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RADIATION MECHANISMS IN DML LOUDSPEAKERS

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1. INTRODUCTION

The Distributed Mode Loudspeaker (DML) appears to operate on completely different rules compared to piston type loudspeakers and its behaviour mysterious. The purpose of this paper is to examine the radiation processes of the DML loudspeaker in order to demystify its transduction mechanisms. The paper will show that it obeys the laws of physics like any other speaker but that it has some interesting properties and material requirements which are unique to it. The paper will first discuss the nature of radiation from panels and then will examine the different wave propagation types in panels. Finally it will show how the DML loudspeaker makes use of both types of propagation to achieve broadband transduction.

2. RADIATION FROM PANELS

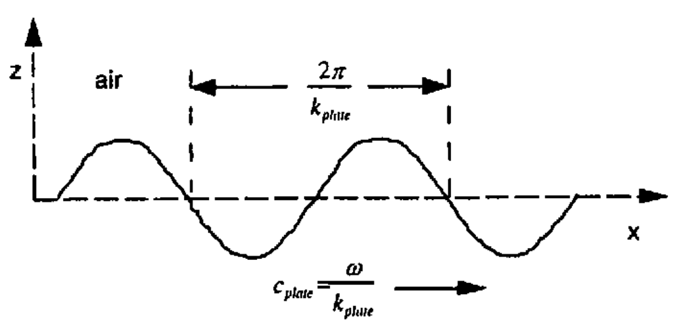

Figure 1 Sound radiation from waves on an infinite plate.

The basic principle of radiation from a plate or membrane is illustrated in figure 1 which shows a travelling wave in an infinite membrane in contact with air. In order for sound energy to be radiated there must be a lateral displacement of the air. Clearly the air and the plate variation in the longitudinal (or x) direction must be equal because they are in contact. However the lateral displacement of the air will depend on the velocity of sound in the air compared to the phase velocity of the wave in the plate. In simple terms the phase velocity in the must be faster than the speed of sound in the air or else the air will be able to move out of the way as the wave passes and therefore not propagate a sound wave. The best way analyse this situation in more detail is to look at it using wavenumbers. By using wave numbers one can show that the wave number in the transverse (or z) direction, which corresponds to the wave propagating in the air, is given by:

$$k_z = \pm\sqrt{k_{air}^2 - k_{plate}^2} \qquad (1)$$

This equation has three possible conditions:

$k_{plate} < k_{air}$ Here the wave causes sound radiation from the plate, a smaller k implies a higher phase velocity at a given frequency. These sound waves propagate away from the plate so the sign in equation 16 is positive. The wave propagates outwards at an angle ϕ from the normal given by:

$$\cos\phi = \frac{k_y}{k_{air}} = \sqrt{1 - \left(\frac{k_{plate}}{k_{air}}\right)^2} \qquad (2)$$

Note that this is a very directional wave propagating in one direction. This means that, contrary to what is publicly stated, it is possible for distributed mode loudspeakers to be directional. It is also interesting to note that the if the panel wave speed is infinite then the direction of propagation is normal to the plate. One can view this condition as the condition required for piston radiation.

The wave impedance for this propagating wave at the plate surface, remembering that the plate is infinite, is given by:

$$Z_{wave} = \frac{\omega\rho_0}{\sqrt{k_{air}^2 - k_{plate}^2}} = \frac{\rho_0 c}{\sqrt{1 - \left(\frac{k_{plate}}{k_{air}}\right)^2}} \qquad (3)$$

This impedance is positive and real and therefore work is done on the air, energy transferred and therefor sound energy propagated.

$k_{plate} > k_{air}$ Here the wave speed in the plate is less than the speed of sound in the air. This means that the result of equation 1 is imaginary and no wave propagation results: that is:

$$k_z = -j\sqrt{k_{plate}^2 - k_{air}^2} \qquad (4)$$

This means that the disturbance of the air decays exponentially away from the plate surface. This is another way of saying that the air can "get out of the way" when the speed of the wave in the plate is less than the speed of sound in air.

The wave impedance at the surface is given by:

$$Z_{wave} = \frac{j\rho_0 c}{\sqrt{\left(\frac{k_{plate}}{k_{air}}\right)^2 - 1}} \qquad (5)$$

This impedance is purely reactive and so no work is done, however it can affect the vibration of the plate via inertial loading if the plate is light enough.

$k_{plate} = k_{air}$ In this condition the two velocities are equal however this condition cannot be satisfied physically because the wave impedance at the surface is infinite.

Thus the implication of these equations are that unless the phase velocity of the wave in the plate is greater than the speed of sound in air it will not radiate efficiently. Furthermore when the plate is radiating efficiently the radiated sound is directional. This presents us with a problem because the density of modes in the plate depends on the speed of sound in the plate. If the wave velocity in the plate must be greater than the speed of sound this will set a lower resonance frequency which will be invariably greater than the lowest audio frequency for reasonably sized plates. In order to have a mode at 34 Hz while still remaining above the critical frequency requires a plate dimension of greater than 10m! Note that this requirement is the same whether bending or transverse shear waves are involved. Clearly this is not the whole story because plates do radiate when the plate velocity is less than the speed of sound in air. The reason for this is that the above analysis assumes a plate of infinite extent. However if the plate is of a finite size there are effects due to the boundaries which are explained in the next section.

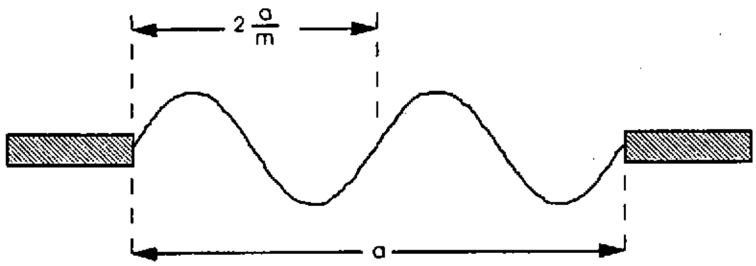

Figure 2 A finite width radiating strip mode.

2.1 Sound radiation from a finite width strip

In an infinite panel waves propagate in one direction forever and exist over an infinite area. However in a finite panel the waves only exist over a finite area and have zero amplitude elsewhere. Figure 2 shows this for the one dimensional case of an infinite strip in an infinite baffle. We can analyse this situation as a superposition of forward and backward travelling waves and treat them separately. However we need to have some way of expressing the fact that the waves are of zero amplitude outside the plate. One way of doing this is to analyse it from a Fourier point of view. Just as in the time domain we can construct a pulse from an infinite summation of sine waves we can construct a finite width propagating wave from an infinite number of propagating waves of the same frequency but different phase velocities. These form components with different values of k and the summation of these components is known as a wavenumber transform. Mathematically we can say that the waves on the plate are given by:

$$f(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}F(k)\exp(jkx)dk \qquad (6)$$

$$F(k)=\int_{-\infty}^{\infty}f(x)\exp(-jkx)dx \qquad (7)$$

The negative values for k correspond to waves propagating in the other direction.

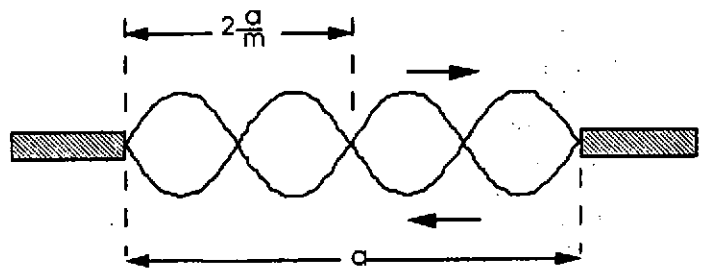

Figure 3 The mode shape for an m^{th} order mode.

Consider the mode shape in the x direction shown in figure 3 for an m^{th} order mode. The normal velocity for a strip is give by:

$$v(x,t) = \begin{cases} v\sin\left(\frac{m\pi x}{a}\right)\exp(j\omega t) & 0 \le x \le a \\ 0 & 0 > x > a \end{cases} \tag{8}$$

and the corresponding wavenumber transform is given by:

$$V(k_x) = \int_0^a v\sin\left(\frac{m\pi x}{a}\right)\exp(-jk_x x)dx \tag{9}$$

This is equivalent to the Fourier transform of a rectangularly weighted sine pulse which is a sinc function centred around the frequency of the sine wave. As we are interested in the radiated energy we need to examine the squared modulus of this wavenumber transform and this is given by:

$$|V(k_x)|^2 = |v|^2 \left[\frac{\frac{2m\pi}{a}}{k_x^2 - \left(\frac{m\pi}{a}\right)^2}\right]^2 \sin^2\left(\frac{k_x a - m\pi}{2}\right) \tag{10}$$

Equation 10 is called a modulus-squared spectrum and is proportional to energy. This representation is highly useful for examining the radiation efficiency from a finite width strip. Recall that the condition for radiation is that the phase velocity of the waves must be greater than the speed of sound in air, that is $|k_x| < k_{air}$. For a given mode frequency ω_m the value of $\pm k_{air}$ ($\pm k_{air} = \pm \omega_m / c_{air}$) will be a particular value on the k_x axis of the modulus-squared spectrum. Values of k_x which are less than this will contribute to radiation whereas values of k_x above this value represent reactive energy which creates nearfield disturbances near the plate but does not contribute to radiation. Thus one can mark out a region of the modulus-squared spectrum which contributes to this radiation that is the total radiated energy will correspond to the total area under the modulus-squared spectrum from $-k_{air}$ to $+k_{air}$ and the radiation efficiency will be the ratio of this area to the total area under the modulus-squared spectrum, as shown in figure 4. The modulus-squared spectrum can also be used to examine the effect of varying the plate parameters on there radiated power. Figure 5 shows the effect of increasing the phase velocity of the wave in the plate while retaining the same mode order and shape. As this would correspond to a higher frequency the value of $\pm k_{air}$ will also increase, as shown in figure 5 and thus the radiated energy will also increase. If the phase velocity of the plate wave increases enough the main lobe of the modulus-squared spectrum will form part of the radiated power. At this point there will be a rapid increase in the radiated power as shown in figure 6 which shows the area under the modulus-squared spectrum from $-k_x$ to $+k_x$ as a function of k_x. This condition correspond to a supersonic phase velocity for plate wave propagation. Clearly if this condition occurs over only part of the frequency range a nonuniform radiated frequency response will result. Figure 7 show the effect of a having different mode order, and therefore different frequency, in a strip of constant width and plate phase velocity. The important thing to note is that although the value of $\pm k_{air}$ increases the centre of the main lobe also increases in proportion. Figure 8 shows the effect of a wider width of the strip while retaining the same plate wave phase velocity. From these figures we see that the total radiated energy changes slowly. This is a surprising result that is not immediately obvious. However it can be explained by examining figures 9 and 10 which show that the part of the plate which contributes to the farfield radiation is the quarter cycle at the strip edges. This is because for subsonic plate velocities the cycles within the plate cancel each other out. Note that this is a far field effect in fact in the nearfield the whole plate is contributing to an evanescent field which decays exponentially away from the plate. Thus the effect of increasing the width of the strip is small. However, note that as the strip gets wider the width of the main lobe and sidelobes get narrower and in the limit the main lobe and sidelobes become a Dirac delta at the $\pm k_{plate}$ value corresponding to the modal frequency when the strip width becomes infinite. Under these conditions the radiation efficiency is zero if $\pm k_{air}$ is less than $\pm k_{plate}$ which corresponds to the results obtained earlier for an infinite plate.

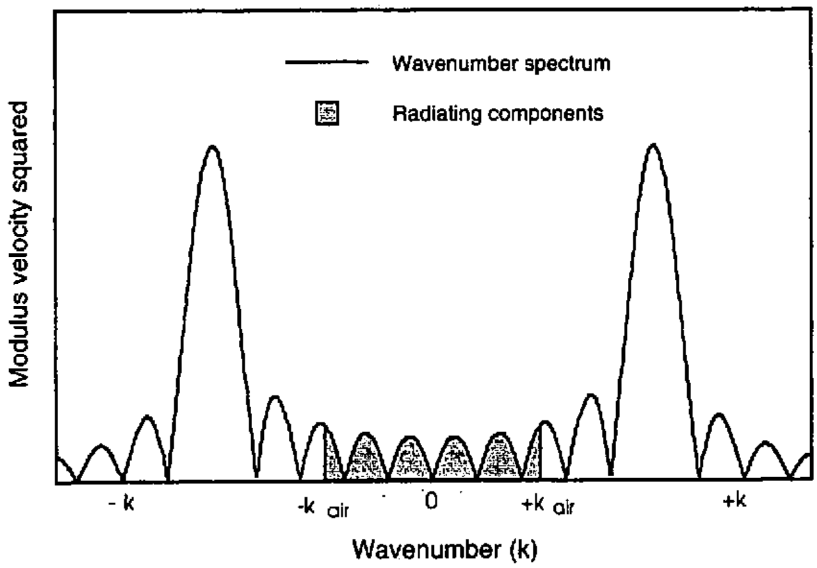

Figure 4 The radiating wavenumber spectrum components of a mode.

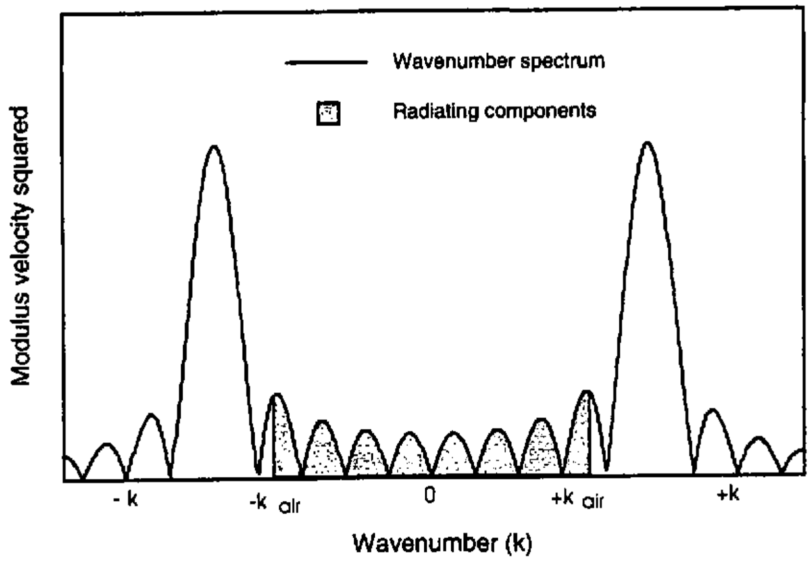

Figure 5 The radiating wavenumber spectrum components for the same mode at a higher frequency and hence higher plate velocity.

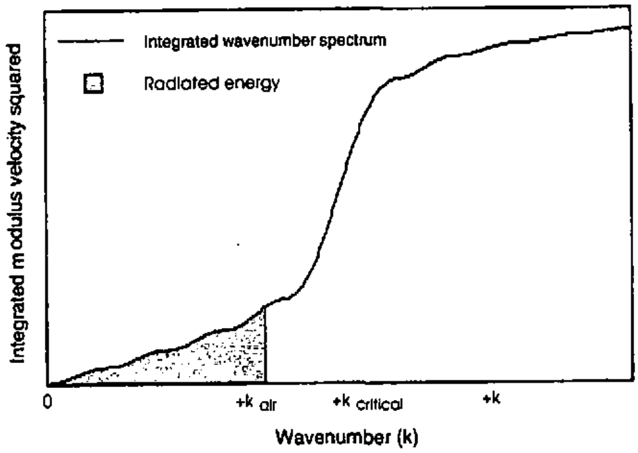

Figure 6 The integrated wavenumber spectrum of a mode.

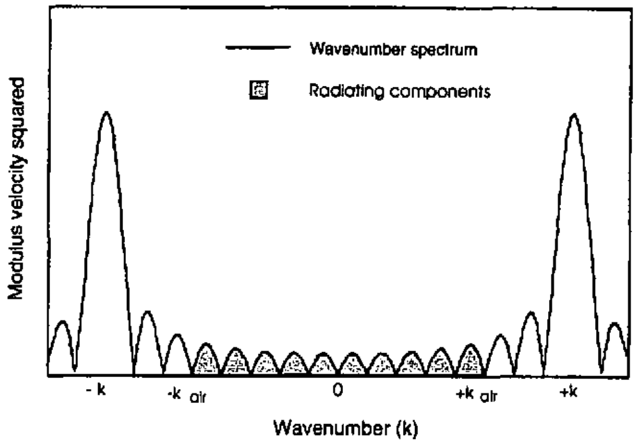

Figure 7 The radiating wavenumber spectrum components of a higher order mode.

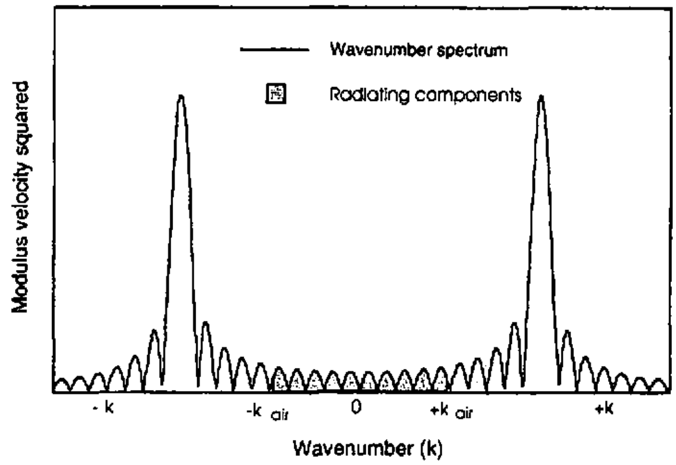

Figure 8 The radiating wavenumber spectrum components of the same mode with a wider strip.

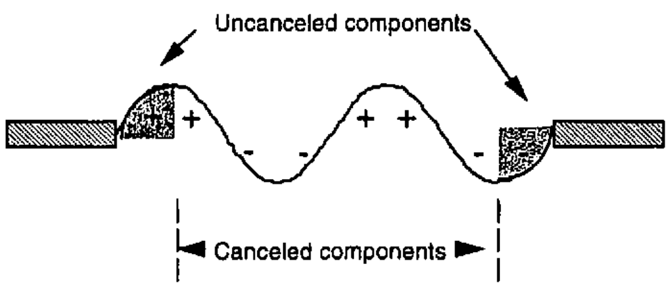

Figure 9 The effective radiating components of an even subsonic mode.

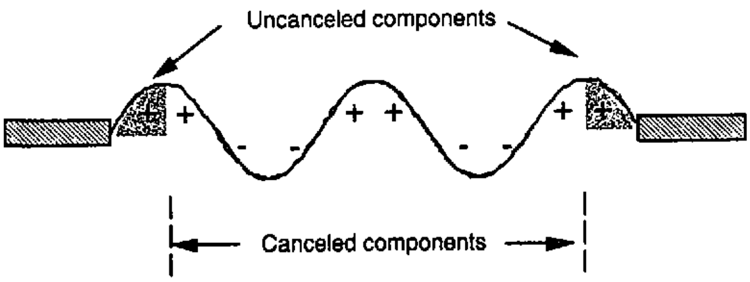

Figure 10 The effective radiating components of an odd subsonic mode.

2.2 Sound radiation from a finite plate

For a rectangular plate the wavenumber transform is similar to that of a strip, that is a sine squared shape centred at the wavenumber values corresponding to the modal frequencies in both the *x* and the *y* directions. The main difference being that the sine squared function is now two dimensional and the centre of the main lobe is now at the intersection of the k_x and k_y values for the mode. As in the strip case values of *k(x,y)* which are less than the value of $\pm k_{air}$ will contribute to radiation. In the two dimensional case this condition describes a circle of radius k_{air} around the origin. Unlike the one dimensional strip it is possible for one dimension to have a supersonic wave velocity while the other does not. In addition both dimensions may be subsonic or supersonic. In fact any condition which satisfies the following condition is possible:

$$k_x^2 + k_y^2 < k_{air}^2 \qquad (15)$$

3. VIBRATIONS IN PANELS

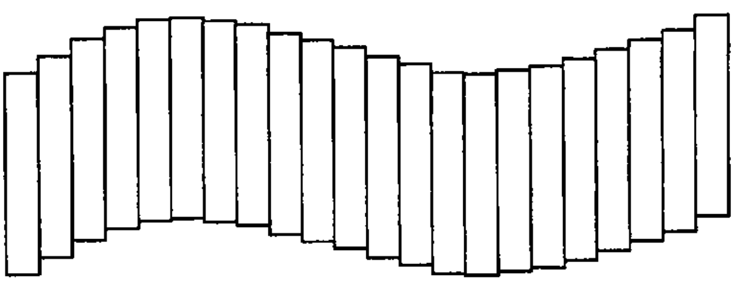

Figure 11 A transverse shear wave.

3.1 Transverse shear waves

There are two types of waves that are useful for sound radiation because they have significant transverse displacements and so can interact with the surrounding air to radiate energy. The first type is the transverse shear wave. For transverse shear waves the velocity of propagation is given by:

$$c_{transverse\ shear} = \sqrt{\frac{G}{\rho}}$$

Where $G =$ the shear modulus of the material (in $\mathrm{Nm^{-2}}$) (16)

and $\rho =$ the density of the material (in $\mathrm{kgm^{-3}}$)

This equation gives a phase velocity that is independent of frequency. and so is non-dispersive.

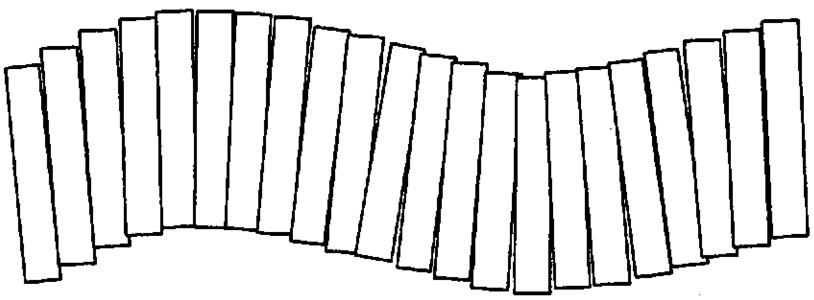

Figure 12 A bending (flexural) wave.

3.2 Bending (flexural) waves

The second type is the bending (flexural) wave which are neither pure longitudinal nor pure transverse waves. They are instead a combination of the two. Examination of figure 12 shows that in addition to the transverse motion there is also longitudinal motion which increases to a maximum at the two surfaces. Also on either side of the centre line of the bar the longitudinal motions are in antiphase. The net result is a rotation about the midpoint, the neutral plane, in addition to the transverse component. The formal analysis of this system is complex as in principle both bending and shear forces are involved. However, providing the shear forces contribution to transverse displacement is small compared to that of the bending forces the following expression for the phase velocity of a bending wave can be derived:

$$c_{bending} = \sqrt{\omega}\left(\frac{D}{m}\right)^{\frac{1}{4}}$$

Where $D =$ the bending stiffness of the plate (in Nm) (17)

and $m =$ the mass per unit area (in $\mathrm{kgm^{-2}}$)

Equation 17 is significantly different from that for transverse shear waves. In particular the phase velocity is frequency dependent, and increases with frequency. This results in dispersive propagation of waves with different frequencies travelling at different velocities. Therefore waveshape is not preserved in bending wave propagation. One can hear the effect of this if one listens to the "chirp" sound emitted by ice covering a pond when hit by a thrown rock. The dispersion and the quartic root arise because, unlike transverse shear waves, the spatial derivative in the wave equation is fourth order instead of second order because the bending wave is an amalgam of longitudinal and lateral waves.

A major assumption behind equation 17 is that the shear contribution to the lateral displacement is small. This is likely to be true if the radius of the bend is large with respect to the thickness of the plate, that is at long wavelengths. However, when the radius of the bend is of a similar size to the thickness this condition is no longer satisfied and the wave propagated asymptotically approaches that of a transverse shear wave. This gives an upper limit on the phase velocity of a bending wave which is equal to that of the transverse shear wave in the material. The ratio between the shear and bending contributions to transverse displacement is approximately:

$$\frac{contribution_{shear}}{contribution_{bending}} \approx \left(\frac{h}{\lambda_{bending}}\right)^2$$

$$\text{Where } h = \text{ the thickness of the plate} \tag{18}$$

$$\text{and } \lambda_{bending} = \text{ the wavelength of the bending wave}$$

From equation 18 the contribution of the shear contribution is less than 3% when $\lambda_{bending} > 6h$.

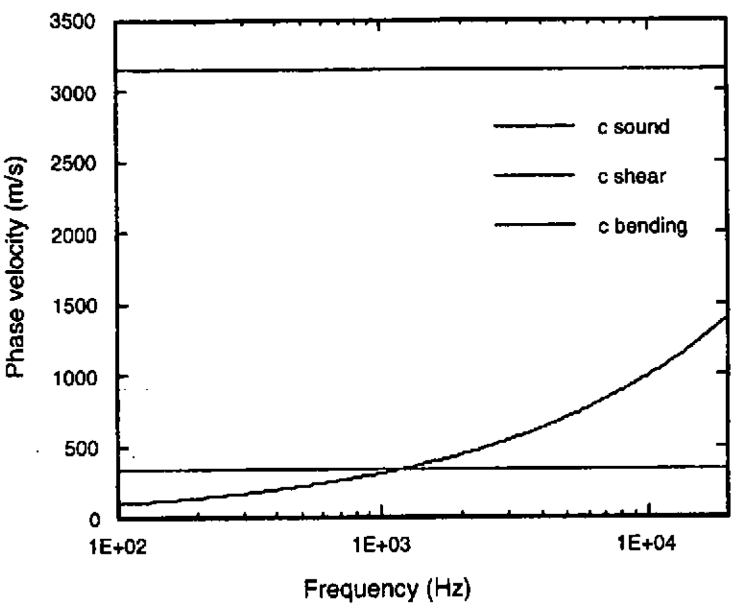

Figure 13 Phase velocity versus frequency for different types of wave propagation

Figure 13 shows the variation in phase velocity for the different types of wave propagation as a function of frequency compared with that of the speed of sound in air in an aluminium plate. The important thing to note is that the velocity of the shear waves are higher than the speed of sound and therefore, as we will see later, will couple efficiently to air whereas bending waves have a region where the phase velocity is lower than the speed of sound in air and therefore will tend to couple less efficiently into air. However the higher phase velocity of the transverse shear wave will result in a lower mode density in a given sized plate compared to bending waves.

The dispersive nature of bending waves has a significant effect on both the acoustic radiation processes and mode structure in a plate. However due to the dispersive nature of the wave propagation it is difficult to analyse their behaviour using wavelength. A more useful approach is to use the wavenumber of the wave given by:

$$k = \frac{\omega}{c} \tag{19}$$

Where $k =$ the wavenumber of the wave

$\omega =$ the angular frequency of the wave

and $c =$ the phase velocity of the wave

This encapsulates any dispersive effect s and can be used directly to calculate various aspects of wave propagation in, and acoustic radiation from, plates.

The equations for wavenumber for transverse shear and bending waves are:

$$k_{transverse shear} = \omega\left(\frac{\rho}{G}\right)^{\frac{1}{2}} \tag{20}$$

$$k_{bending} = \sqrt{\omega}\left(\frac{m}{D}\right)^{\frac{1}{4}} \tag{21}$$

Two points are of note from equations 20 and 21 the first is that the wavenumber of lateral shear waves is proportional to frequency and as one would expect from a non-dispersive wave. For a bending wave however, the wavenumber rises only as the square root of frequency. In both cases the coefficient is inversely proportional to the phase velocity so a low slope implies a high phase velocity. It is often helpful to plot wavenumber versus angular frequency in a dispersion diagram. Figure 14 shows the dispersion curves for different wave types in an aluminium plate along with the dispersion curve for sound in air.

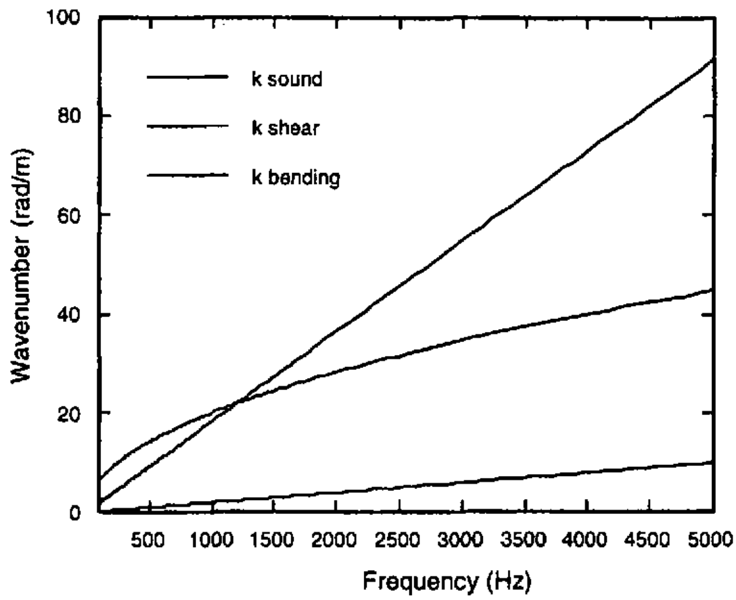

Figure 14 Wavenumber versus frequency for different types of wave propagation

4. THE EFFECT OF DISPERSION ON RADIATION EFFICIENCY

As we have seen earlier as the wave velocity increases the radiation efficiency also increases. Therefore the effect of the dispersion of the bending wave is to cause the plate to have a low radiation efficiency at low frequencies, where the wave velocity is subsonic. As the frequency increases the wave travels faster and transduces more effectively. In the limit it becomes supersonic and the plate becomes a very effective radiator. Unfortunately this behaviour results in a frequency response which, assuming equal transfer of energy at each frequency, rises to a peak. This is clearly undesirable and is addressed in DML speaker via the use of sandwich type materials.

4.1 Sound radiation from finite sandwich plates.

For non homogeneous materials, in particular those consisting of two thin plates separated by a core of lower shear stiffness, like the DML loudspeaker, the radiation mechanisms are the same. However the variation of plate phase speed with frequencies is different and this does affect the frequency response of the radiation. This is because there is no longer a single propagation mechanism for the plate wave. At low frequencies the plate wave is a bending wave whose velocity is determined by the bending stiffness of the whole panel. Because the panel will be quite thick, 12mm or more, and because the core has a relatively low shear modulus, the bending wave will become a shear wave, as shown in equation 18, whose velocity is determined by the core shear modulus. This will happen at intermediate frequencies. At higher frequencies the bending wave propagation again occurs due to the individual faceplate bending stiffness. The net result of all this is shown in figure 15 which shows a typical dispersion curve for such a structure. The major feature to note is that the combination of these mechanisms results in a speed of sound which is less than the speed of sound in air for a wider frequency range than would be the case if the panel was homogeneous. This means that, by appropriate choice of material properties, the plate can be operated just below critical frequency for a wider frequency range. The effect of varying the core shear stiffness is shown in figure 16 which shows that increasing the core stiffness improves the radiation efficiency, because it increases the plate phase velocity.

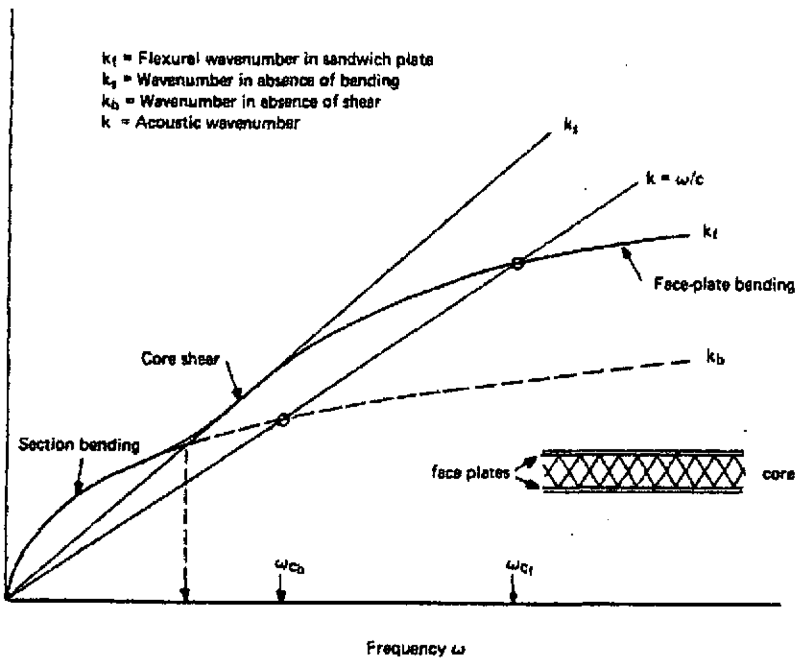

Figure 15 A typical dispersion curve for a sandwich construction plate (from [3])

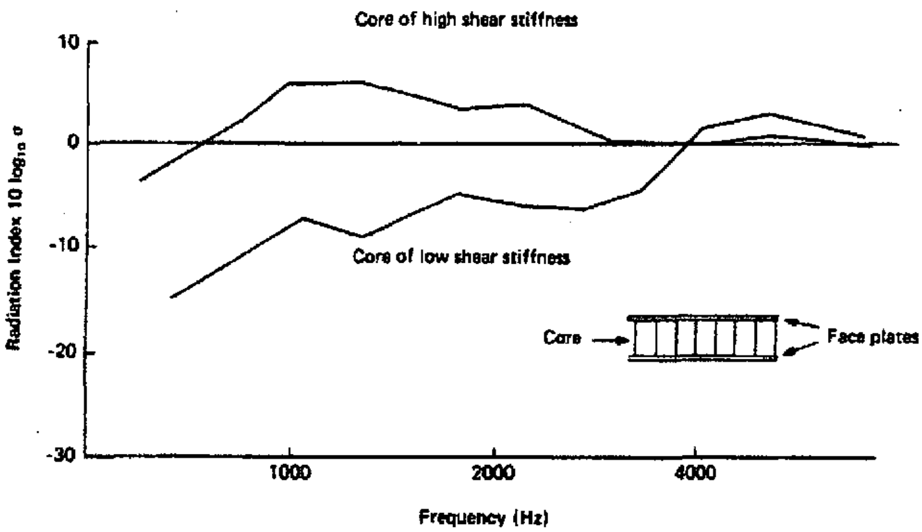

Figure 16 The effect of core shear stiffness on the radiation efficiency of a baffled plate (from [3])

5. CONCLUSION

The Distributed Mode Loudspeaker's radiation mechanism has been analysed. It has been shown that a combination of wave propagation mechanisms is responsible for its broad frequency response. To achieve an even response is a challenge and requires careful design of a composite materials overall bending modulus, core shear modulus, faceplate bending modulus, and thickness of the whole assembly.

6. REFERENCES

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