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Modulated Phase Reflection Gratings for Even Diffusion Characteristics

James A S Angus

Department of Electronics, University of York, York , England.

INTRODUCTION

Studios, concert halls, control and dubbing rooms, reverberation chambers, or any place where high quality acoustics are required, need to have a diffuse reverberant sound field. The means of achieving this has vexed acousticians throughout history. Traditionally, plaster mouldings, niches and other decorative surface irregularities have been used to provide diffusion in an "ad hoc" manner. More recently diffusion structures based on patterns of wells whose depths are formally defined by an appropriate mathematical sequence have been proposed and used [1-7].

In many practical applications, especially those which require a large area of diffusion treatment, these structures have been concatenated together to form larger areas of diffusion. Unfortunately the effect of this is to narrow the diffusion pattern of the individual diffusers into a finite number of directions that depend on the sequence length. As the length of the original diffusing sequence can be quite small (lengths of seven, eleven, etc.) this can quite disturbing.

A previous paper [7] presented a solution to this problem, modulated phase reflection gratings. These were based on the idea of modulating existing diffusion structures with other sequences to achieve the desired diffusion pattern. The technique resulted in large area diffusion structures that did not suffer from having narrowed diffusion patterns but which could be implemented easily using existing diffusers.

This paper will show how such composite structures can used to extend and improve the performance of existing diffusers. In particular the paper will show that it is possible to develop diffusing structures, based on existing components, with a specific performance. The modulated sequences discussed will include those that achieve even diffusion through the use of binary quadratic residue sequences, The theory, design, advantages and limitations of these structures will be discussed and simulation results of their performance will be presented.

THEORY

There is a well-known theorem in both antenna theory and Fourier optics [8,9] which states:

$$P(\sin\alpha) = \frac{1}{\lambda}\int_{-\infty}^{\infty} E_r(x,0)\exp(-jkx\sin\alpha)dx \qquad (1)$$

Which can be written as:

$$P(\sin\alpha) = \frac{1}{2\pi}\int_{-\infty}^{\infty} E_r(x,0)\exp(-jkx\sin\alpha)dkx \qquad (2)$$

This can be compared with the equation:

$$F(\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}f(t)\exp(-j\omega t)dt \tag{3}$$

where:

$$\sin\alpha=\omega$$

$$kx=t$$

Equation (3) is the well-known Fourier transform and so equation (1) can be seen to represent a Fourier transform as well. Equation (1) therefore states that the power as a function of sinα (the polar pattern) is related by the Fourier transform to the complex pressure distribution at the object radiating the sound. This means that if we want a uniform diffusion of sound from a reflecting surface we must have a set of complex pressures whose Fourier transform is uniform. Such a requirement is met by phase reflection gratings [1-5].

These gratings alter the phase of the wave front, using the different delays of wells of different depths. They work because they perturb the phase of the wave front such that the Fourier transform of the reflected wavefront is uniform, with the possible exception of dc. It is possible to show that the Fourier transform of a single version (no repeats) of the quadratic residue and primitive root sequences are uniform, except for a reduction in the dc component, that is, the specular direction, in the case of the primitive root sequence. This is ideal behaviour for such surfaces but as their complexity increases with the modulus most practical sequences are based on small prime numbers such as 7, 11, etc. Therefore in many cases where a large area needs to be covered the small sequences are concatenated to for larger areas. Indeed some products are designed to facilitate this.

However the effect of repeating the sequence is to introduce periodicity into the sequence and this will in turn produce harmonics in the Fourier transform. These harmonics represent more concentrated reflection of energy into the diffraction orders of the grating and their sharpness is proportional to the number of repeats. That is, the more repeats of the sequence the narrower the lobes from the diffraction angles. Although the repeated sequence is still diffusing, this behaviour is undesirable as it is similar to a faceted mirror.

DISCUSSION

Therefore there is a problem, small diffusers work well and are practical for both construction and installation but we need to concatenate them to cover large areas and this results in the defects mentioned earlier.

The problem is caused by the fact that the periodic sequence has distinct line components (harmonics) and these cause our narrow diffraction lobes. If we could find some way of removing or spreading out these harmonics then, as a consequence of equation (1) we would have a uniform diffuser. Spread spectrum systems [10], which are used in communications, have a similar requirement in that they require that a sinusoidal carrier (a single harmonic) be spread to cover a range of frequencies. They achieve this by modulating the carrier with a pseudorandom "spreading sequence". This results in a spectrum which is noise like and therefore has an approximately uniform frequency spectrum. In our diffusing context a single harmonic represents a narrow diffraction pattern whereas a uniform spectrum represents an even diffusion as a function of angle.

Modulated Phase Reflection Gratings for Even Diffusion Characteristics

In a previous paper [7] a method of modulating existing concatenated diffusion structures using binary pseudorandom "spreading sequence" was proposed. This resulted in a wide area diffuser that had a uniform, noise like angular spectrum and so did not suffer the narrow diffraction lobes of concatenation. These new gratings called Modulated Phase Reflection Gratings (MPRG's) are briefly described next.

MODULATED PHASE REFLECTION GRATINGS

A common method of spreading the carrier in spread spectrum systems is to use a binary pseudorandom sequence to modulate the carrier waveform by multiplying the carrier by the sequence. This has the effect of letting the carrier through with no alteration when the sequence is one and phase inverted when it is zero. Clearly one must have one cycle of the carrier per bit of the sequence for this to work well. If we consider one repeat of a quadratic residue sequence as equivalent to a cycle of the carrier then this would be equivalent to multiplying the quadratic residue sequence by the pseudorandom sequence. This will have the effect of letting the quadratic residue sequence through with no alteration when the sequence is one and inverting it when the sequence is zero. Therefore the treatment on the wall would consist of concatenated quadratic residue sequences with some being inverted versions of the basic sequence. The choice of whether one uses the normal or the inverted version would be determined by the binary pseudorandom spreading sequence. This technique is also known as Sequence Inversion Keying (SIK). Note this method is different to that of Peter d' Antonio's fractal based diffusers [4] which use the *summation of scaled versions of the sequence* to achieve a broader frequency range of diffusion, as opposed to the *multiplication of different sequences* to reduce the narrowing of the diffusion lobes due to repetition.

Of the many possible binary sequences M-sequences would seem to be a good starting point as they have desirable Fourier properties. There are many other bi-level sequences that have flat Fourier transforms but M-sequences are well documented. Figure 1 shows how this might look in practice, the shaded strips represent inverted versions of the basic quadratic residue sequence, and the modulating spreading sequence is a 15 bit m-sequence.

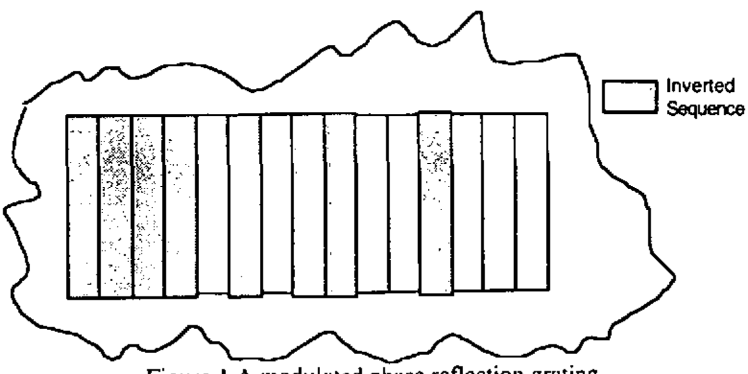


Figure 1 A modulated phase reflection grating.

How do we invert a basic quadratic residue sequence? The previous paper [7] showed that the inverted sequence is equivalent to the one that exists behind every quadratic residue diffuser as it describes the space that is left behind the well defined by the sequence. In other words, to invert a quadratic residue diffuser, simply turn it over! Figure 2 shows this in practice for a length 5 sequence. This means that to obtain a modulated diffuser all one has to do is to use a basic quadratic residue diffuser and its upside down version in conjunction with an m-sequence.

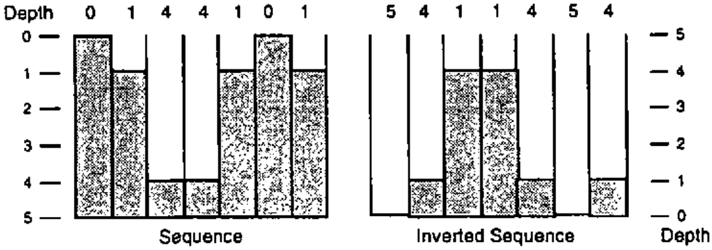


Figure 2 Normal and inverted quadratic residue diffusers.

Note however, that the full depth of the modulus must be used, which is 5 units in our example. For example, the popular length 7 sequence, 0,1,4,2,2,4,1,..., when inverted gives 7,6,3,5,5,3,6,..., so to use this as a modulated grating the non inverted sequence must be spaced from the wall by (7-4) 3 units as shown in figure 3.

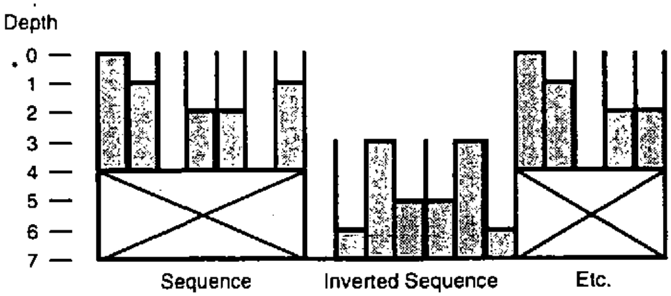


Figure 3 Spacing required for length 7 sequence.

MPRG'S RESULTS AND DISCUSSION

The simulated response of a modulated phase reflection grating, using a length seven quadratic residue basic sequence and a length fifteen m-sequence as the spreading sequence, is shown in figure 4. Its response as a function of angle for single, periodic, and modulated configurations is shown in figure 4a for the lowest design frequency. Figures 4b and 4c show the polar plots for a single and a modulated grating at the same frequency respectively.

From these graphs one can see that the modulated phase reflection grating works very well. The effect of the modulation extends over the frequency range of the basic quadratic residue sequence as expected from the Fourier properties of the composite sequence. Figures 4a to 6a show that the effect of the pseudo random modulation is to make the diffusion pattern more closely approach that of a single sequence.

However one can also observe, from the figures that the effect of modulating a quadratic residue sequence with an m-sequence is to suppress some of the reflected energy in the specular direction. This is also to be expected, as the m-sequence has a smaller dc component compared with its other harmonics. It also suggests that one could develop other modulating sequences that achieve a desired reflection characteristic. For example a modulating sequence with a with a zero dc component that could suppress all reflections in the specular direction.

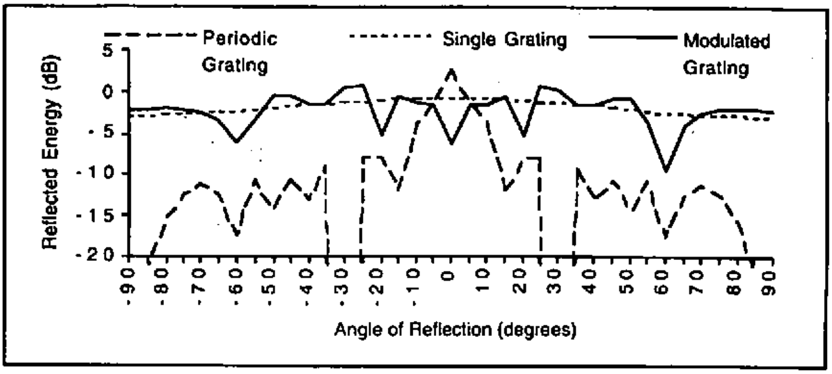
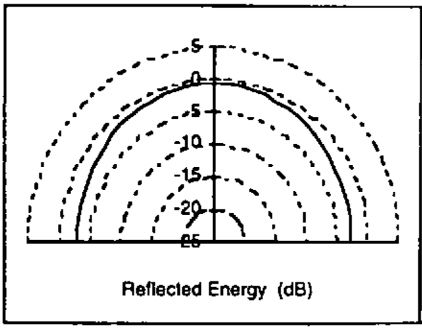


Figure 4a Reflected energy for an m-sequence modulated grating at LF.



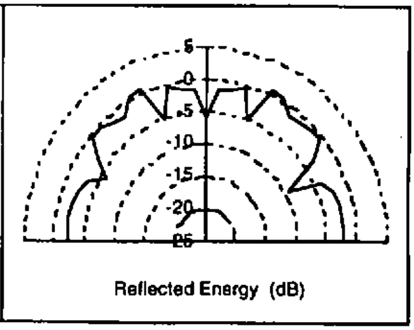


Figure 4b Single grating at LF Figure 4c M-Sequence grating at LF

5 MODULATING SEQUENCES WITH SPECIFIC PROPERTIES

The convolution theorem states that multiplication (modulation) in the time domain is equivalent to convolution in the frequency domain. In our diffusing context this means that the effect of the modulating sequence on the polar pattern can assessed by either convolving the angular spectrum, or multiplying the autocorrelation function, of the modulating sequence with that of the diffuser being modulated.

We desire even reflection over all angles including the specular direction, the ideal acoustical matte surface. Therefor it would be useful to investigate the correlation properties of other modulating sequences to see if they can provide more even diffusion.

Binary Modulating Sequences for Even Diffusion

In order to achieve the first ideal pattern we need a diffuser sequence in which the aperiodic autocorrelation function should be an impulse at zero delay time and zero at other delays. In practice this is not possible so we need to look for sequences with low values of sidelobes in their aperiodic, or single sequence, autocorrelation functions. The quadratic residue sequences that are currently used are fairly optimum in this respect. As the composite aperiodic autocorrelation function of a modulated sequence is the product of the individual aperiodic autocorrelation functions of the two sequences we need to find binary sequences that have low sidelobe levels in their aperiodic autocorrelation functions. M-sequences have peak sidelobe levels that are approximately $\sqrt{N}$ lower than the main peak, where N is the length of the sequence, and so are not the most optimal sequences to use for modulated gratings. The effect of high sidelobe levels in the sequence is to introduce ripple in the polar response of the composite sequence. A lower sidelobe level will result in a smoother polar response that more closely approaches that of the single basis quadratic residue sequence.

Binary sequences with low sidelobe levels are well known to radar designers [11] as they are used for pulse compression and low sidelobe levels are crucial to achieving good performance. There are three types of sequences that seem to offer potential gains in our context.

- **Binary Quadratic Residue Sequences.** Binary quadratic residue sequences are binary sequences that are formed by setting an element in the sequence to 1 whenever it forms a quadratic residue of a given modulus. This is most easily achieved by generating a non-binary quadratic residue sequence and then using the numbers, other than zero, so generated as an index to say which bits should be set. For example the length 7 non-binary quadratic residue sequence is 0,1,4,2,2,4,1,0,... thus in the length 7 binary quadratic residue sequence bits 1,2 and 4 would be one and the rest zero giving 0,1,1,0,1,0,0 as the sequence. These sequences have similar properties to m-sequences.
- **Complementary Sequences.** Complementary sequences consist of two different sequences of the same length N. However the aperiodic autocorrelation sidelobes of one sequence are the exact negative of the aperiodic autocorrelation sidelobes of the other sequence. This means that if they are both used together the sidelobes cancel but the peaks add thus giving an optimum sequence of length 2N. In our context this can be achieved by placing them side by side. However the fact that they must be separated spatially will detract from their performance somewhat.
- **Barker Codes.** Barker codes are a set of optimal binary sequences whose aperiodic autocorrelation sidelobe levels are the lowest possible at N times lower than the main peak, where N is the length of the sequence. Unfortunately there are only a small number of them and the longest one is only 13 bits long. However in many contexts the number of repeated diffusers may be less than this. There are also other optimum sequences whose sidelobe levels are nearly as good, being N/2 or N/3 times lower than the main peak, and these would allow for longer sequences.

All the above sequences can be easily applied as modulating functions to a basic quadratic residue diffuser and we would expect to see an improvement over m-sequences in the polar performance.

RESULTS

The simulated response of several modulated phase reflection gratings, using a length seven quadratic residue basic sequence and a variety of binary sequences as the spreading sequence, is shown in figures 5 to 7. The response as a function of angle for single, periodic, and modulated configurations is shown in figures 5a to 7a. For each binary sequence the lowest design frequency is shown. Figures 5b to 7b and 5c to 7c show the polar plots for single and modulated gratings at the same frequencies as the corresponding figures 5a to 7a.

Binary Modulating Sequences for Even Diffusion

Figures 7 to 15 show the results for three different possible sequences for even diffusion. The sequence length was kept as similar as was possible so that the results could be compared.

- **Binary Quadratic Residue Sequences.** Figure 5 shows the response of a length 13 binary quadratic residue sequence as the modulating sequence. Of particular note is that the ripple in the polar response is greater than that of the m-sequence shown earlier, but it is more even. Also even at midrange frequencies the sequence is spreading at all angles and compares well with the single grating response.
- **Complementary Sequences.** Figure 6 shows the response of a length 16 complementary sequence, made up of two complementary length 8 sequences, as the modulating sequence. These show a performance that is intermediate between the barker sequences and the quadratic residue sequences. They have a higher ripple than the Barker codes but less than the quadratic residue sequences. They also suppress angles near the specular direction, unlike the quadratic residue sequences.
- **Barker Codes.** Figure 7 shows the response of a length 13 Barker code as the modulating sequence. The smoothness of this response is excellent and is due to the good sidelobe performance of this code. Unfortunately its response at non-integer frequencies is not as good, as shown in figure 8. In this case there is additional suppression of angles near to the specular direction compared with the single grating response.

All the above sequences perform well. However, the smoothness of the Barker code response is exceptional and it would seem to be the sequence of choice. If the behaviour of this sequence near the specular direction, at non integer frequencies, is unacceptable then the binary quadratic residue sequence would seem to be the best choice, because it maintains uniform diffusion over the full frequency range of the modulated diffuser.

CONCLUSION

By modulating a basic quadratic residue diffuser with pseudorandom binary sequences one can remove the lobe narrowing which normally occurs when such diffusers are concatenated. The technique is simple to apply as it involves simply inverting conventional quadratic diffusers depending on whether the modulating sequence is one or zero. The resulting composite sequences have a diffusion performance that approaches that of a single sequence. However they can cover a much larger surface area. By using modulating sequences with low sidelobes in their aperiodic autocorrelation function, such as Barker codes, excellent diffusion performance can be achieved. These techniques add additional materials to the acoustic designer's armoury, for tackling real acoustic designs that have physical and practical, as well as theoretical, constraints.

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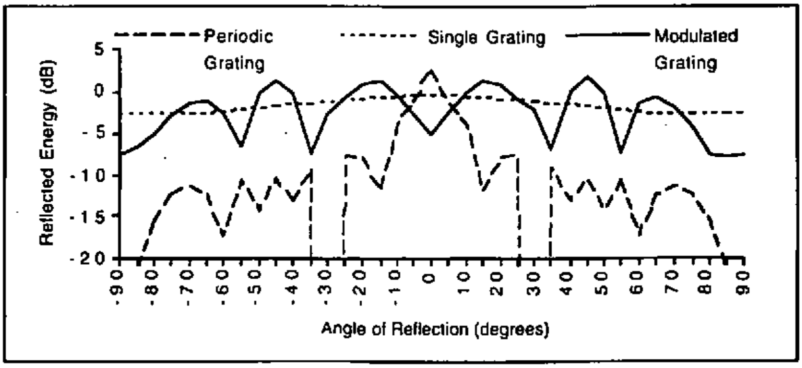

Figure 5a Reflected energy for a binary quadratic modulated grating at LF.

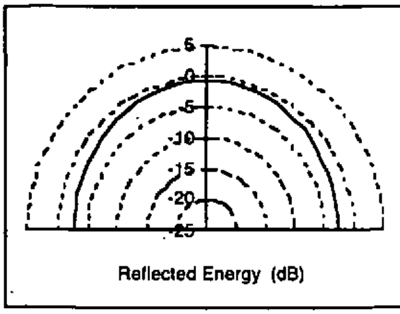

Figure 5b Single grating at LF

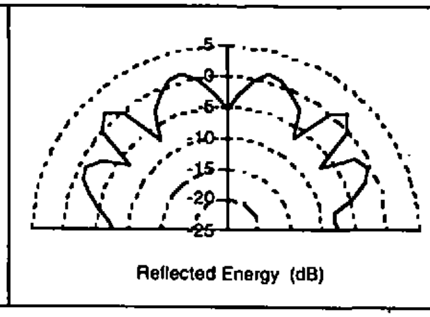

Figure 5c Quadratic modulated grating at LF

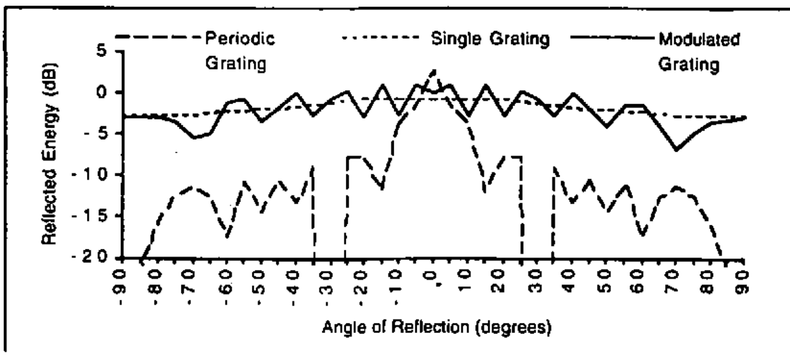

Figure 6a Reflected energy for a complementary code modulated grating at LF.

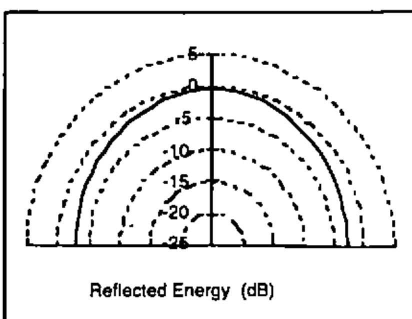

Figure 6b Single grating at LF

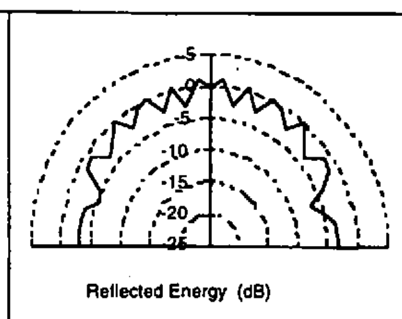

Figure 6c Complementary code grating at LF

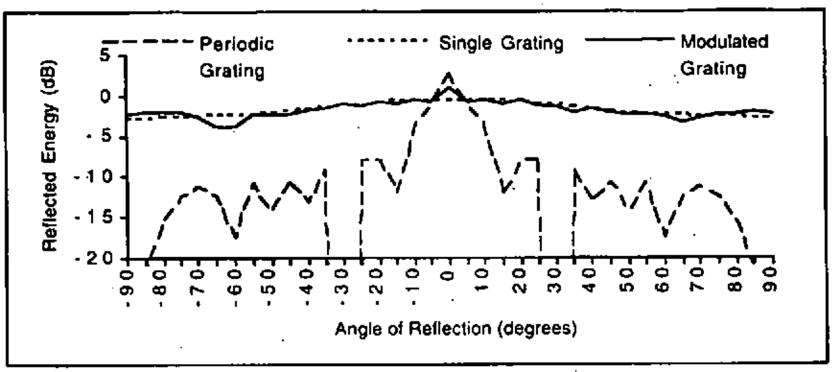

Figure 7a Reflected energy for a Barker modulated grating at LF.

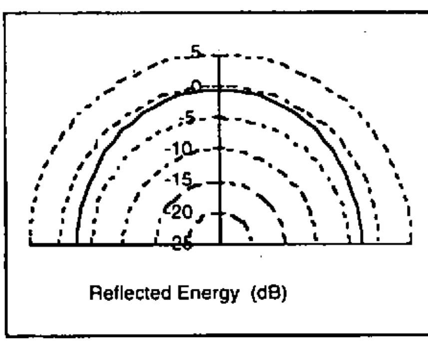

Figure 7b Single grating at LF

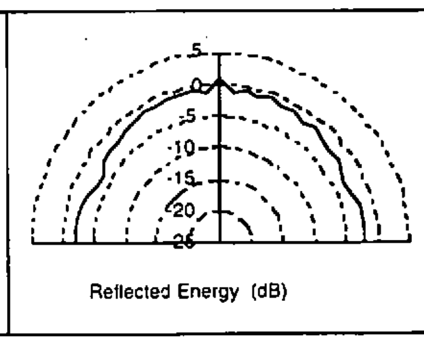

Figure 7c Barker modulated grating at LF

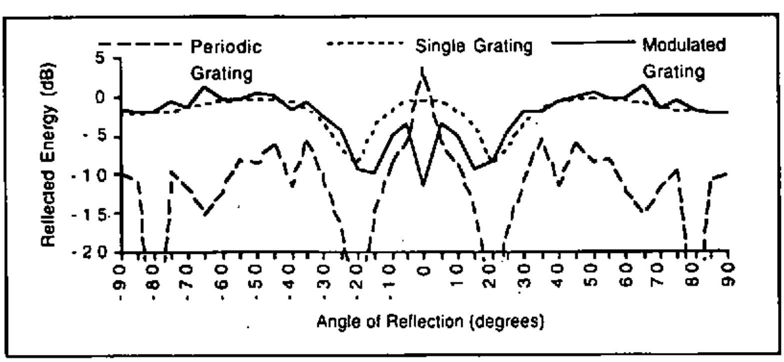

Figure 8 Reflected energy for a Barker modulated grating at MF.