REVISITING SABINE'S FORMULA

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1 INTRODUCTION

In 1992, I proposed a generalized Sabine formula that developed reverberation time over a power series of the average reflection coefficient on the boundaries of an ergodic enclosure[1]. Building on the work of Joyce[2] and Kuttruff[3], this generalized formula considers the long-term distribution of reflections along one single ray as a Lévy process, for which Paul Lévy[4] developed the complete characterization. As a consequence, the decay process reduces to the analytical continuation to real values of the characteristic function of a Lévy process.

Two years later, using numerical simulations carried out by Mortessagne et al.[5,6] on stadia, which are made of squares prolonged by two half-discs on two opposite sides, I reduced the development to just two terms that made it possible to monitor reverberation times up to large absorptions in these specific 2D enclosures.

The present paper revisits this development in the case of 2D and 3D spherical and rectangular ergodic enclosures that, according to Joyce[2], follow Lambert law reflections. As explained by Kuttruff[3], reverberation is piloted by the first two statistical moments of the free path distribution between successive reflections. Analytical formulations of these two moments have been derived for 2D and 3D spherical and rectangular ergodic enclosures; but their complexity, especially for the 3D rectangular enclosure, is beyond the scope of this conference paper, which only presents results. However, it should be noted that the complex analytical integration recovers the usual values of the mean free paths – $\pi S/P$ for the 2D case and $4V/S$ for the 3D case, where V is the volume, S the surface, and P the perimeter of the enclosure – as demonstrated by Joyce[2]. In fact, the mean free paths were calculated as a verification of the integration procedures.

The paper starts with the reverberation formulae of 1992 and 1994, which are compared to Levy's original expression for a characteristic function and to Kuttruff's formula[3]. The case of high mean absorption coefficients is also discussed, when Kuttruff's formula breaks down, as well as the case of different absorption coefficients on different boundaries. This makes it possible to specify the limit of validity when mean absorption becomes large. It then turns to the analytical formulae of the mean free paths and mean quadratic free paths for 2D and 3D, spherical and rectangular ergodic enclosures, and to the corresponding formulae that can then be derived from Lévy's expression. In the end, it revisits Kuttruff's formula and shows that more accurate integration leads to an expression that does not break down for large absorptions coefficients.

Finally, the implications of this 'revised theory' for absorption measurements in reverberation chambers and for auditorium design are discussed.

2 MY 1992 AND 1994 REVERBERATION FORMULA

2.1 Generalized Reverberation Formula

As explained above in the introduction, my generalized reverberation formula from 1992 was based on the expression of the characteristic function of a Lévy process, which generalizes Poisson processes while keeping independent events that take place in successive instants. Following

Kuttruff[3], I considered the probability distribution $P(n|t)$ of reflections along a ray during the time span t. Due to the ergodic hypothesis, all rays are equivalent. Then, according to Lévy, the logarithm of the characteristic function of such a process, better known as the cumulant function, is expressed by:

$$\psi(z,t) = ln\left(\sum_{n=0}^{\infty} e^{izn}\, P(n|t)\right) = m(t)iz - g(t)\frac{z^2}{2} + \int_{-\infty}^{+\infty}\left(e^{izu} - 1 - \frac{izu}{1+u^2}\right) dN(t,u)$$

where $m(t)$, $g(t)$ and $N(t,u)$ are continuous in t, null for $t = 0$; and where $N(t,u)$ is a non-decreasing function of u in each interval $(-\infty, 0)$ and $(0, \infty)$, null at infinity, and satisfying for all t $\int u^2\, dN(t,u)$ finite on all finite intervals. From that expression and symmetry considerations due to ergodicity since time averages and ensemble averages are equal, $m(t)$, $g(t)$ and $N(t,u)$ must be proportional to t, which leads to infer that the decay rate takes the form:

$$\delta(t) = -\frac{\partial\psi(ln(R),t)}{\partial t} = -m\, ln(R) - g\frac{\left(ln(R)\right)^2}{2} - \int_{-\infty}^{+\infty}\left(e^{ln(R)u} - 1 - \frac{ln(R)u}{1+u^2}\right) dN(u)$$

where iz has been replaced by $ln(R)$, R being the mean reflection coefficient. In ref.[1], I further assumed that g is null, which leads to the following formula for the reverberation time:

$$RT = \frac{13.8}{-m\, ln(R) + \sum_u (1 - R^u)\, n_u}$$

when u takes discrete positive values only. Note that Kuttruff's formula is equivalent to retaining the two first terms only in the cumulant function, assuming that $N(u) = 0$ for all u, leading to a different expression of the reverberant time:

$$RT_K = \frac{13.8}{-m\, ln(R)\left(1 + \frac{g\, ln(R)}{2m}\right)}$$

which diverges for large values of R since $ln(R)$ is always negative.

2.2 Deriving the terms in the formula

The different terms in the reverberation time formula can be derived from the standard deviation of path fluctuations. Indeed, when $m(t)$ and $g(t)$ are both null and u takes only one value, $P(n|t)$ takes the explicit form:

$$P_u(n|t) = e^{-tN(u)}\frac{\left(tN(u)\right)^n}{n!}$$

Here, u corresponds to an "amplitude" – or *order* – of reflection, which can be different from 1, so that the effective number of reflections, in the usual sense of the term, is equal to nu. From this expression, the probability of waiting time t for observing the n^{th} reflection of order u is given by:

$$P_u(t|n) = e^{-tN(u)}\frac{\left(tN(u)\right)^{n-1}}{(n-1)!}N(u)$$

since there were only $(n-1)$ reflections during time t, the n^{th} one occurring at instant $t = dt$. The mean value and the variance of this waiting time are therefore given by:

$$< t > = \frac{n}{N(u)}\ , \qquad \sigma_u^2(n) = \frac{n}{N^2(u)}$$

Taking into account the effective number of reflections, the mean waiting time $\bar{t}$ and its variance σ^2 between effective reflections are given by:

$$\bar{t} = \frac{< t >}{nu} = \frac{1}{uN(u)}\ , \qquad \sigma^2 = \frac{\sigma_u^2(n)}{nu} = \frac{1}{uN^2(u)} = \frac{\bar{t}}{N(u)}\ , \text{ and } \gamma^2 = \frac{\sigma^2}{\bar{t}^2} = u$$

where γ^2 is the relative variance. In the case of stadia, numerical simulations from Mortessagne et al.[5,6] lead to $N(u) = 111.3$ for $u = 0.576$. Considering that the value of u is very close to ½, I refined the analysis by considering a mean value m, attested by further data sent by Mortessage and Legrand. It consists in modifying the effective number of reflections to take into account the influence of m during the mean waiting time $< t >$: the effective number of reflections becomes equal to $nu + m < t > = n\left(u + \frac{m}{N(u)}\right)$, so that the mean waiting time and variances between effective reflections become:

$$\bar{t} = \frac{< t >}{n\left(u + \frac{m}{N(u)}\right)} = \frac{1}{m + uN(u)}\,, \qquad \sigma^2 = \frac{\sigma_u^2(n)}{n\left(u + \frac{m}{N(u)}\right)} = \frac{\bar{t}}{N(u)}\,, \quad \gamma^2 = u + \frac{m}{N(u)}$$

In other words, the variance keeps the same expression as before. The data from Mortessagne et al.[5,6] now lead to $N(u) = 111.3$ and $m = 8.46$ for $u = 0.5$. Note that Sabine formula corresponds to $u = 1$.

Mortessagne et al.[5,6] compared their numerical simulations to Kuttruff's formula. This time, the probability distribution $P(n|t)$ of reflections along a ray during the time span t takes the form:

$$P(n|t) = \frac{1}{\sqrt{2\pi g t}} e^{-\frac{(n-mt)^2}{2gt}}$$

so the probability of waiting time t for observing the n^{th} effective reflection is now given by:

$$P(t|n) = \frac{1}{\sqrt{2\pi\gamma^2 t}} e^{-\frac{(t-\frac{n}{m})^2}{2\gamma^2 t}}$$

where $\gamma^2 = \frac{g}{m^2}$ is the relative variance of the process. However, this is not the approach taken by Kuttruff, who postulates a Gaussian probability distribution for $P(t|n)$:

$$P(t|n) = \frac{1}{\sqrt{2\pi n}\,\gamma\bar{t}} e^{-\frac{(t-n\bar{t})^2}{2n\gamma^2\bar{t}^2}} \propto P(n|t)$$

where $\bar{t}$ is the mean waiting time between successive reflections as before, and γ^2 still is its relative variance. Kuttruff then points out that n can be replaced by its mean value $t/\bar{t}$ in the denominators, leading to the approximation:

$$P(n|t) \approx \frac{1}{\sqrt{2\pi\, t\gamma^2/\bar{t}}} e^{-\frac{(n-t/\bar{t})^2}{2t\gamma^2/\bar{t}}} = \frac{1}{\sqrt{2\pi g t}} e^{-\frac{(n-mt)^2}{2gt}}$$

Thus, $m = 1/\bar{t}$ and $g = \gamma^2/\bar{t}$ in the formula for RT_K, which takes the expression given by Kuttruff:

$$RT_K = \frac{13.8}{-m\,ln(R)\left(1 + \frac{\gamma^2}{2}ln(R)\right)}$$

Table 1 presents the reverberation times calculated for different values of the mean absorption coefficient $\alpha = (1 - R)$. They were computed with the data from Mortessagne et al.[5,6], and compared with Sabine's, Eyring's and Kuttruff's reverberation times. The latest one diverges for $\alpha > 0.981$, that is, for the last column of Table 1. The data correspond to a mean waiting time of 0.0156 s, or a mean free path of 5.3 m, that is, a stadion built on a square with 4.85 m long sides. The speed of sound was taken as 340 m/s. Note that Eyring reverberation time RT_E corresponds to keeping the first term only in the denominator of Kuttruff's formula.

The last line of Table 1 is a revised version of Kuttruff's formula, that is discussed in Sect. 5.

Table 1: reverberation times for the stadion of Mortessagne et al.[5,6].

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
RT_S	2.153	1.076	0.718	0.538	0.431	0.359	0.308	0.269	0.239	0.215
RT_E	2.043	0.965	0.604	0.421	0.311	0.235	0.179	0.134	0.093	0.0
RT_K	2.107	1.031	0.673	0.494	0.388	0.319	0.274	0.249	0.278	–
$RT_{u=0.576}$	2.106	1.028	0.668	0.486	0.377	0.302	0.248	0.205	0.169	0.124
$RT_{u=1/2}$	2.090	1.012	0.651	0.469	0.359	0.284	0.228	0.184	0.144	0.0
RT_{K2}	2.104	1.023	0.660	0.476	0.364	0.286	0.228	0.180	0.136	0.0

As can be seen in Table 1, the last four reverberation formulae are equivalent within 5% accuracy up to $\alpha = 0.5$. Note that $RT_{u=0.576}$ suffers from the same drawback as Sabine formula: it gives positive reverberation time for full absorption.

3 2D AND 3D ERGODIC ENCLOSURES

We now turn on to more complex enclosure shapes, namely circular and rectangular enclosures.

3.1 Circular enclosures

In his paper on the effect of surface roughness on reverberation time, Joyce[2] explicitly developed the case of the 3D spherical enclosures, and gave qualitative results for the 2D circular enclosures. He was interested in the transition from random reflection – Lambert law – to specular reflection. Here, I only use the results for random reflection.

In that case, the mean free path is equal to $\bar{l} = \frac{4}{3}R$, where R is the radius of the sphere, and the mean quadratic path to $\overline{l^2} = 2R^2$, which gives a variance $\sigma^2 = \frac{2}{9}R^2$, or a relative variance of $\gamma^2 = \frac{\overline{l^2} - \bar{l}^2}{\bar{l}} = \frac{1}{8} = 0.125$ according to Kuttruff[3]. The calculation is straightforward and is not reproduced here. In the case of the circular disc of radius R, these values are respectively $\bar{l} = \frac{\pi}{2}R$, $\overline{l^2} = \frac{8}{3}R^2$, $\sigma^2 = \left(\frac{8}{3} - \frac{\pi^2}{4}\right)R^2$, or a relative variance of $\gamma^2 = \left(\frac{32}{3\pi^2} - 1\right) = 0.0807 \approx \frac{1}{12.4}$. Note that mean free paths and mean quadratic paths must be converted to time with the help of the speed of sound c, taken as 340 m/s in the calculations.

Table 2 gives the same reverberation times as Table 1, calculated for a disc for different values of the mean absorption; and Table 3 the same data for the sphere. In both case, the mean free path was kept to 5.3 m, as in Table 1. Thus, Sabine's and Eyring's reverberation times are the same as for the stadion (Table 1). However, the values of u have to be adapted to each case: for the disc, $u = 0.0807$ leads to $N(u) = 794.3$; but choosing $u = 1/13 = 0.0769$ leads to the same value of $N(u) = 794.3$ with $m = 0.0608$. For the sphere, $u = 0.125$ leads to $N(u) = 512.8$; and choosing $u = 1/9 = 0.111$ leads to the same value of $N(u) = 512.8$ with $m = 0.0608$.

Table 2: reverberation times for a disc of radius 3.37 m.

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
RT_S	2.153	1.076	0.718	0.538	0.431	0.359	0.308	0.269	0.239	0.215
RT_E	2.043	0.965	0.604	0.421	0.311	0.235	0.179	0.134	0.093	0.0
RT_K	2.052	0.97	0.61	0.43	0.32	0.244	0.188	0.143	0.103	–
$RT_{u=0.0807}$	2.052	0.974	0.612	0.430	0.319	0.244	0.188	0.143	0.102	0.017
$RT_{u=1/13}$	2.051	0.973	0.612	0.429	0.319	0.243	0.187	0.142	0.102	0.0
RT_{K2}	2.052	0.973	0.612	0.430	0.319	0.243	0.187	0.142	0.102	0.0

Table 3: reverberation times for a sphere of radius 4 m.

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
RT_S	2.153	1.076	0.718	0.538	0.431	0.359	0.308	0.269	0.239	0.215
RT_E	2.043	0.965	0.604	0.421	0.311	0.235	0.179	0.134	0.093	0.0
RT_K	2.057	0.978	0.617	0.435	0.325	0.249	0.193	0.149	0.109	–
$RT_{u=0.125}$	2.057	0.978	0.617	0.435	0.324	0.249	0.193	0.148	0.108	0.027
$RT_{u=1/9}$	2.054	0.975	0.614	0.432	0.321	0.246	0.190	0.145	0.104	0.0
RT_{K2}	2.057	0.978	0.617	0.435	0.324	0.248	0.191	0.146	0.105	0.0

In this case of low relative variances, the last four formulae remain very close to each other up to $\alpha = 0.9$. Note that Kuttruff's formula only diverges for $R = (1 - \alpha) < 1.72\text{x}10^{-11}$ for the disc, and $R < 1.125\text{x}10^{-7}$ for the sphere, which are almost zero.

As pointed out by Joyce[2], the advantages of the disc and the sphere are that computing mean free paths and mean quadratic paths is straightforward. On the other hand, their disadvantages are that the relative variance γ^2 cannot be varied. This is why rectangular enclosures are now considered.

3.2 Rectangular enclosures

The cases of 2D and 3D rectangular enclosures require more complex integration. Basically, a two-step integration procedure is used for 2D rectangular enclosures, first on directions for rays starting on a running point on one side, taking into account a Lambert distribution of direction (cosine law), then on the running point along the side. The procedure must be repeated on the four sides of the rectangle. For 3D rectangular enclosures, a four-step integration procedure is used, first on a rectangular slice of the parallelepiped, using the same steps as for 2D enclosure, then along the direction perpendicular to the slice, and finally rotating the result around azimuths. All results are analytical, except for one integral in the last step of the 3D integration.

This procedure does recover the expected mean free paths in both 2D and 3D cases, namely $\bar{l} = \frac{\pi L h}{2(L+h)}$ in the 2D case, and $\bar{l} = \frac{4Llh}{2(Ll+lh+hL)}$ in the 3D case, where L, l, and h are respectively the length, the width, and the height of the enclosure. The mean quadratic paths take much more complex expressions: in the 2D case, it becomes:

$$\overline{l^2} = \frac{2}{3(L+h)}\left[(L^2+h^2)^{\frac{3}{2}} - (L^3+h^3)\right] + \frac{h^2}{(L+h)}\left[L\,ln\left(\frac{L-h+\sqrt{L^2+h^2}}{L+h-\sqrt{L^2+h^2}}\right) - \frac{L^2}{\sqrt{L^2+h^2}-h} + 2h\right] + \frac{L^2}{(L+h)}\left[h\,ln\left(\frac{h-L+\sqrt{L^2+h^2}}{h+L-\sqrt{L^2+h^2}}\right) - \frac{h^2}{\sqrt{L^2+h^2}-L} + 2L\right]$$

Computation of the 3D case is under current development. Note that the 2D expression for l^2 diverges as $h^2 ln(2\,L/h)$ for large values of the ratio (L/h); in other words, the very long paths dominate for elongated enclosures.

Tables 4 and 5 present the same reverberation times as in Table 1, respectively for a square ($\gamma^2 = 0.205$), and a rectangle of ratio 1:10 ($\gamma^2 = 0.353$). Note that much larger γ^2 values are obtained that in the case of the disc and the sphere. In fact, any ratio can be achieved, provided the rectangle is elongated enough. Note also that Kuttruff's formula diverges for $R < 6.0\text{x}10^{-5}$ in the case of the square, and $\alpha > 0.997$, in the case of the rectangle of ratio 1:10.

Table 4: reverberation times for a square of 6.75 m long sides.

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
RT_S	2.153	1.076	0.718	0.538	0.431	0.359	0.308	0.269	0.239	0.215
RT_E	2.043	0.965	0.604	0.421	0.311	0.235	0.179	0.134	0.093	0.0
RT_K	2.066	0.987	0.627	0.445	0.334	0.259	0.204	0.160	0.122	–
$RT_{u=0.205}$	2.065	0.987	0.626	0.444	0.333	0.258	0.202	0.157	0.117	0.044
$RT_{u=1/5}$	2.064	0.986	0.625	0.443	0.332	0.257	0.201	0.156	0.116	0.0
RT_{K2}	2.065	0.986	0.625	0.442	0.331	0.255	0.199	0.153	0.112	0.0

Table 5: reverberation times for a rectangle of ratio 1:10 (short side 3.7 m long).

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
RT_S	2.153	1.076	0.718	0.538	0.431	0.359	0.308	0.269	0.239	0.215
RT_E	2.043	0.965	0.604	0.421	0.311	0.235	0.179	0.134	0.093	0.0
RT_K	2.082	1.004	0.644	0.463	0.354	0.280	0.227	0.187	0.157	–
$RT_{u=0.353}$	2.082	1.003	0.642	0.461	0.350	0.275	0.219	0.175	0.137	0.076
$RT_{u=1/3}$	2.077	0.999	0.638	0.456	0.346	0.270	0.215	0.170	0.131	0.0
RT_{K2}	2.081	1.001	0.639	0.456	0.345	0.268	0.211	0.165	0.122	0.0

In these cases of larger relative variances, the differences between the last 4 reverberation formulae are much larger. They reach more than 10% for $\alpha = 0.9$.

For 3D parallelepipeds, Kuttruff[3] has carried out Monte-Carlo simulations for some special case of relative room dimensions. The numerical mean free paths are very close to the theoretical values; and γ^2 varies between 0.342 for a cube to 0.613 for a flat room of ratio 1:10:10, which are roughly twice as large values as for the corresponding rectangular rooms. Tables 6, 7 and 8 present the same reverberation times as in Table 1, respectively for the cube, ratio 1:2:5 ($\gamma^2 = 0.403$), and ratio 1:10:10. In these case, Kuttruff's formula diverges for α larger than 0.997, 0.993, and 0.962 for respectively the cube and the rooms of dimensions 1:2:5 and 1:10:10.

Table 6: reverberation times for a cube of 8 m long edges.

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
RT_S	2.153	1.076	0.718	0.538	0.431	0.359	0.308	0.269	0.239	0.215
RT_E	2.043	0.965	0.604	0.421	0.311	0.235	0.179	0.134	0.093	0.0
RT_K	2.081	1.003	0.643	0.462	0.352	0.279	0.225	0.185	0.154	–
$RT_{u=0.342}$	2.080	1.002	0.641	0.459	0.349	0.274	0.218	0.174	0.135	0.074
$RT_{u=1/3}$	2.079	1.000	0.639	0.457	0.347	0.272	0.216	0.172	0.132	0.0
RT_{K2}	2.080	1.000	0.638	0.456	0.344	0.267	0.210	0.164	0.122	0.0

Table 7: reverberation times for a room of ratio 1:2:5 (short edge 4.5 m long).

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
RT_S	2.153	1.076	0.718	0.538	0.431	0.359	0.308	0.269	0.239	0.215
RT_E	2.043	0.965	0.604	0.421	0.311	0.235	0.179	0.134	0.093	0.0
RT_K	2.088	1.010	0.650	0.470	0.361	0.288	0.236	0.198	0.174	–
$RT_{u=0.403}$	2.087	1.009	0.648	0.466	0.356	0.281	0.226	0.182	0.143	0.087
$RT_{u=1/3}$	2.073	0.995	0.634	0.452	0.341	0.265	0.209	0.165	0.125	0.0
RT_{K2}	2.086	1.006	0.644	0.461	0.349	0.272	0.215	0.168	0.126	0.0

Table 8: reverberation times for a flat room of ratio 1:10:10 (3.2 m high).

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
RT_S	2.153	1.076	0.718	0.538	0.431	0.359	0.308	0.269	0.239	0.215
RT_E	2.043	0.965	0.604	0.421	0.311	0.235	0.179	0.134	0.093	0.0
RT_K	2.112	1.036	0.678	0.500	0.394	0.327	0.283	0.264	0.318	–
$RT_{u=0.613}$	2.110	1.032	0.672	0.491	0.381	0.307	0.253	0.210	0.175	0.132
$RT_{u=1/2}$	2.087	1.009	0.648	0.466	0.356	0.280	0.224	0.180	0.140	0.0
RT_{K2}	2.107	1.027	0.664	0.479	0.367	0.289	0.230	0.182	0.138	0.0

As the relative variance increases further in these cases, the last four reverberation formulae diverge at larger absorption. However, the last two formulae agree within 2% at all absorptions.

4 DISCUSSION

4.1 Influence of relative variance

In the previous examples, care was taken that the mean free path and the mean waiting time were kept to the same values. As a consequence, Sabine and Eyring reverberation times are the same for all enclosures. The other reverberation times vary, as they depend on the relative variance γ^2. However, the dispersions are less than 5% within one same enclosure for the usual range of absorptions ($\alpha \leq 0.5$), and increase with γ^2 for large absorption coefficients, where they can reach more that 20%.

Note that Kuttruff reverberation time diverges for very small values of the reflection coefficient R – large absorption. But for practical applications, this only happens for mean absorption coefficient larger than 0.9. Only disproportionate rooms, with $\gamma^2 > 0.86$, reach this divergence for $\alpha < 0.9$; for a rectangular 2D room, it corresponds to a dimension ratio larger than 45:1, which indeed has no practical use.

4.2 Validity for large absorption

The different reverberation formulae presented in this paper are only valid for ergodic room, as they are all based on the asymptotic distribution of reflections along one single ray. Indeed, only one ray needs being considered in ergodic room, as any randomly chosen starting position evolves in a ray that eventually comes infinitesimally near any other combination of position and direction. As the trajectories of the rays do not depend on the value of the reflection coefficient, the distribution of reflections is independent of absorption. As a consequence, the reverberation formulae are valid for all absorption values.

It should be note, however, that the formulae make use of the *asymptotic* distribution of reflections. It can therefore happen that energy along the ray has decayed almost to zero before the asymptotic distribution is approached. This typically happens in the case of very large absorptions. In other words, reverberation formulae, although still valid in principle, cannot be applied to very large absorptions. This seriously reduces the importance of the divergence of Kuttruff's formula discussed in the previous section.

4.3 Non-uniform absorption

The case of non-uniform absorption derives from the previous discussion: as long as the asymptotic distribution of reflections is attained before energy along the ray has decayed almost to zero, the different formulae studied in this paper are valid. One must only attribute a certain probability to each absorption coefficient, and when absorption is not angle dependent, the obvious choice is the proportion of boundary surface corresponding to each absorption coefficient. With this choice, a mean absorption coefficient is readily obtained, with which the different formulae can be computed.

5 REVISITING KUTTRUFF'S FORMULA

In Sect. 2.2, I explained that Kuttruff has chosen a different approach than starting with the probability distribution $P(n|t)$ of reflections along a ray during the time span t. Instead, he starts with the probability distribution $P(t|n)$ of travelling times t spanning n reflections. The central theorem of probability leads him to postulate a Gaussian probability distribution for $P(t|n)$:

$$P(t|n) = \frac{1}{\sqrt{2\pi n}\,\gamma\bar{t}} e^{-\frac{(t-n\bar{t})^2}{2n\gamma^2\bar{t}^2}}$$

This distribution can also be considered as equal to $P(n|t)$, but Kuttruff has then some problem for deriving its cumulant function, and therefore replaces n by its mean value $t/\bar{t}$ in the denominators.

In fact, the cumulant function of this distribution of n given t can be exactly computed, under the assumption that n is a continuous variable and not a discrete one. One obtains:

$$\psi(z,t) = ln\left(\int_0^\infty e^{izn}\,P(n|t)dn\right) = -t\frac{\sqrt{1+2\gamma^2 iz}-1}{\bar{t}\gamma^2}$$

When replacing iz with $ln(R)$, one obtained a 'revised' Kuttruff formula:

$$RT_{K2} = \frac{13.8\,\bar{t}\gamma^2}{\sqrt{1-2\gamma^2 ln(R)}-1}$$

which reduces to Kuttruff's original formula to the second order in $ln(R)$ for small reflection coefficients. However, compared to the original formula, the revised formula does not diverge for large reflection coefficients.

The revised Kuttruff formula has been inserted as the last line of Tables 1 to 8. However, I did not check whether the revised formula reduces to the general form of Lévy process, though I suspect it is the case.

6 CONCLUSION

This study of the general form for a reverberation formula was started in the hope of obtaining formulae piloted by dimension. When this idea turned out to be unsuccessful, the profile of the probability distribution for short free paths was investigated. Indeed, this probability is proportional to path length for very short free paths, therefore vanishes for null paths, contrarily to most published distribution of free paths, for example in Kuttruff[3]. It was thus assumed that Sabine's formula, that relies on a strictly positive probability for null paths, could not be the most general reverberation formula and had to be replaced by a revised reverberation formula, the formulation of which depends on the standard deviation of the free paths. This is indeed the path followed in this paper.

Assuming that the asymptotic distribution of reflections along one single ray controls reverberation, which is the case for ergodic enclosures[1,2], the generic decay rate for such enclosures is presented in Sect. 2.1. It is then simplified according to different assumptions, and the corresponding reverberation times are presented for a few enclosures with simple shape, both in 2D and 3D cases. Disappointing are the small variations between shape and dimensions, as the piloting factors are the mean waiting time and its relative variance. Thus, reverberation does not depend on dimension.

There remain to check the different formulae with Monte-Carlo simulations, following Kuttruff's footsteps. This will be the topic of a forthcoming publication.

In *conclusion*, the present study proves once more that Sabine's and Eyring reverberation formulae are crude approximations that do not take into account the shape of the enclosure. It is however possible to improve these formulae, taking into account the relative variance of the free paths, or equivalently, the waiting time between two successive reflections. Almost all improved formulae are compatible to the generic model of a Lévy process for reflection distribution along any ray; and they deliver very similar results as long as the enclosure is not too disproportionate. It is therefore not possible to recommend one specific formula, either for reverberation chambers or for auditoria, but only that both Sabine's and Eyring's formulae should be avoided.

7 REFERENCES

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