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BEAMWIDTH DELIMITED POWER RESPONSE

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1. INTRODUCTION

The measurement that is most commonly utilized to quantify the performance of a loudspeaker is a far field axial response normalized to some reference distance and input level. While this measurement "provides the basis for a consistent evaluation of a loudspeaker" as stated by Vanderkooy and Lipshitz, it "does not represent all the features that influence sound quality." They argue that the "sound quality, or timbre," provided by a loudspeaker is "dependent on the direct plus reverberant sound [1]." A loudspeaker's contribution to the reverberant sound in a given space is the net result of the energy that is transmitted in all directions. This combination of the loudspeaker's axial and off-axis radiation into three-space is known as the power response, which serves as a more accurate indication of the total energy being radiated by the loudspeaker.

The power response is deemed a better measure of performance due to its inclusion of a larger quantity of information. Rather than basing the performance of the loudspeaker upon its response at one point in space, it allows for an average of the total power generated by the loudspeaker in all directions. The power response "...(also called the energy response), represent[s] the radiated power over 4π steradians as a function of frequency." [2] In essence, the power response indicates a normalized frequency response averaged over all possible directions of radiation. This can be visualized as the energy transmitted through a sphere having a surface area of unity.

$\Omega = s / r^2$ (in steradians, s = surface area, r = radius of sphere)

if $s = 1m^2$ then $r \approx 0.282m$ ($s = 4\pi r^2$)

therefore $\Omega_{total\ sphere} = 4\pi r^2/r^2 = 4\pi$ steradians

A complete power response measurement is most valuable when a fairly omnidirectional device is operated in a reverberant space. Many professional loudspeakers diverge from these criteria. Professional loudspeakers are operated in enormous areas (frequently outdoors), and the horn and flare loaded devices that are utilized in the pro industry are, by design, highly directional over a fairly broad frequency range. As a consequence, the amount of sound that the loudspeaker contributes to the reverberant field can be minimal. An axial response measurement remains inadequate, yet a complete power response measurement includes information that is somewhat extraneous.

For these reasons, a new method has been developed for accurately representing the performance of a highly directive loudspeaker. We call this measurement the Beamwidth Delimited Power Response (BDPR). The BDPR provides a weighted average frequency response over a defined coverage area. It is an accurate representation of the performance of a loudspeaker over its intended beamwidth, but also can be

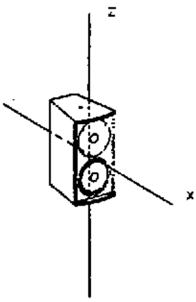


Figure 1

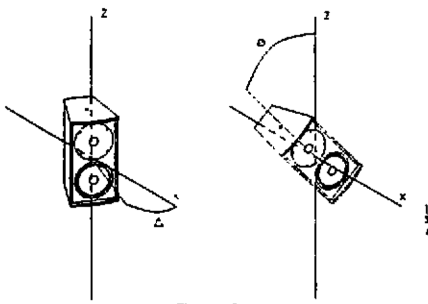


Figure 2

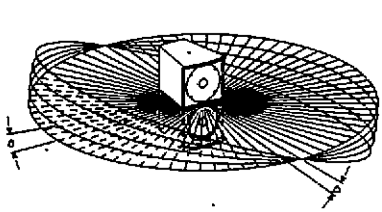


Figure 3

viewed in conjunction with a complete power response measurement in order to determine the ratio of sound radiated through the intended beamwidth to the total energy radiated from the loudspeaker. When used in conjunction with standard polar representations, the BDPR provides the system designer with valuable information regarding the magnitude, location, and frequency content of any lobes which occur outside of the intended beamwidth. Many methods can be utilized to obtain the beamwidth delimited power response, one that can be readily obtained from existing data requires abstracting the relevant information from three dimensional polar data gathered in accordance with the current measurement standard.

2. DATA ACQUISITION

In order to thoroughly understand the calculations used to arrive at the beamwidth delimited power response, one must first conceptualize the technique currently utilized to acquire a complete spherical set of polar data. To obtain a spherical representation of loudspeaker performance, a loudspeaker must be revolved about two axes while the microphone is held in a fixed position. One axis extends from top to bottom passing through the acoustic origin of the loudspeaker (z axis), while the other axis passes through the center of the cabinet from front to back (x axis) [figure 1]. As the cabinet is revolved about the x axis, an angle with respect to the z axis is created (ø), and when the cabinet is rotated about the z axis, an angle with respect to the x axis is created (Δ) [figure 2]. In order to obtain data resolved to 5° (the current measurement standard), ø is held constant while Δ is increased in 5° increments until the loudspeaker has completed a full 360° revolution. If one visualizes the gathering of data in terms of the lines on a globe, this first series of measurements provides data that lies along one of the longitudinal lines (specifically, the prime meridian). The loudspeaker is returned to position zero, ø is incremented 5°, and once again Δ is incremented in 5° steps until another of the polars' longitudinal lines has been created [figure 3]. Each ø is now associated with 72 distinct Δ measurements. This process is repeated until the complete set of polar data has been acquired.

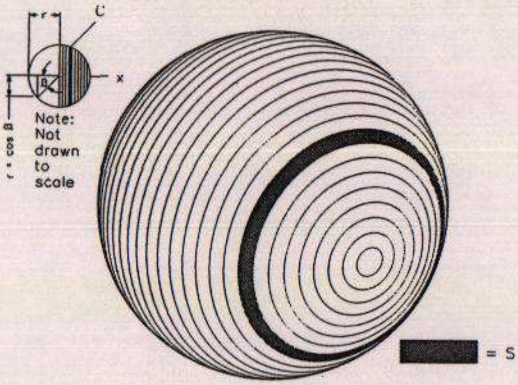


Figure 4

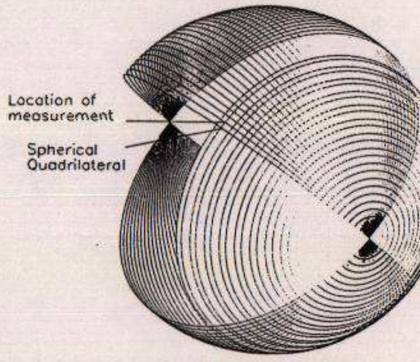


Figure 5

3. THE POWER RESPONSE

This current measurement system serves as an accurate means of quantifying loudspeaker radiation in equiangular increments. Complication arises however, because the measurements are not evenly spaced on the surface of the sphere through which the sound is being transmitted (an example of evenly spaced points on a sphere is the dimple pattern on a golf ball). Each measurement in the complete polar is called upon to represent the response of the loudspeaker over some area of the sphere which has a surface area of unity. These individual areas, or spherical quadrilaterals, do not have uniform size. Therefore, the measurements must be weighted so that each point's true contribution to the power response is accounted for in the complete polar. By utilizing the calculus of a surface of revolution, it is possible to calculate the areas between latitudinal lines on a sphere having a surface area of unity [figure 4]. These calculations of a bounded surface of revolution are as follows:

The surface of revolution is defined by the arc of the circle $x^2 + y^2 = r^2$.

This arc of the circle C can be represented parametrically by the equations

$x = r * \cos \beta$ $\quad$ $y = r * \sin \beta$ $\quad$ $\beta_1 <= \beta <= \beta_2$

The area, S, is then defined by (all integrals from β_1 to β_2):

$$S = 2\pi \int g(\beta) * [(dx/d\beta)^2 + (dy/d\beta)^2]^{1/2} d\beta$$

$$S = 2\pi \int (r * \sin \beta) * [(-r * \sin \beta)^2 + (r * \cos \beta)^2]^{1/2} d\beta$$

$$S = 2\pi r \int (\sin \beta) * [r^2 * (\sin^2 \beta + \cos^2 \beta)]^{1/2} d\beta$$

$$S = 2\pi r \int r * \sin \beta \, d\beta$$

$$S = 2\pi r^2 \int \sin \beta \, d\beta$$

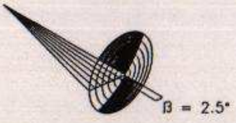


Figure 6

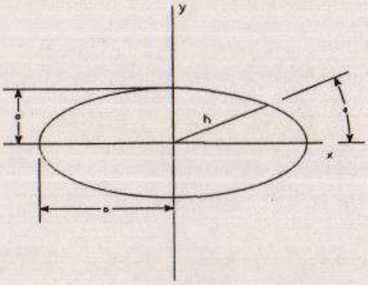


Figure 7

For a sphere having a surface area of unity: $4\pi r^2 = 1$

Therefore $r = [1/4\pi]^{1/2}$, which when substituted into the expression for S yields: $S = 1/2 \int \sin ß \, dß = 1/2 [\cos ß_2 - \cos ß_1]$

Each of these areas or slices of the sphere can in turn be subdivided into 72 equal parts due to the nature of the consistent 5° spacing of ø in the measurements. The corners of these spherical quadrilaterals are not defined by four measurement points, but rather are defined as the area surrounding a single point of measurement that includes ±2.5° of Δ and ø [figure 5].

The first area of interest is the area surrounding the pole. The measurement that was taken at the pole will be used to represent the response over some finite area. There exists a ring of 72 measurements 5° away from this pole measurement. Therefore, the pole measurement is to represent the included area from ß = 0 to ß = 2.5°, or half the distance to the next measurement [figure 6]. Each measurement included in the ring of 72 measurements at 5° represents the response of 1/72 of the total area from 2.5° to 7.5°. This procedure is continually implemented until the complete sphere has been realized. In this way the sum total of all quadrilateral weight factors is 1 (the surface area of the sphere). Therefore, the sum total of all of the weighted measurements taken for the complete set of polars will be normalized to the same reference distance and input level as the original measurements (ideally 2.83V/1meter). This decibel level is by no means absolute, but allows for a comparison of axial and power response measurements on a single graph.

The aforementioned sum total of all weighted measurements is the power response. If the intended coverage pattern of the device is known, however, then the actual frequency response of the device over its intended coverage (the beamwidth delimited power response) can be determined. Conversely, by removing this beamwidth delimited power response information from the complete power response, one can determine how much undesirable energy is being transmitted to other areas of a given venue.

4. CALCULATION OF AN ELLIPTICAL LOBE

The first thing that must be determined before the beamwidth delimited power response can be arrived at is which data points should be included in the power summation. In the case of a horn which has identical intended coverage both horizontally and vertically, the angular relationship can be easily defined. For instance a 90° x 90° horn would simply include all measurements taken for ±45° of Δ for all values of ø. If, however, the horn does not have identical intended horizontal and vertical coverage or perhaps does not even possess symmetry with respect to the horizontal or vertical axis, then an angular transition must be calculated which will define the bounds of the final beamwidth delimited power response.

The closest approximation to the transition from differing horizontal and vertical coverage is an elliptical lobe. In actuality, at lower frequencies, the transition will resemble a rectangle with rounded off corners and at higher frequencies, the transition will more closely approximate a diamond. The elliptical approximation, however, typically holds for the majority of the vocal bandwidth and in general provides an adequate match [3]. (It should be noted that if one wished to define a frequency dependent envelope for a more accurate evaluation, it is entirely possible and merely involves a slightly more complex algorithm.) An ellipse is defined as follows [figure 7]:

$$(x^2 / a^2) + (y^2 / b^2) = 1$$

Where a is one half of the vertical coverage and b is one half of the horizontal coverage. If all points on the ellipse are expressed in spherical coordinates, then the distance h between the origin and a point on the ellipse can be defined in terms of a, b and the angle ∂. This derivation is realized below:

$$x_{ellipse} = h * \cos \partial$$

$$y_{ellipse} = h * \sin \partial$$

$$[h^2 * \cos^2\partial / a^2] + [h^2 * \sin^2\partial / b^2] = 1$$

$$h^2 * (\cos^2\partial / a^2 + \sin^2\partial / b^2) = 1$$

$$h^2 * [(\cos^2\partial * b^2 / b^2a^2) + (\sin^2\partial * a^2 / b^2a^2)] = 1$$

$$h^2 = b^2a^2 / (\cos^2\partial * b^2 + \sin^2\partial * a^2)$$

$$h = (b^2a^2 / (\cos^2\partial * b^2 + \sin^2\partial * a^2))^{1/2}$$

These calculations yield the appropriate Δ angular coverage of a horn for each individual value of ø. These angles define the bounds of the beamwidth delimited power response. In the case of an asymmetrical horn, elliptical lobes should be calculated for the transition between horizontal and vertical angles for each quadrant individually.

The above calculations will yield an exact angular relationship for all values of ø. The data that has been gathered however, is only resolved to 5°. This leads to the problem of how to deal with data acquired along the edge of the elliptical envelope. For example, if the intended coverage of a horn at ø = 10° is

52.6°, then all measurements along the 10° longitudinal line within ±26.3° should be included. Data in the vicinity of this edge, however, has only been acquired for ±25° and ±30° respectively. The most accurate way to deal with this error is to project the ellipse onto the surface of the sphere and calculate the area of the region bounded by the spherical quadrilateral that lies within the elliptical projection. The accuracy benefit of this calculation is offset entirely by its difficult implementation. Rather, one can accept this minimal error and attempt to minimize it through a proper implementation of a rounding scheme. (Note: this error is much akin to quantization error in digital logic, but rather than being bit limited we are data point limited. The obvious way to truly minimize the error is to increase the number of data points by resolving the data to smaller degree increments. In fact, it is precisely this "quantization" error that allows one to, in essence, ignore the edge correction error - for it is in fact rendered incalculable due to the error introduced as a result of the course spatial resolution of the data.) Each spherical quadrilateral represents ±2.5° of surface area surrounding each data point. Therefore, in the case of the ±26.3° beamwidth, the ±25° data points actually represent ±27.5° of surface area. Therefore, inclusion of the ±25° data points is sufficient (in fact it is excessive). Even if the intended coverage of the horn at this value of ø was less than, but not equal to 30°, than less than half of the ±30° quadrilaterals should be included in the measurement. Therefore, to minimize error, only up to the ±25° data points should be included. In essence, one merely has to round all angular measurements down in order to minimize the edge error associated with the given angular resolution.

After all of the calculated beamwidths for each value of ø have been rounded down to the nearest measured data point, then the envelope of the beamwidth delimited power response has been defined. A power summation of all of these data points will yield the average frequency response of a loudspeaker over its intended coverage.

5. MEASUREMENT NORMALIZATION FOR COMPARATIVE DATA

Based upon the above theory, it can be readily seen that the envelope for a 120° x 40° horn will be considerably larger than the envelope for a 30° x 40° horn. Therefore, when the energy transmitted within the respective envelopes is summed, the larger envelope will yield larger SPL values than the smaller envelope, making it difficult to compare dissimilar horn responses on a similar scale. In order to compensate for this, the data gathered at each point within the envelope must be weighted again, based upon the respective spherical quadrilateral area as compared to the area of the envelope rather than with respect to the entire sphere.

This can be implemented as a re-weighting of the previously calculated weights for each included quadrilateral. The area of the quadrilaterals included within the envelope are merely summed, and then each respective quadrilateral area is divided by the area of the envelope. This normalizes each BDPR measurement to an envelope of unity regardless of the coverage of the horn loaded system, which provides a convenient means of directly comparing data from dissimilar loudspeakers.

Fig. 8 Beamwidth Delimited Power Response

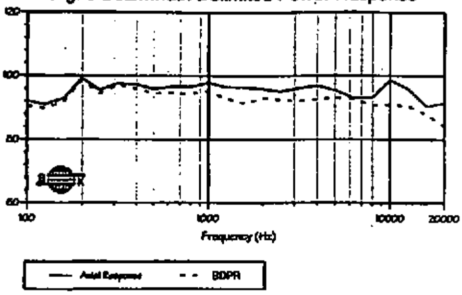
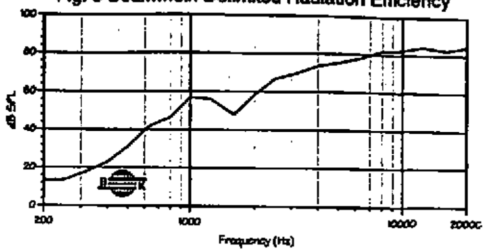


Fig. 9 Beamwidth Delimited Radiation Efficiency



6. POWER SUMMATION

Finally, the BDPR can be calculated. All measurements that are contained within the envelope must first be expressed in terms of pressure in Pascals as opposed to dB SPL. This is accomplished as follows:

pressure (Pa) = 20μPa * 10(dB SPL value / 20)

Each 1/3 octave band pressure frequency response is then multiplied by its appropriate weight. These weighted measurement are then summed over the entire envelope. The resultant frequency response is then expressed in terms of dB SPL through the reverse implementation of the formula expressed above, or:

dB SPL = 20 * log(pressure (Pa) / 20μPa)

This resultant frequency response is the beamwidth delimited power response, or the frequency response of the system averaged over the intended angular coverage envelope of the system.

7.ENERGY OUTSIDE OF THE INTENDED COVERAGE ENVELOPE

The beamwidth delimited power response can also be modified to determine what percentage of the total energy transmitted by a loudspeaker is contained within the coverage area. Conversely, these calculations can show the frequency content and level of the energy that is radiated in undesirable directions. This is perhaps most readily seen when viewed as the ratio of the energy contained within the intended coverage envelope to the total radiation of the device into three space. We call this measurement, the beamwidth delimited radiation efficiency (BDRE), for it is a true measure of a horn's ability to perform its required task. It should be noted that this type of graph should only be viewed in conjunction with horizontal and vertical polars - otherwise erroneous conclusions could be drawn. For instance, if a device is intended to cover 90° x 40°, but is rather more like 60° x 25° in true coverage, then a significant portion of the energy generated by the loudspeaker would be contained within the intended beam, but a large percentage of that energy would be isolated in an even more narrow lobe. A possible way around this limitation is to calculate the BDRE for the intended coverage and also for diminishing coverage envelopes to determine if the progression of energy concentration is consistent with smaller envelopes. While the BDRE alone is limited in its ability to provide absolute information, it readily reveals areas where there are changes in the coverage of a device, pointing out frequency regions over which a designer can expect to encounter difficulties. At the very least, it allows for a quick determination of the caliber of a loudspeaker, especially when viewed in direct comparison with competitive product.

In order to arrive at these calculations, one must eliminate the secondary weighting algorithm (the algorithm that normalized the measurements to an envelope of area unity). Then the BDRE calculation is simply the ratio of the summed data contained within the envelope to the sum total of the energy being radiated by the loudspeaker. This beamwidth delimited radiation efficiency can then be plotted as a percentage versus frequency.

8.CONCLUSION

No single measurement in isolation can completely quantify the performance of a loudspeaker. The Beamwidth Delimited Power Response and Beamwidth Delimited Radiation Efficiency allow for a quick and easy evaluation of the consistency of a loudspeaker's radiation into three space and its ability to contain energy within its intended coverage. This allows the system designer to attain a level of comfort with a device even before complete simulation of a particular venue, and certainly provides system designers without access to modeling tools a more in depth look at the three dimensional behavior of a directional device.

Figures 8 and 9 depict representative BDPR and BDRE graphs for sample loudspeakers. In figure 8, the BDPR is compared to the axial response of a given loudspeaker. Frequency response variations that are present in the axial response are smoothed by the lack of energy at those frequencies off axis. Regardless of which graph looks better, however, the mere fact that they are significantly different emphasizes the importance of observing more than just the response of the loudspeaker at one point in space. Figure 9 depicts the BDRE for a sample loudspeaker. The pattern control through the mid and high frequencies is made apparent by the gently rising percentage of energy contained within the spatial envelope. The difficulty at crossover, however, is dramatically brought out by the presence of the deep null through the mid-range. The horizontal and vertical beamwidth and polar plots for this particular loudspeaker looked quite good, but this measurement should raise a flag that there is an apparent widening of the pattern at crossover through the corners of the horn loaded devices.

[1] Vanderkooy, John and Stanley P. Lipshitz "Power Response of Loudspeakers with Noncoincident Drivers - The Influence of Crossover Design" JAES Vol. 34, No. 4, 1986 April

[2] Ibid.

[3] Personal correspondence via e-mail with David Gunness, Senior Design Engineer, EAW, September 1996.