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AMPLITUDE GRATINGS AS ACOUSTIC DIFFUSERS

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1. INTRODUCTION

Studios, concert halls, control and dubbing rooms or any place where high quality acoustics are required need to have a diffuse reverberant sound field. The means of achieving this has vexed acousticians throughout history. Traditionally, the use of plaster mouldings, niches and other decorative surface irregularities have been used to provide diffusion in an "ad hoc" manner. More recently diffusion structures based on patterns of wells whose depths are formally defined by an appropriate mathematical sequence have been proposed and used [1-4].

However, in many practical applications the acoustic designer is faced with limited space in which to work. Unfortunately, diffusers take up space and the designer may face an uncomfortable compromise on the effectiveness of the diffusion and floor area of the final design. A particular area of concern is at low frequencies where one often needs additional diffusion but where the size of such structures is prohibitive.

The purpose of this paper is to present an alternative form of diffusion structure based on absorption, rather than phase reflection, gratings. The paper will first discuss how conventional diffusers work. It will then discuss the theory, design, advantages and limitations of amplitude grating structures. Simulation results of their performance will then be presented. The paper will show that it is possible to develop diffusing structures which take up less space at low frequencies than conventional phase reflecting structures, providing one allows them to have some absorption. Finally we will discuss the implication of the results for the design of studios, performance spaces and reverberation chambers.

2. HOW CONVENTIONAL DIFFUSERS WORK

Consider a hard surface consisting of bumps of height d. Also consider an acoustic wavefront approaching it from a normal direction. The way this wavefront is reflected will depend on the height of the bumps relative to its wavelength. Let us consider three cases:

i) In the case of $d \ll \lambda$ the surface will behave like a flat surface and specularly reflect the wavefront.

ii) The case of $d = \lambda/4$ the wavefronts which are reflected from the front of the bumps are reflected $\lambda/2$ earlier than those from the surface. This means that in the normal direction the wavefronts cancel and so no sound pressure is propagated in this direction. However, there has been no energy loss in the system so the wavefront must be reflected in some direction. In fact as one moves away from the normal direction the relative path lengths between the bump and the surface become less and the amplitude of the wavefront increased as one moves off the normal direction. This is the basic principle behind diffusion using hard reflectors. That is, the diffusing surface modifies the phase of the wavefronts so that the reflected wave such that it must propagate in other directions than the specular direction.

iii) In the case of $d = \lambda/2$ the wavefronts from the bumps and surface are delayed by λ and so arrive back in phase. Thus the bumps disappear and the surface behaves as if it were flat. That is, it behaves like a specular reflector.

So, one has a problem, a regular sequence of bumps will diffuse but only at frequencies at which it is an odd multiple of $\lambda/4$. Note also that these frequencies will depend on the angle of incidence of the incoming wavefront.

What is required is a pattern of bumps which alter the phases of the incident in such a way that two objectives are satisfied.

i) The sound is scattered in some "optimum" manner.

ii) The scattering is optimum over a range of frequencies.

These objectives can be satisfied by several different sequences, however they share two common properties.

i) The Fourier transform of the sequence is constant except for the dc component which may be the same or lower. This satisfies objective (i) because it can be shown that reflection surfaces with such a property scatters energy equally in all directions. The effect of a reduced dc component is to further reduce the amount of energy which is reflected in the specular direction.

ii) The second desirable property of these sequences is that the Fourier transform is unaffected if the wavelength of the incident sound varies. This has the effect of changing the scale of the sequence but again one can show that the resulting sequence still has the same properties as the original sequence.

Both the above properties arise because the sequences work by perturbing the wavefronts over a full cycle of the waveform.

To make this a little clearer, let us consider the two sequences which are used for diffusers.

i) **Quadratic residue sequences**

well depth = n^2 mod p where p is a prime number if p = 5 this gives a set of well depths of

0, 1, 4, 4, 1, 0, 1 ....etc.

so the sequence repeats with a period of 5;

ii) **Primitive root sequences**

well depth = a^n mod p where p is a prime and a is a suitable constant called a primitive root. For a = 2 and p = 5 we get the sequence

1, 2, 4, 3, 1, 2 ......etc.

Here we have a sequence which has a period of 4 (5 - 1).

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At the lowest design frequency for these examples a well of depth 5 would correspond to $\lambda/2$. Note that when the frequency gets high enough so that $\lambda/2$ becomes equal to the minimum difference in depths (1) then the surface again becomes equivalent to a flat surface.

3. DISCUSSION

As we have seen, these sequences achieve their performance by spreading the phase of the reflected wavefront over at least one cycle of the incident wavefront. In order to do this, their maximum depth must be $\lambda/2$ at the lowest design frequency. This means that to achieve diffusion a reasonable depth is required. For example, to have effective diffusion down 500Hz a depth of 34cm (13.5 inches) is required. To get down to 250Hz one would need double this depth. However, as we have seen, a simple bump of $\lambda/4$ can provide diffusion, albeit somewhat frequency dependently. This is half the depth of the above sequences and represents the ultimate limit for a diffusing object.

It is possible have sequences which achieve the phase scatter required for good diffusion using a depth closer to $\lambda/4$ at the lowest frequency [4] and so allow better performance diffusers in restricted spaces. However even $\lambda/4$ at low frequencies is often too large to be useful. What one really requires is a diffuser which is effective without using any depth! This can be achieved in a limited way by noting that phase is not the only way of perturbing a wavefront in fact amplitude variation also perturbs a wavefront and can be used to provide diffusion via amplitude reflection gratings.

4. AMPLITUDE REFLECTION GRATINGS

There is a well known theorem in both antenna theory and Fourier optics [5,6] which states:

$$P(\sin\alpha) = \frac{1}{\lambda}\int_{-\infty}^{\infty} E_x(x,0)\exp(-jkx\sin\alpha)dx \qquad (1)$$

Which can be written as:

$$P(\sin\alpha) = \frac{1}{2\pi}\int_{-\infty}^{\infty} E_x(x,0)\exp(-jkx\sin\alpha)dkx \qquad (2)$$

This can be compared with the equation:

$$F(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t)\exp(-j\omega t)dt$$

where: (3)

$$\sin\alpha = \omega$$
$$kx = t$$

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Equation (3) is the well known Fourier transform and so equation (1) can be seen to represent a Fourier transform as well. Equation (1) therefore states that the power as a function of sinα (the polar pattern) is related by the Fourier transform to the complex pressure distribution at the object radiating the sound. This means that if we want a uniform diffusion of sound from a reflecting surface we must have a set of complex pressures whose Fourier transform is uniform. We have already seen that phase reflection structures can achieve this requirement. However if one could have an amplitude weighting, that is a pattern of absorbers, which gave a flat Fourier transform then we could achieve a similar effect. Note that the dc component of the pressure distribution corresponds to the response in the specular direction.

The most obvious sequences to consider are binary, that is they contain the only levels 0 and 1 where 1 represents reflection from a hard surface and 0 represents absorption from some form of absorbing material. Clearly not all acoustic absorbers are 100% absorbing but this can be simply allowed for by using (1-absorption) instead of zero in the sequence. The net effect of less than 100% absorption would be to increase the level of the specular component. Of the many possible binary sequences M-sequences would seem to be a good starting point as they have desirable Fourier properties. There are many other bi-level sequences which have flat Fourier transforms but M-sequences are well documented.

Thus what we are proposing is a surface treatment which consists of strips of absorbing material whose width is less than $\lambda/2$ at the highest frequency of use laid out in a pattern in which strips of absorber represent zero and strips of reflecting wall represent 1 (see figure 1). Note that because we are not depending on depth we do not have a low frequency limit to the range of diffusion only a high frequency limit which is a function of the width of the strips.

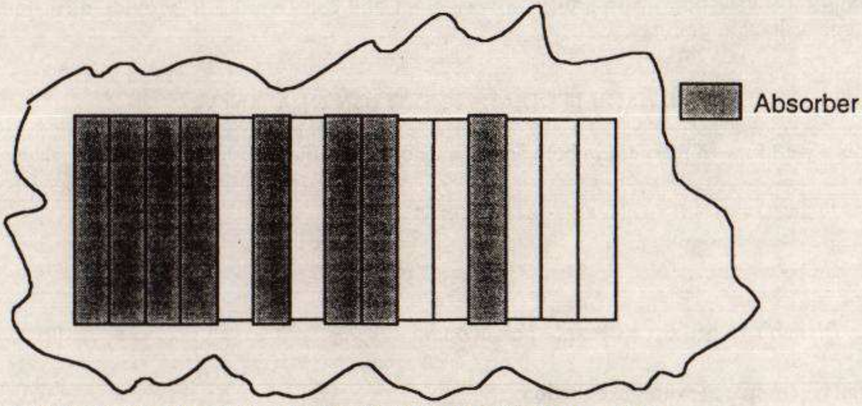


Figure 1. An Absorption Reflection Grating (length = 15)

5. RESULTS AND FURTHER WORK

A thirty one bit long sequence was simulated and its response as a function of angle is shown in figures 2 and 3 for two different wavelengths. From these graphs one can see that the absorption grating still responds more strongly in the specular direction. However it scatters energy more effectively than either the reflecting surface or a surface with the equivalent absorption spread evenly. The scattering sidelobes are -7dB down on the specular direction whereas the largest sidelobe is -13dB down for the smooth reflecting surfaces. The reason for the strong specular component is the fact that the dc level of a sequence containing ones and zeros cannot be zero. Phase reflection gratings can achieve negative numbers and so can have a smaller component in the specular direction. Even so there are some benefits to be had from the modest amounts of scattering that these gratings give especially a low frequencies where the absorption could be provided by resonant absorbers with a small depth.

It would also be possible to interleave high frequency and low frequency absorption (by using one as a one and the other as a zero in an M-sequence) and get the diffusion benefit because at low frequencies the high frequency absorber would be reflecting and at high frequencies the low frequency absorber would be reflecting thus forming complementary sequences. One could also develop sequences which had a higher average absorption which would reduce the specular reflection at the cost of some increased absorption however it would still be higher than the sidelobes.

The only way to achieve less reflection in the specular direction would be to use a $\lambda/4$ bumps and then apply the amplitude grating to both the peaks and the troughs of the surface this gives a three level sequence which suppresses the specular reflection, although only over a narrow range of frequencies. The effect of doing this is shown in figure 4 and on can see that the result is much better with a distinct null in the specular direction. Unfortunately this diffuser will occupy ¼ of space but it may be an easier structure to integrate into a room as the ¼ bumps could be evenly spaced.

Figure 2. Reflected Energy Ratio vs Angle (lambda = 0.25)

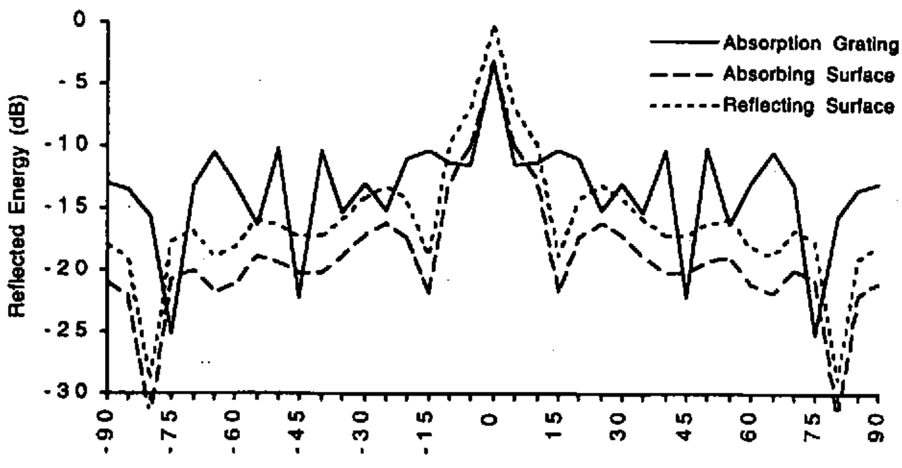


Figure 3. Reflected Energy Ratio vs Angle (lambda = 0.5)

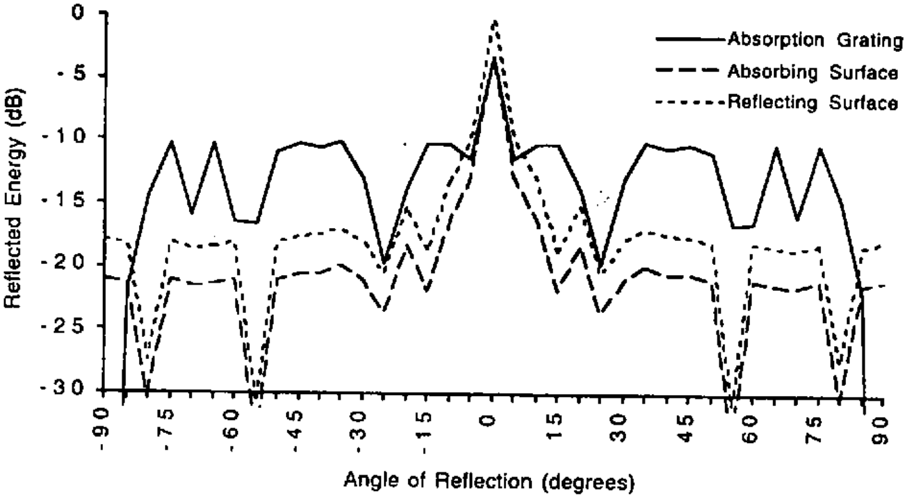
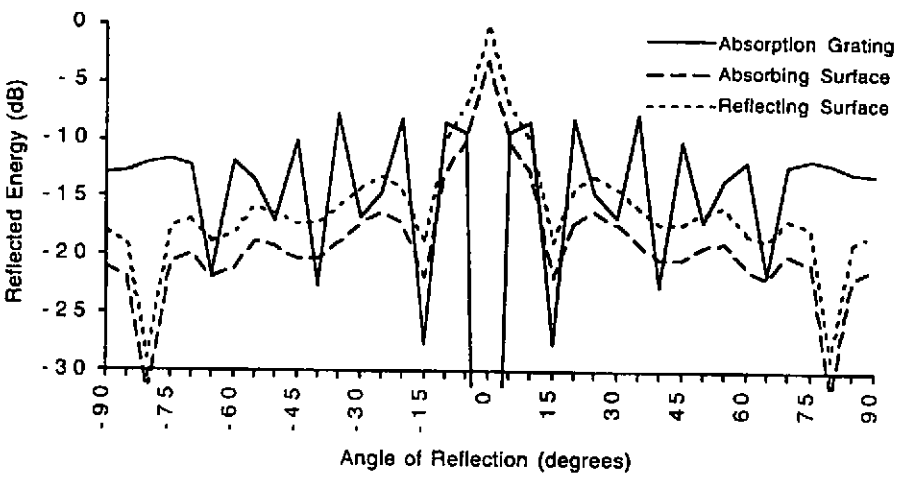


Figure 4. Reflected Energy Ratio vs Angle (lambda = 0.25) for Inverted Absorption Gratings



6. CONCLUSION

By placing absorbing material in pseudorandom strips one can achieve both absorption and a modest amount of diffusion. The diffusion is frequency independent and can be extended to low frequencies with a little cost in depth. This adds a new tool to the acoustic designers armoury for tackling real acoustic designs which have physical as well as theoretical constraints.

7. REFERENCES

[1] Schroeder, M R, "Diffuse sound reflection by maximum-length sequences." *Journal of the Acoustical Society of America*, **57**, January 1975.

[2] Schroeder, M R. "Progress in architectural acoustics and artificial reverberation: Concert Hall acoustics and number theory," *Journal of the Audio Engineering Society*, **32**, No 4, April 1984, pp 194-203.

[3] D'Antonio, P and Konnert, J H, "The reflection phase grating diffuser: design theory and application," *Journal of the Audio Engineering Society*, **32**, 4, April 1984, pp 228-236.

[4] Angus, J A S, "Alternative diffuser sequences," *Proceedings of the Institute of Acoustics*, **14**, part 5, October 1992, pp 193-202.

[5] Smith, M S. "Introduction to Antennas", pub Macmillan 1988.

[6] Steward, E G, "Fourier Optics: An Introduction", 2nd Ed., pub Ellis Horwood, 1987.