wave converge towards the approximate posterior, with the acceptance condition on each wave becoming more restrictive as the algorithm progresses. Starting from the prior as in the classical ABC scheme, it is these intermediate distributions that are sampled from on each subsequent wave, making use of previous inference to influence decisions on where to sample new candidates from, restricting the search space. In addition to this, accepted samples are assigned weights inversely proportional to $D(Y, Y^*)$ as part of a move kernel, which influences sampling of θ on the next wave. It is not necessary to give details of the full procedure for these proceedings. However, full details of the procedure can be found in [5].

For clarity, the above mentioned schemes are classically used in scenarios where the parameters θ to be inferred are continuous variables. The parameters to be inferred in the current work are different configurations of the 36 boundary conditions lining the plates length. This can be written as $\Theta = \{0, 1\}^{36}$, each θ a high dimensional binary variable, describing each boundary condition as free (0), or fixed (1) by the associated clamping section. For this reason, it was necessary to modify the ABC-SMC scheme to operate on a different type of variable, rather than the classical continuous case. This modified scheme has been coined *Combinatorial* ABC-SMC, due to the search taking place in a space of different combinations of binary conditions. Similarly to ABC-SMC, samples are projected through a sequence of intermediate distributions with decreasing thresholds. It is construction of these intermediate distributions that is altered to accommodate a different type of variable being sampled. Put simply, for any given boundary section, the probability that this boundary will be fixed in the next wave is given by the proportion of instances that boundary was deemed fixed in the current wave. This can be repeated for each boundary section, as independence between boundaries has been assumed for the current case. The authors believe that 5 cm is a great enough length for the sensor displacement to decrease (as seen in Figure 1) so that a specific clamp section may not be affected by the condition of its neighbours. The authors believe this is a relevant assumption to carry out the inference needed in the current work.

4 Results

Before results are displayed, it shall be quickly explained how combinatorial ABC-SMC has been utlised to resolve the uncertainty presented in Section 1. As mentioned previously, each complete configuration of possible boundary conditions for the plate can be encoded as a parameter $\theta \in \{0, 1\}^{36}$. In the literature, there does not exist a forward model f capable of recovering the transverse displacement of a thin plate subjected to an external force with these complex boundary conditions. Therefore, artificial data curves Y^* were produced through use of the FEM software COMSOL with a numerical model replicating the experimental setup. This model produced data Y^* resulting from the same type of excitation responsible for Y, but with a choice of what configuration of boundary conditions to use when constraining the plate. The sampling procedure is as follows: Starting with an equal probability of each boundary being fixed or free, a configuration of conditions θ^* is sampled. This is inputted into the generative model f^*, and the maximum transverse displacement Y^* of the plate is computed with the given constraints. If $D(Y,^*) \leq \alpha_1,\ \theta^*$ is accepted into the first intermediate distribution, if not, it is discarded and $p\,(\,\theta\,)$ is sampled from again. In this work, this process was repeated until 500 samples were collected in each wave. The intermediate distribution was computed as outlined above, and then the scheme repeats. For clarity, it was learned that 5 waves of combinatorial ABC-SMC was sufficient to provide successful inference in this instance.

Combinatorial ABC-SMC was ran to infer the true configuration of clamping on the plate for the data Y in Figure 1(a). A least squared metric was used to assess similarity between Y and simulated data Y^* produced by the numerical model. One can see how simulated data Y^* of a similar shape and position to Y in Figure 1(a) would minimise D in this form. The approximate posterior obtained at the end of the scheme is presented in Figure 2(a). Figure 2(b) displays the 500 accepted Y^* deemed acceptable to approximate the experimental data on the final wave.



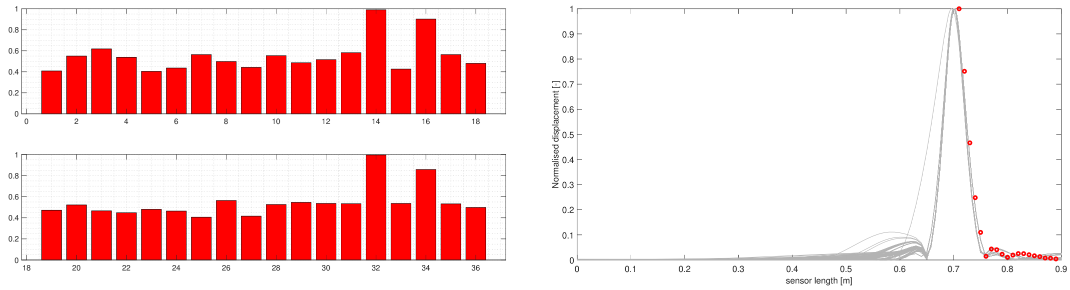
(a) (b)

Figure 2: Approximate posterior distribution for boundary condition inference after 5 waves of ombinatorial ABC-SMC, each index represents a particular boundary condition along either side of the plate length in Figure 2(a) Accepted curves for the final wave of combinatorial ABC-SMC using a difference of area metric are displayed in Figure 2(b) Experimental data are shown with circle markers.

From the above figures it can be noted that the form of D in this run successfully accounted for the shape of simulated data, only accepting samples producing curves with very similar trajectories in the experimental domain. It can then be seen which configurations of boundary conditions are capable of producing these curves, implying which configuration of boundary conditions were most likely responsible for the experimental data. The scheme suggests that the clamps 9-11 and 27-29 (either side of the plate along the 40-55cm range) in the experimental setup are completely loose, allowing this portion of the plate boundary to move freely. One would assume this may be due to repeated forcing at the shaker location from multiple experimental runs resulted in loosening of the clamps in this range.

An additional run of combinatorial ABC-SMC was ran against the data in Figure 1(b) with an altered numerical model to reflect the change in excitation impact. This was done to test if the scheme would be capable of recovering similar conclusions from different experimental data, mainly concerning the boundary conditions at the impact location, which differs between the two sets of data. These data follow a more complex looking displacement profile, for this reason the metric used on these data was altered significantly in an effort to only accept boundary condition configurations capable of producing displacement profiles displaying the secondary peak visible in Figure 1(b). When comparing Y and simulated Y^* from Figure 1(b), a least squared metric was used over 3 separate sections of the plate length, in an effort to ensure the absolute distance between the two data sets was averaged over subdomains to capture the complex behaviour of Y. An additional term was added as an aid to only accept samples producing secondary peaks in the 0.75-0.8m range as seen in Y. The approximate posterior obtained at the end of the scheme is presented in Figure 3(a). Figure 3(b) displays the 500 accepted Y^* deemed acceptable to approximate the experimental data on the final wave.

Similarly to the results in Figure 2, it can be seen that the form of D used in this instance found success accepting simulated data very similar to experimental data by the end of the scheme. Clamps 15 and 33 at the 70-75 cm position appear to have little effect on the plate response, despite the shaker position covering this area. Instead, clamps 16 and 34 are now deemed almost certainly fixed. This covers the positions of 75-80 cm along the sensor, which is the location of the secondary peak. It was concluded that these clamps most likely fixing the plate in place was a limiting factor in the height of this secondary peak, also causing the lack of influence of clamp positions 15 and 33.



(a) (b)

Figure 3: Approximate posterior distribution for boundary condition inference after 5 waves of combinatorial ABC-SMC, each index represents a particular boundary condition along either side of the plate length in Figure 3(a) Accepted curves for the final wave of combinatorial ABC-SMC using a difference of area metric in Figure 3(b) Experimental data are shown with circle markers.

5 Conclusion and future work

Through the presented results, it has been shown how the combinatorial extension of classical ABC-SMC can be utilised when considering inference problems of binary variables. It has also been emphasised that extreme care needs to be taken when considering the most appropriate form of D. This needs to accurately quantify similarity between two sets of data, which may be difficult to formulate depending on the type of data being compared. The algorithm was effective in learning the most likely configuration of boundary conditions on the plate given the experimental data, matching assumptions made by the authors. While researchers may not need to use the presented methodology in the exact context that is shown here, the authors hope these proceedings encourage readers to consider where this methodology, or other augmented versions of ABC may prove useful in their own research.

Current and future research being carried out utilises ABC-SMC in the context of fibre-optic sensing. Experimental data have been collected by the authors from a fibre-optic cable attached to the interior wall of a partially-filled pipe at a discrete set of angular locations on the pipe cross-section for a given flow regime. Equivalent numerical LES data for the same flow regime have been produced. Current research attempts to utilise ABC-SMC in conjunction with numerical data to see if this methodology is capable of recovering the angular position at which different sets of experimental data were recovered from.

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