SPARSE LOUDSPEAKER ARRAYS AND THEIR DESIGN

JAS Angus School of Acoustics and Electronic Engineering, University of Salford, Salford, Greater Manchester, M5 4WT, England

1. INTRODUCTION

An early example of an array loudspeaker was the column loudspeaker. In this arrangement a number of small loudspeakers were arranged in a closely space line. Because of the extended length of the source in one plane directivity control was achieved in that plane. However, the beam pattern would get progressively more directive with frequency. They could also exhibit unwanted side lobes at higher frequencies, which reduced their utility. Techniques were developed to reduce this behaviour, usually by applying the necessary frequency dependent weighting, or tapering, using simple electrical circuits. Methods of steering these line speakers were also developed either, by using simple analogue delay techniques, or by using the inherent phase shifts in the filters used to taper the array. However, the limitations, and cost, of these methods limited their broad application. Other less expensive methods, such as constant directivity horns, were developed to achieve the need for controlled directivity over a broad frequency range.

With the advent of relatively inexpensive digital signal processing array loudspeakers have become more popular. They offer unprecedented control and have been widely used. However, both the new systems, and the old column loudspeakers, suffer from the problem of being undersampled at some point in their frequency range. That is, above some frequency, the spacing between the drivers is greater that half the wavelength of the sound being produced. This results in ***spatial aliasing*** and results in a loss of control of the beam pattern. To avoid spatial aliasing requires a huge number of small loudspeakers, resulting in a prohibitive cost for the array. Some ad-hoc techniques have been developed, such as logarithmic spacing; have been developed to reduce this problem. However, a better understanding of how to subsample the array in order to achieve a desired level of sidelobe performance would be useful.

This paper looks at the problem of achieving controlled directivity from array loudspeakers when the density of drivers is less than the minimum required to avoid spatial aliasing. It first examines the basic theory behind array loudspeakers and then goes on to look at the effects of spatial aliasing. Methods of reducing this, including spatial filtering are then discussed. Final various strategies for designing sparse arrays are described.

2. THEORY

In order to understand sparse loudspeaker arrays we must first look at the theory behind array loudspeaker design.

2.1 Conventional Array Loudspeakers

Consider an, evenly spaced, linear array of perfect point source radiators, as shown in figure 1. If we are an infinite, or at least very large, distance away, we can make the following approximations:

1. The wavefronts are planar, and therefore all the radiators will have the same angle of incidence (θ) to the far off point.

2. The differences in path-lengths are so small that only the initial phase difference, due to θ, affects the received amplitude.

These approximations are known as the farfield assumptions and, in theory, will be satisfied providing one is a reasonable distance from the speaker array. Note that it was generally assumed that these conditions were satisfied in the normal audience-speaker arrangements. However, recent work with large arrays [refs] casts some doubt on this.

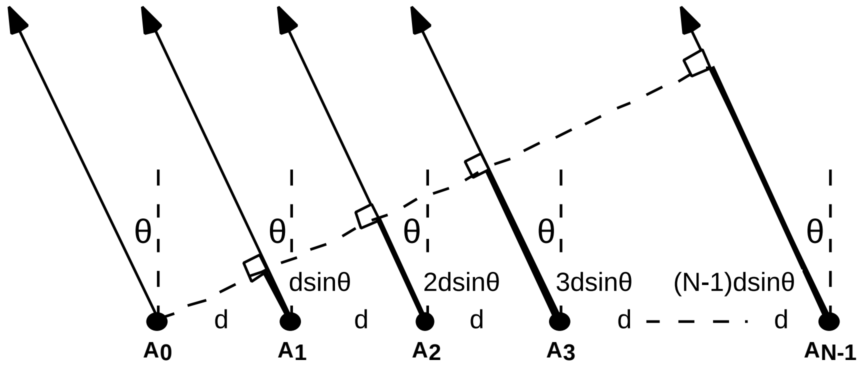

Figure 1 A linear array of N point sources.

Assuming, for the moment, that the far-field assumptions are satisfied we can say the following about our linear array of ideal point sources.

1. The far-field response will be given by the sum of the individual point sources with an additional phase delay/advance due to due to θ, which is the angle from the normal, as shown in figure 1.
2. The phase delay due to θ will be given by:

$$Phase\ delay = nd\sin q \tag{1}$$

Where n is proportional to the point source number, as shown in figure 1.

For the example shown in figure 1, this results in an equation for the far-field polar response, at a frequency who's wavenumber is ***k***, which is:

$$P(\theta_k) = A_0 e^{-j(0)\,kd\sin\theta} + A_1 e^{-j(1)kd\sin\theta} + A_2 e^{-j(2)kd\sin\theta} + A_3 e^{-j(3)\,kd\sin\theta} + \cdots + A_{N\text{-}1} e^{-j(N-1)kd\sin\theta} \tag{2}$$

Where the wave number ***k*** is given by:

$$k = \frac{2\pi}{\lambda} = \frac{\omega}{c} = \frac{2\pi f}{c}$$

This can be rewritten as:

$$P(\theta_k) = \sum_{n=0}^{n} A_n e^{-jnkd\sin\theta} \tag{3}$$

If we make $\Omega = kd\sin\theta$ then Equation 3 can be rewritten as:

$$P(\theta_k) = \sum_{n=0}^{n} A_n e^{-jn\Omega} \tag{4}$$

Equation 4 is in fact a Discrete Fourier Transform (DFT) in which $\Omega = kd\sin\theta$. This means that the farfield polar pattern of an array of point sources is related to the applied signals by a Fourier Transform relationship and therefore all the theorems that apply to the Discrete Fourier Transform apply to the array loudspeaker. As we shall see later this has some important consequences.

2.2 The Visible Region

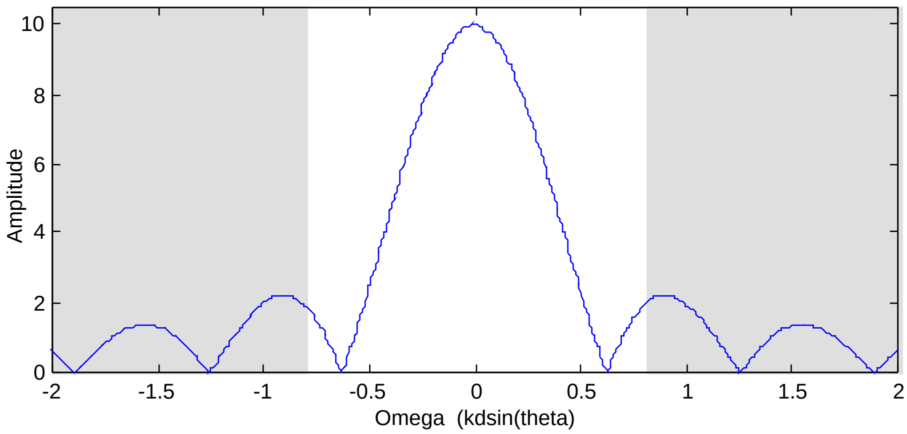

Figure 2 The visible region of an array speaker in Ω space (***kd*=**0.79).

Although, in theory, the Ω variable in equation 4 can range from −∞ to +∞, in reality it cannot. In fact, because sinθ cannot exceed ±1, there is only a limited range of Ω that makes any physical sense. This region is known as the "visible region" and, because $\Omega = kd\sin\theta$, the visible region corresponds to $-kd \leq \Omega \leq kd$. This is shown in figure 2 for a 10-element array, with the elements spaced 4.3cm apart, at 1khz (***kd*=**0.79). If we double the frequency to 2kHz then ***kd*** doubles (***kd*=**1.58) and the visible region also doubles, as shown in figure 3.

As the visible region corresponds to the angles between ±90º of the normal direction the effect of doubling the visible region also implies a narrowing of the main lobe, if its shape does not change as the visible region increases, as in our examples.

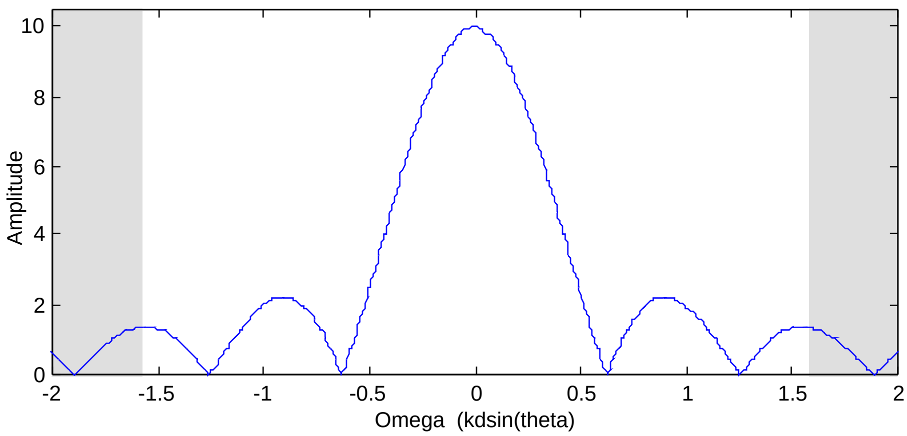

Figure 3 The visible region of an array speaker in Ω space (***kd*=**1.58).

3 SUB-SAMPLED ARRAYS

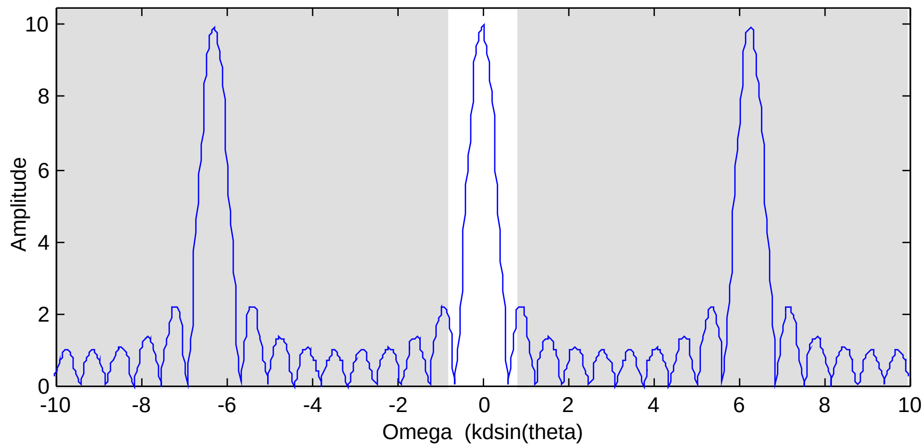

Figure 4 The visible region of an array speaker in a larger Ω space (***kd***=0.79).

When the frequency gets high enough so that the spacing between the array elements becomes greater than half a wavelength the array becomes undersampled. Under these conditions one gets ***spatial aliasing*** which results in multiple main lobes. Figures 4, 5 and 6 illustrate this. Figure 4 shows the 1kHz example with the Ω scale expanded. The first thing to note is that the visible region still covers the same region as that of figure 2. The second thing to note is that the expanded scale reveals the multiple peaks that indicate spatial aliasing.

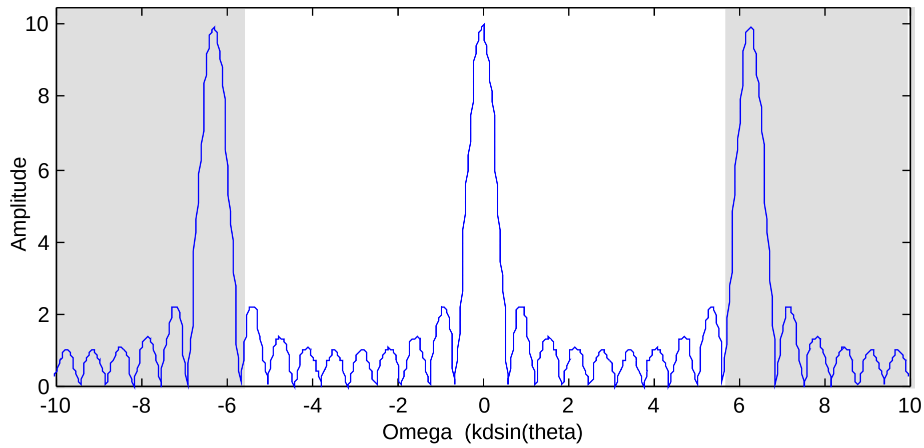

Figure 5 The visible region of an array speaker in a larger Ω space (***kd***= 7.85).

Figure 5 shows the visible region when the frequency equals 7kHz (***kd***=5.5). Here we can see that, although the aliased main lobe is not visible there is an increase in sidelobe levels due to the spatial aliasing. Figure 6 shows the visible region when the frequency equals 10kHz (***kd***= 7.85). Here we can see that the aliased main lobe is now visible and there is an large increase in the sidelobe levels due to the spatial aliasing.

Ideally, we want pattern control over the entire audio frequency range. However, even if we make the speaker spacing 4.3cm, which is infeasible small because we would need a large number to achieve low frequency pattern control, we still have significant aliasing at 10kHz. Therefore, we need to come up with strategies to make use of sparse arrays if useful performance is to be achieved.

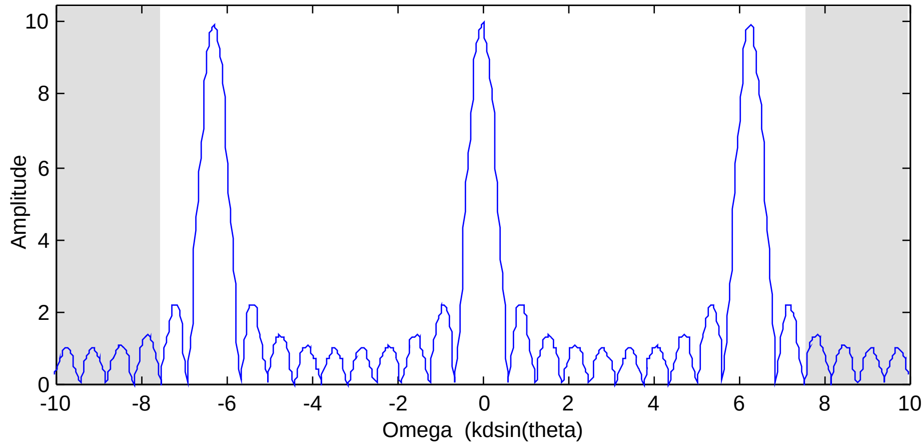

Figure 6 The visible region of an array speaker in a larger Ω space (***kd*=** 7.85).

4 SUB-SAMPLED ARRAY DESIGNS

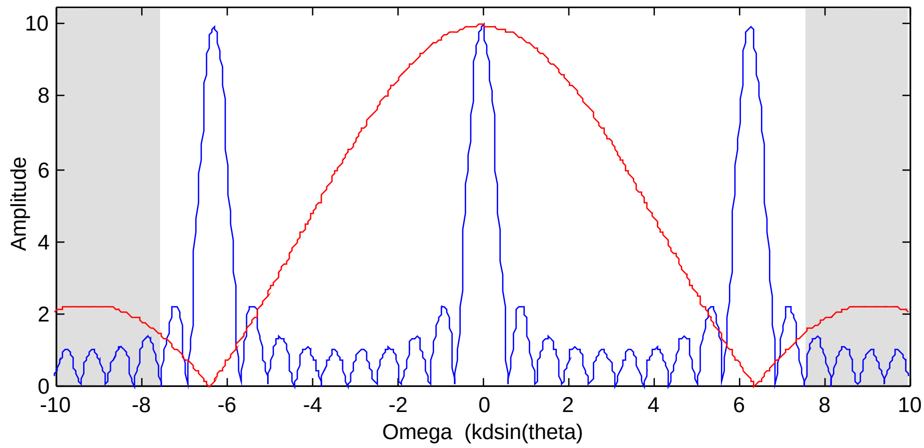

Figure 7 An array speaker and a continuous source equal to the spacing (***kd*=** 7.85).

4.1 Acoustic Spatial Filtering

One way of reducing the effect of spatial aliasing is to use directive loudspeakers, instead of point sources, as the array elements. If one uses directive sources then their polar patterns will act as a form of spatial filter. That is the off axis sidelobes will be reduced by the of axis reduction in sound level that a directive source affords. Figure 7 shows our array response at 10kHz (***kd*=** 7.85) with the response of a continuous line source, of length equal to the element spacing, superimposed upon it. Of particular note is that the zeros of the continuous line source fall on the aliased main lobes from the point source array. Because the farfield polar pattern of an array of point sources is related to the applied signals by a Fourier Transform relationship, all the theorems that apply to the Discrete Fourier Transform apply to the array loudspeaker. This means that the theorem that convolution in one domain is equal to multiplication in the other domain applies to this situation. Replacing each of the point sources with a continuous line source is equivalent to convolving it with the point array. Therefore, the effect of replacing the point source with the continuous sources is to multiply their farfield patterns together. This ***pattern multiplication*** is well known and the effect is for our example is shown in figure 8. One can see that the aliased main lobes have been eliminated. In fact, the response has become equivalent to a continuous line source of the same extent as the array. Clearly using directional sources, such as constant directivity horns, can also be used to achieve similar effects. It is this that result in the success of large arrays based on them because, providing they horns have directivity control before spatial aliasing occurs. Once the directivity of the individual elements is considered the need for curved arrays also becomes apparent, as the spatial filtering effect of the sources must also be factored in.

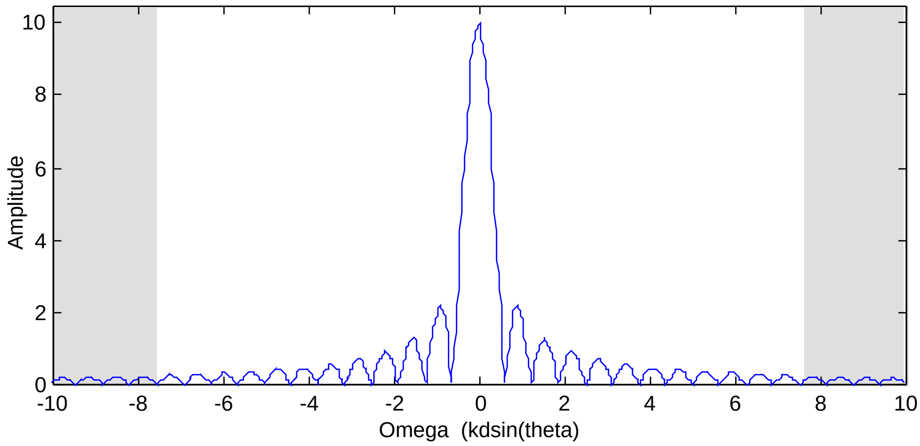

Figure 8 An array speaker and a continuous source equal to the spacing (***kd*****=** 7.85).

4.2 Logarithmic Spacing

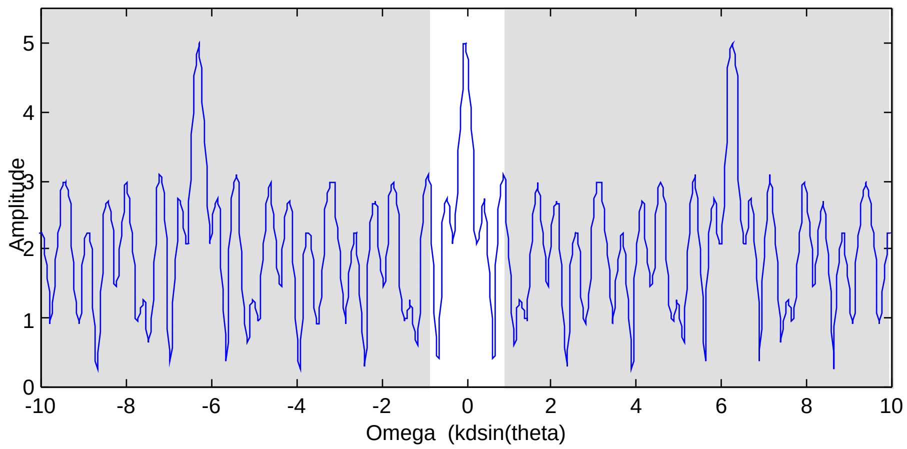

Figure 9 A, 5 element, logarithmically spaced array, spacing doubles between sources (***kd*****=**0.79).

One possible spacing for an array is logarithmic spacing, in which the spacing between the sources exponentially increases away from the centre element. Figure 9 shows the amplitude of an array in which the spacing doubles between each source. Two points are of note. Firstly, the centre lobe is very narrow because the extent is equivalent to a length 16 linearly spaced array. Secondly, the main lobe peak is only 5 because there are only 5 active elements in the array, compared to 16 in the linearly spaced array of the same size. In some senses figure 9 is disappointing, because there are still alias lobes and the sidelobe level is not that good. However, the main lobe is very narrow giving good directivity at low frequencies (or low ***kd***). In general, such arrays are used where the array is frequency tapered, that is, outer elements are progressively turned off as the frequency increases. In these circumstances, the aliasing is less of a problem, although the sidelobe level can be.

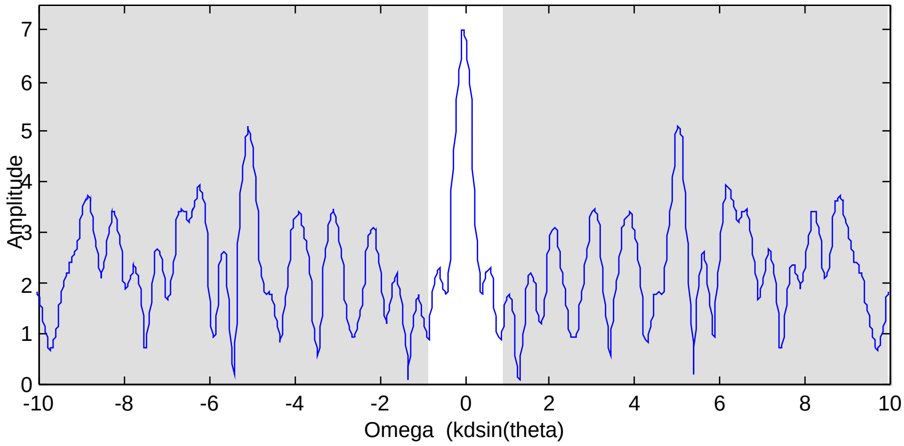

Figure 10 A 7 element logarithmically spaced array, non-integer spacing between sources (***kd***=0.79).

One of the reasons for the poor performance of the array in figure 9 is the fact that all the spacings are integers. This means that ripples in the polar pattern due to missing elements tend to add constructively, especially at the alias spatial frequencies. Figure 10 shows the effect of using spacings based on non-integer spacings (1.0, 1.28, 1.65, 2.12, 2.72, 3.5). The length of the array is equivalent to a 12 element linearly spaced array and contains 7 sources. One can see that the performance has improved a little particularly near the main lobe.

4.3 Prime Spacing

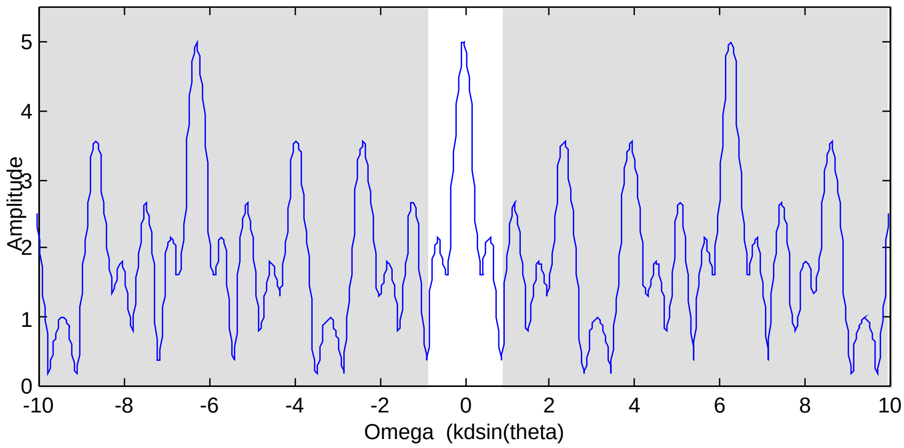

Figure 11 A prime spaced array, spacing between sources is [1,2,3,5] (***kd***=0.79).

Another way of populating the arrays is to use prime numbers to define the spacing between the sources. The result of this is shown in figure 11. Again, the performance is not very good. With many sidelobes. Closer examination reveals that the spacing defined in this fashion results in the powers of n applied to the sum exponentials in equation (2) have common factors. A better choice is to arrange for the exponential powers to be prime numbers which results in the spacing [1,1,1,2,2,4]. The result of this is shown in figure 12 and one can see that the sidelobe levels have improved.

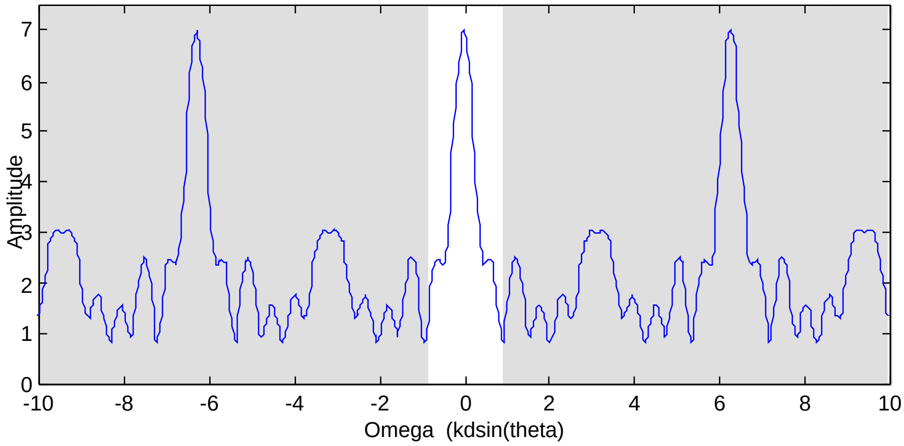

Figure 11 A 7 source spaced array, spacing gives prime exponential sum factors (***kd***=0.79).

4.4 Fibbonaci Spacing

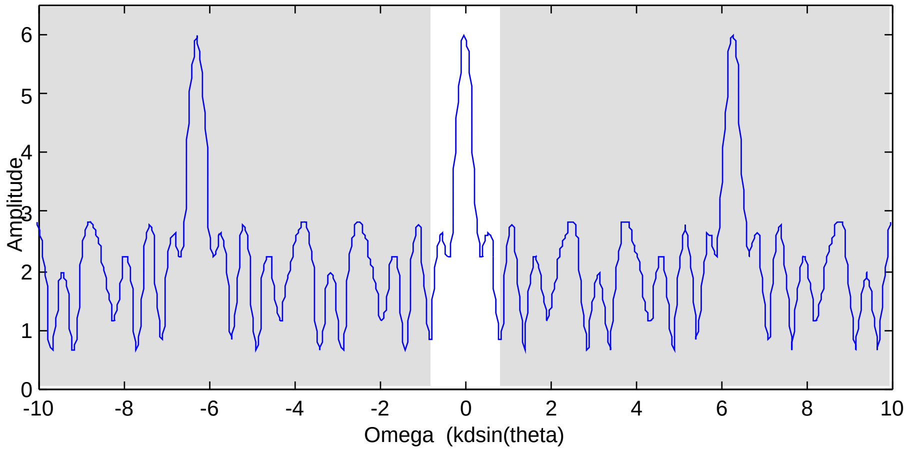

Figure 12 A 6 source Fibbonaci spaced array, spacing is [1,1,2,3,5] (***kd***=0.79).

A final possible spacing is to use a Fibbonaci sequence and the result is shown in figure 12. Performance is similar to that of the prime exponential factor array.

5 CONCLUSION

This paper has looked at the problem of achieving controlled directivity from array loudspeakers when the density of drivers is less than the minimum required to avoid spatial aliasing. It first examines the basic theory behind array loudspeakers and then goes on to look at the effects of spatial aliasing. Methods of reducing this, including spatial filtering have been discussed. Finally, various strategies for designing sparse arrays were described. Although sparse arrays allow larger arrays with less sources a price is paid in sidelobe level.

6 REFERENCES

[1] Ureda, M., "Pressure Response of Line Sources", 113th Convention Los Angeles, California 2002 October 5-8, preprint 5649.