FINITE ELEMENT MODELLING OF THE PERFORMANCE OF A HIGH FREQUENCY HYDROPHONE

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1. INTRODUCTION

With the development of underwater positioning and image systems, there is a requirement for a high quality measuring hydrophone to cover the frequency range 100 – 500 kHz. Likewise, a stable reference hydrophone working over this frequency range is required for the dissemination of traceable standards for pressure. The required technical specifications for this kind of hydrophone have been described elsewhere [1] together with a review of the possibility of using different piezoelectric materials, such as piezoelectric composites and piezoelectric polymers, as the active element.

In order to understand and optimise the performance of a hydrophone it is important to have a realistic theoretical model of its behaviour. This paper describes the numerical modelling of a simple hydrophone design based on a lead metaniobate slab 1 x 1 x 0.4 mm in size encapsulated in a polyurethane rubber (PR1570) cylinder of diameter 3 mm. Initial measurement of the sensitivity and directionality of this type of design revealed some unexpected characteristics. The numerical modelling has been used to provide an understanding of these experimental results and to predict the effect of different geometries and encapsulants on the response of the hydrophone. This work shows that finite element techniques may be profitably applied to the analysis of high frequency devices as well as to the analysis of low frequency devices.

There are a number of general purpose finite element software packages commercially available that are capable of modelling the acoustics and vibration of a submerged piezoelectric structure. These can be classified into one of two approaches depending on how they model the acoustic propagation. The first approach is to use a combined finite element and boundary element technique such as that available in PAFEC; the other approach is to model the fluid by finite elements and surround these by absorbing boundary elements such those available in ANSYS. The theories of these two approaches have been well documented in, for instance, references [2] and [3]. An evaluation and comparison of the capability of these codes to model a hydrophone has been performed by using PAFEC and ANSYS to model the acoustics and vibration of a submerged piezoelectric slab [4] as part of a long term project. This paper mainly describes the use of a 2-D model in ANSYS since this code contains a two dimensional piezoelectric element which is not available in the PAFEC software package.

2. MODELLING STRATEGY

2.1 General

The 2-D finite element model is designed to correspond to the cross-sectional form of the hydrophone (see Figure 1). Physically, the 2-D model assumes that the length of the ceramic sensor and the coating material are infinite along the direction of hydrophone length (z). In practice, the hydrophone's ceramic slab is only 1 mm long so that 2-D modelling will provide only an approximate simulation of the performance of the practical hydrophone. When compared with 3-D modelling, however, 2-D modelling has two advantages: firstly, it avoids truncation effects resulting from the cutting of the finite element model at some arbitrary length (since it is not realistic to model the whole hydrophone and support), and secondly, it reduces the size of finite element model and enables parametric studies to be carried out efficiently.

The finite element model was meshed using piezoelectric, solid and fluid elements respectively, with the interaction between the structure and fluid being included by using interface fluid elements. The symmetry of the design means that it is only necessary to model half the cross-section of the hydrophone with a symmetric boundary condition being applied to the line y=0. The basic geometrical parameters of the model used are as follows: slab thickness t=0.4 mm, slab width w=1.0 mm, radius of coating material r_1=1.5 mm and radius of fluid area r_2=10.0 mm. The piezoelectric material was considered to be lead metaniobate made by the Unilator Division of Morgan Matroc Limited. The basic parameters are as below: $s_{11}=17.4\times10^{-12}$, $s_{12}=-4.52\times10^{-12}$, $s_{13}=-5.82\times10^{-12}$, $s_{33}=14.4\times10^{-12}$, $s_{66}=43.84\times10^{-12}$ m^2N^{-1}, $d_{31}=-9.5pCN^{-1}$, $d_{33}=85$ pCN^{-1}, $\varepsilon = 320$ and $\rho = 5900$ kgm^{-3}. The density and sound speed of the surrounding fluid (water) were taken to be 1000 kgm^{-3} and 1460 ms^{-1} respectively. The elastic properties of the coating material PR1570 are dependent on the excitation frequency [5] and are discussed further in section 2.4.

2.2 Absorbing boundary condition

The ANSYS code does not currently contain boundary elements. The effect of an infinite fluid was, therefore, modelled by the use of a finite layer of fluid and an absorbing boundary condition. Physically, this absorbing condition corresponds to a matched impedance for a plane wave normally incident on the boundary. This absorbing condition is far from perfect if the incident wave is not a plane wave or the boundary is curved. In order to achieve a better performance with this absorbing boundary condition the boundary has to be placed at a reasonably large distance from the radiator. However, this distance is limited by the capability of computer and software package, since the total number of elements in the model increases very quickly with this distance. In addition the fluid radius needs to be chosen so that it satisfies the farfield radiation condition for the hydrophone. For the work described in this paper, the outside radius of the fluid was set to be 10 mm after comparing results for radii of 7.5 mm, 10 mm, 15 mm and 20 mm. The use of different outside radii resulted in changes in hydrophone sensitivity of about $\pm$ 0.6 dB.

2.3 Sensitivity of hydrophone

The definition of the sensitivity of a hydrophone implies that a finite element model needs to calculate the response of a hydrophone to an incident plane wave. The absence of a boundary element facility means that this is difficult to achieve directly with ANSYS. Consequently an indirect approach has been adopted that makes use of the reciprocity principle. The transmit response of the hydrophone is obtained first, and then the cylindrical reciprocity factor is used to calculate the receive sensitivity. In this paper, the sensitivity of hydrophone (M) is derived from

$$M\ (dB) = T + 20 \log J_s - 240, \text{ re. } 1\ V\mu Pa^{-1},$$

where T is the transmission response in the cylindrical field

$$T = 20 \log(p r^{1/2} / I),$$

p is the pressure in the q direction at the distance r, I is the excitation current applied to the ceramic sensor and J_s is the reciprocity constant for a cylindrical field given by

$$J_s = 2\ \lambda^{1/2}/(\rho c).$$

2.4 Properties of the coating material

Like other polyurethane materials, the elastic properties of coating material PR 1570 are dependent on the excitation frequency. In order to include this effect in the finite element model, the ANSYS parametric design language was used to create a batch file which modified the elastic constants of the coating material in the finite element database as the frequency changed. The empirical expressions for the elastic properties were based on experimental measurements of the elastic constants of PR1570 [6]. The approximations used to describe the changes in Young's modulus E, Poisson's ratio σ and loss tangent δ over the given frequency range $[f_0 f_1]$ were

$$E = E_0 + (E_1 - E_0)(\log(f) - \log(f_0))/(\log(f_1) - \log(f_0)),$$

$$\sigma = \sigma_0 + (\sigma_1 - \sigma_0)(\log(f) - \log(f_0))/(\log(f_1) - \log(f_0))$$

and

$$\delta = \delta_0 - (\delta_1 - \delta_0)(\log(f) - \log(f_0))/(\log(f_1) - \log(f_0))$$

where f_0 and f_1 are the start and end frequencies, and E_0, E_1, σ_0, σ_1, δ_0 and δ_1 are the start and end values of Young's modulus, Poisson's ratio and loss tangent respectively. The basic data used in the modelling to be described were $E_0 = 2.5 \times 10^8$, $E_1 = 2.7 \times 10^8$, $\sigma_0 = 0.48$, $\sigma_1 = 0.486$, $\delta_0 = 0.1$ and $\delta_1 = 0.09$. These were the estimated values for the elastic constants for PR1570 at about 22 °C. *In the following numerical modelling, the basic data for the geometry and material parameters will be used except when otherwise stated in the figure caption.*

3. COMPARISONS WITH 3-D MODELLING AND EXPERIMENTAL RESULTS

In order to verify the 2-D finite element model, a 1 x 1 x 0.4 mm lead metaniobate slab without the coating material was used as a test case for comparison with a full 3-D model where the sensitivity and directionality results have already been obtained using PAFEC [7]. The slab sizes and the finite element meshes were kept the same as those used in the following calculations so that the accuracy of the meshes was also tested. The directionality results for a 2-D uncoated slab are shown in Figure 2 where the 0^0 direction is the polarising direction (x) of the slab. The results for sensitivity are shown as a function of polar angle θ (along the horizontal axis) and frequency (over the range 100-500 kHz in steps of 10 kHz) along the vertical axis. The symmetry of the model means that results are given for θ over the range 0^0-90^0. The sensitivity is plotted on a decibel scale (see the legend beside the figure for values). The typical directionalities obtained by ANSYS and PAFEC are shown at frequencies 250 and 500 kHz in Figure 3 in polar co-ordinates. The maximum difference of the normalised amplitude between 2-D and 3-D modelling is about 1 dB. Figure 4 shows the comparison of the sensitivity results using ANSYS and PAFEC. The small fluctuations (about 0.5 dB) in the sensitivity curve obtained using ANSYS are attributed to the reflection of waves from the imperfect outer absorbing boundary of the finite element model. The sensitivity obtained by 2-D modelling is 0.5 - 3 dB higher than that obtained by 3-D modelling and the difference increases with frequency.

These modelling results have also been compared with experimental results obtained for a test device by the National Physical Laboratory (which will be presented elsewhere). Considering the physical difference between the finite element model and the practical hydrophone, the modelling results obtained are in general in close agreement with the experimental results. Most of features observed experimentally are seen in the numerical modelling. Together these results show that the 2-D model is a good approximation to the practical device. A parametric study has, therefore, been carried out with this 2-D model to explore the sensitivity of this simple design to design parameters.

4. PARAMETRIC STUDIES AND DISCUSSIONS

4.1. Effects of the elastic and piezoelectric constants of ceramic material

The elastic and piezoelectric constants of ceramics can change between different batches of production, and also change slightly with different poling and processing conditions. The largest changes can be of the order of 20%, 5 % and 10% on the quoted values for dielectric, piezoelectric and elastic compliance constants. A wide range of modelling calculations showed that the directional response of the current 2-D model is not sensitive to changes in the elastic and piezoelectric constants if these changes are less than 20% of the manufacturer's values. This is illustrated in the polar plots of Figure 5 which represent the directional response at a number of typical frequencies for different elastic and piezoelectric constants. In these plots 'Standard' in the legend indicates that the manufacturer's values were used; 'Change d' means that d_{31} and d_{33} were reduced by 15 % and 20 %; 'Change s_{33}' means that s_{33} was increased by 20%; 'Change s_{11}' means that s_{11} was increased by 20% and 'Change s_{12}' means that s_{12} and s_{13} were increased by 20% compared with the standard values. The corresponding sensitivities of the 2-D model of the hydrophone under the same conditions are shown in Figure 6. Decreasing d_{31} and d_{33} reduces the sensitivity by about 1 dB as would be expected. The changes in elastic constants have little influence on the sensitivity results.

4.2. Effects of the elastic constants of the coating material

The effect of a change in the damping constants on the sensitivity are shown in Figure 7 where δ_0 and δ_1 are increased from 0.1 and 0.09 (solid line) to 0.18 and 0.17 (dashed line). The corresponding effects of changing Young's modulus are shown in Figure 8. Increasing the damping factor can significantly change the shape of sensitivity curve, especially at higher frequencies, where it reduces the sensitivity. Reducing Young's modulus mainly results in a shift of the sensitivity curve towards lower frequency. A small increase in Poisson's ratio (σ = 0.485 ~ 0.491) has little effect on sensitivity (see Figure 7; circle-dotted line).

It should be noted that changes in the properties of the coating material can lead to larger changes of sensitivity and directionality than changes in the elastic and piezoelectric constants of the sensor slab for the current hydrophone model. However, this phenomenon is believed to be partially due to the geometry of the current hydrophone in which the cross-sectional area of the coating material is much larger than that of the ceramic. This means that the dynamic characteristics of this model are dominated by the coating material.

In addition, the coating material PR1570 is an isotropic viscoelastic material which has a much larger loss tangent for its shear modulus than for its bulk modulus. The commercial finite element packages are, however, principally designed for working with normal elastic materials which have approximately equal damping constants for the shear and bulk moduli. This leads to a significant difficulty when modelling the dynamic behaviour of viscoelastic materials since only one damping constant is available in the finite element software package. Thus either shear motions are under-damped or compressional motions overdamped. This feature is observed when adjusting the damping constant of the coating material in the finite element model to fit the experimental data over the frequency range of interest.

4.3 Effects of geometrical parameters

The effects of changing the geometry of the hydrophone on the directional response and the sensitivity have also been investigated. Two aspects of the model have been studied; firstly the radius of the coating material and secondly the dimensions of the ceramic slab.

The directional response of the standard model is shown in Figure 9 for comparison with Figure 10 which shows the directional response when the coating layer is reduced to a radius of 1.25mm. The sensitivities of the hydrophone for models with diameters of 1.0, 1.25, 1.5, 1.75 and 2.0 mm are shown in Figure 11. Both the directionality and the sensitivity are dependant on the radius of the coating material.

The effects of the width of the ceramic slab on the directionality are shown in Figure 12, where the width used is 1.8 mm, and on the sensitivity are shown in Figure 13, where the widths used are 1.0, 1.4 and 1.8 mm respectively. The effects of the thickness of the ceramic slab on the directionality are show in Figure 14, where the thickness is 1.0 mm, and on the sensitivity in Figure 15, where results are compared for thicknesses of 0.4, 0.7 and 1.0 mm. As expected, the sensitivity increases as the slab thickness increases.

Overall, the geometrical parameters of the hydrophone model have a large influence on the sensitivity and the directional response because these produce first order changes in the dynamic characteristics. From the designer's point of view, suitable geometrical parameters and damping factors can be chosen to minimise the effect of the resonant modes of the hydrophone structure over the working frequency range.

5. CONCLUSIONS

A 2-D finite element model has been used to perform a comprehensive investigation of the directional response and sensitivity of a slab hydrophone with the effects of geometry and material parameters being explored. The applicability and capability of this 2-D model has been verified by comparing the results with those for a 3-D uncoated slab model. Parametric studies show that the elastic properties of the coating material and dimensions of hydrophone dominate the performance of this kind of design. It may be possible to optimise the performance of this simple design by adjusting the dynamic characteristics of the coating material. The work has indicated the need for improved finite element models that are capable of fully modelling the viscoelastic behaviour of materials with two damping constants and appropriate frequency dependant behaviour. The results also show that the finite element technique may now be applied effectively to high frequency devices.

6. ACKNOWLEDGEMENTS

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7. REFERENCES

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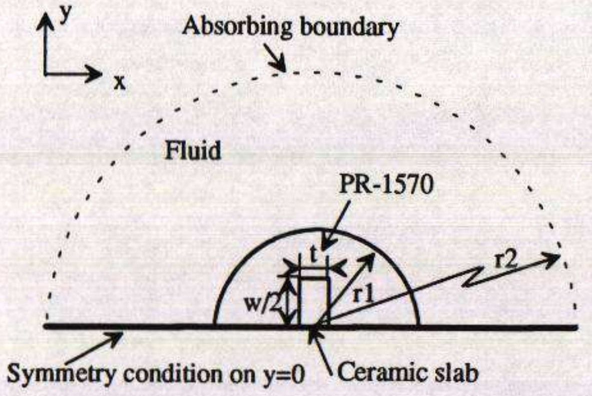

Figure 1. 2-D finite element model of hydrophone.

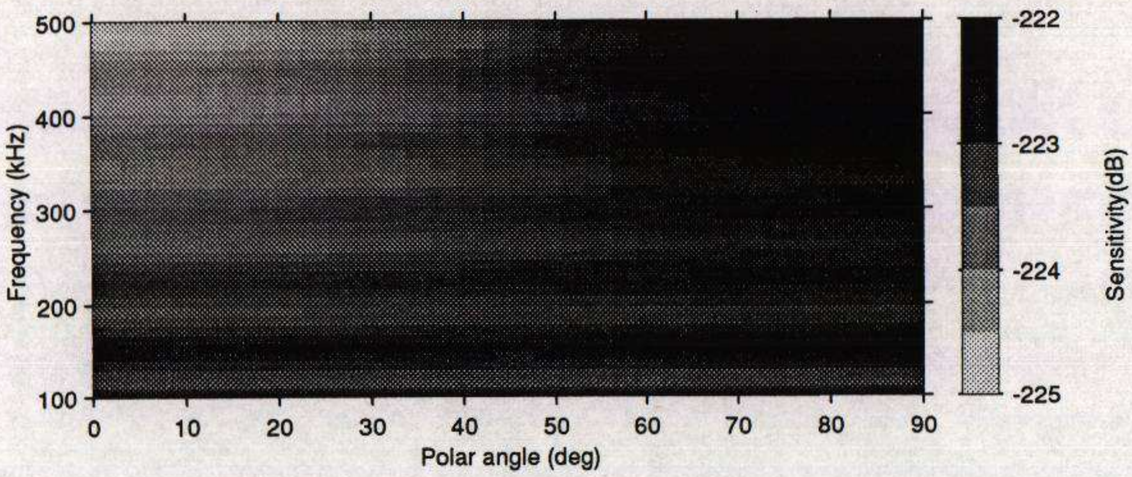

Figure 2. Directional response of a 1 x 1 x 0.4 mm slab, without coating material, obtained using a 2-D model in ANSYS.

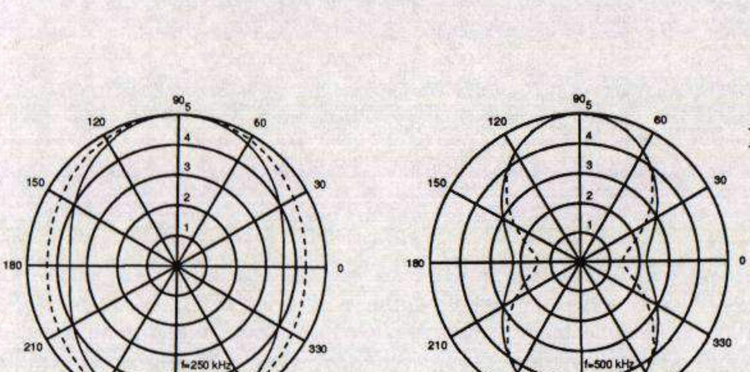

Figure 3. Comparison of the directional response of 1x1x0.4 mm slab without coating material, obtained using ANSYS and PAFEC; Solid line: ANSYS; Dashed line: PAFEC.

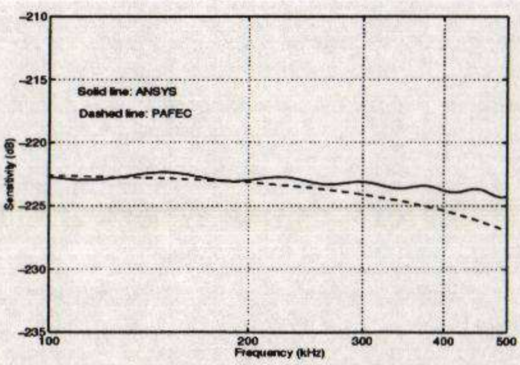

Figure 4. Comparison of the sensitivity of 1x1x0.4 mm slab, without coating material, obtained using ANSYS and PAFEC

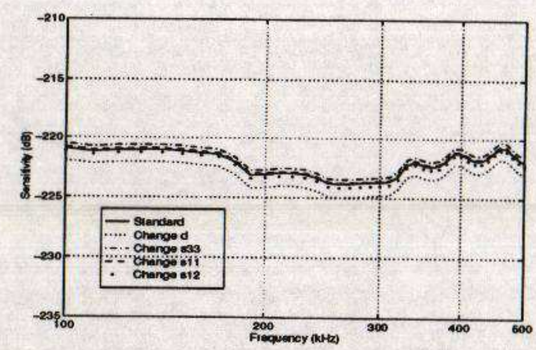

Figure 6. Comparison of the sensitivity of the hydrophone for different elastic and piezoelectric constants, where the legend is the same as in Figure 5.

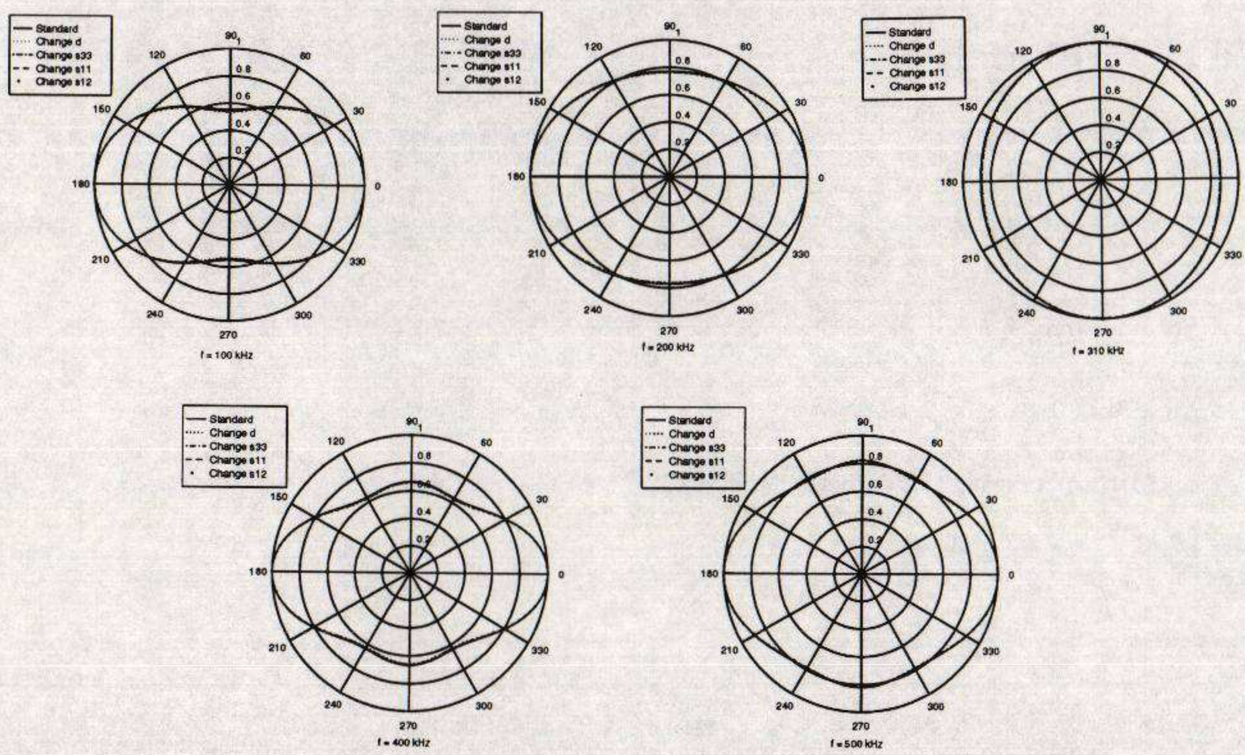

Figure 5. Comparison of the directional response of the hydrophone for different elastic and piezoelectric constants. 'Standard' in legend indicates that the basic values are used; 'Change d' means that d_{31} and d_{33} are reduced by 15 % and 20 %; 'Change s_{33}' means that s_{33} is increased by 20%; 'Change s_{11}' means that s_{11} is increased by 20% and 'Change s_{12}' means that s_{12} and s_{13} are increased by 20% .

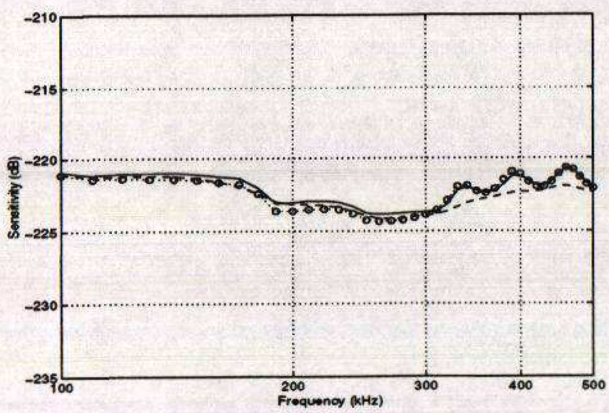

Figure 7. Comparison of the sensitivity of the hydrophone for different values of loss tangent δ and Poisson's ratio.
Dashed line: $\delta_0 = 0.18$ and $\delta_1 = 0.17$;
solid line: $\delta_0 = 0.1$ and $\delta_1 = 0.09$;
circle-dotted line: Poisson's ratio 0.485~0.491.

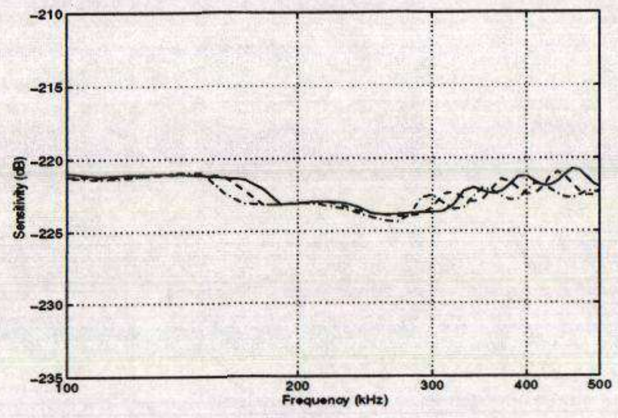

Figure 8. Comparison of sensitivity of the hydrophone for different values of Young's modulus. $E_0 = 2.5\times10^8$ and $E_1 = 2.7\times10^8$ (solid line); $E_0 = 2.2\times10^8$ and $E_1 = 2.4\times10^8$ (dashed line); $E_0 = 1.9\times10^8$ and $E_1 = 2.1\times10^8$ (dot-dashed line).

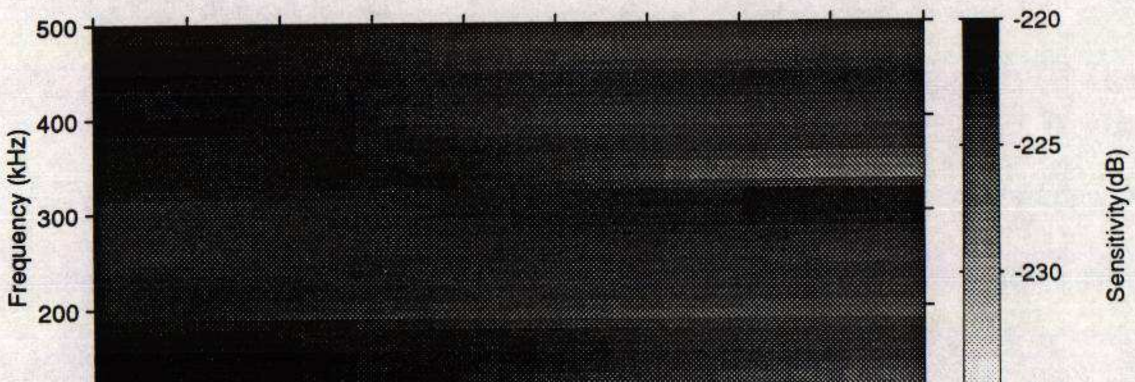

Figure 9. The directional response of the hydrophone model when the basic geometry and material parameters are used.

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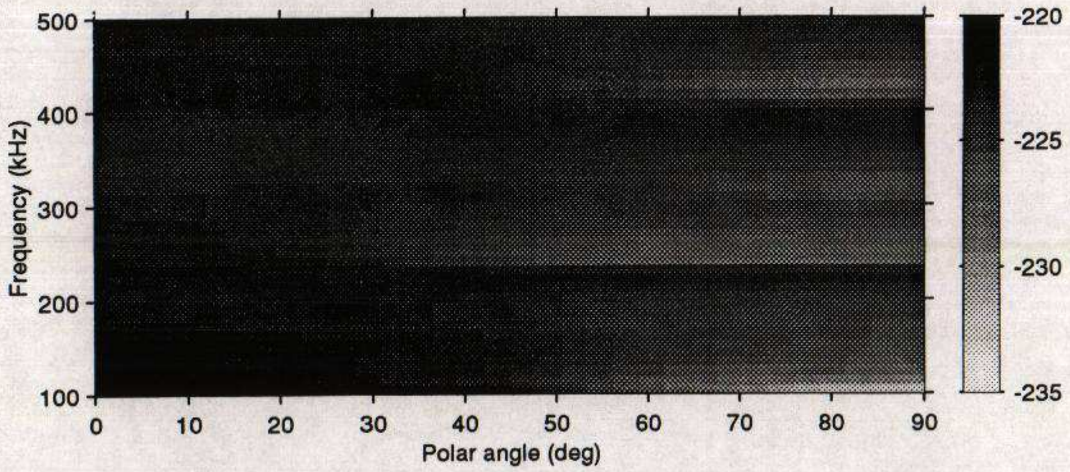

Figure 10. The directional response of the hydrophone model where the radius of the coating material is 1.25 mm.

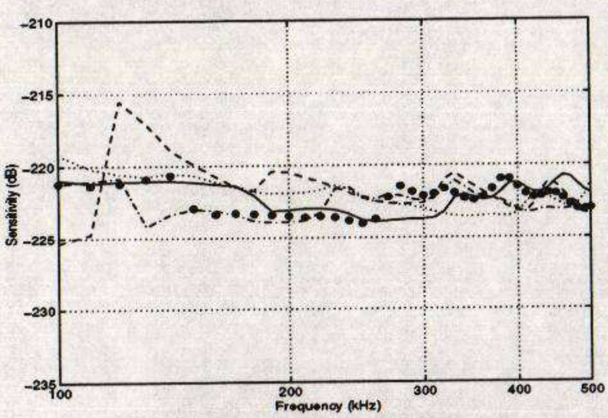

Figure 11. Comparison of the sensitivity of the hydrophone model for different radii of the coating material; solid line: 1.5 mm; dotted line: 1.25 mm; dashed line:1.0mm;dot-dashed line:2.0 mm; and circles: 1.75 mm.

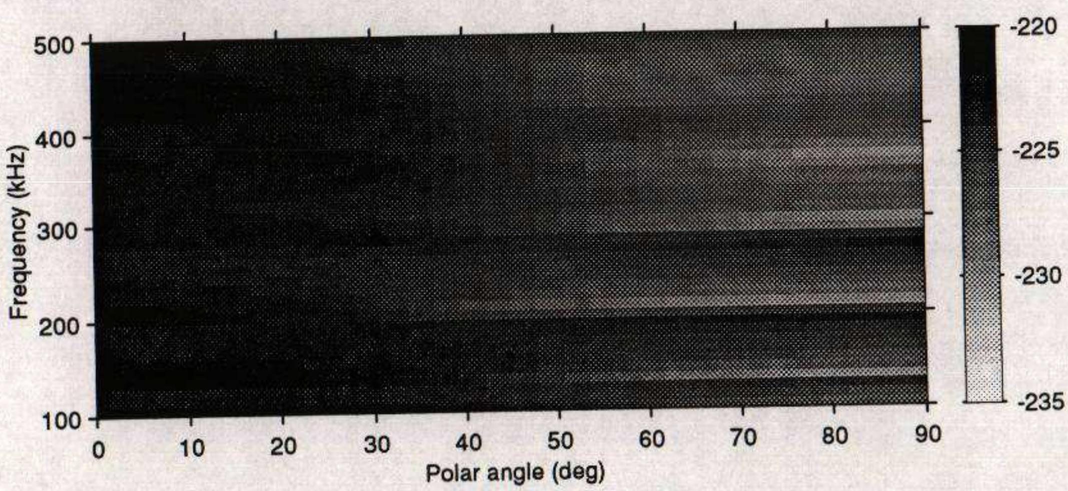

Figure 12. The directional response of the hydrophone model where the width of the slab is 1.8 mm.

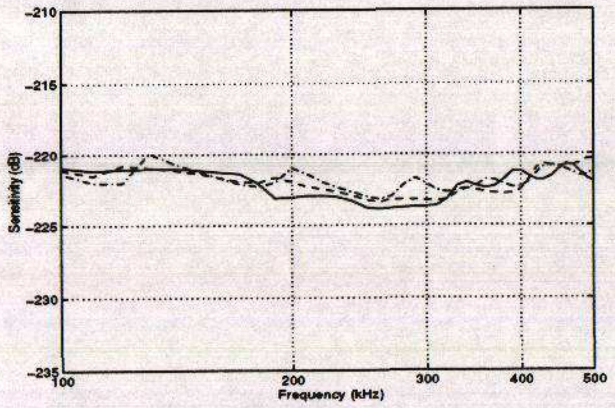

Figure 13. Comparison of the sensitivity of the hydrophone model for different widths of the piezoelectric slab; solid line: 1.0 mm; dashed line:1.4 mm and dot-dashed line 1.8 mm.

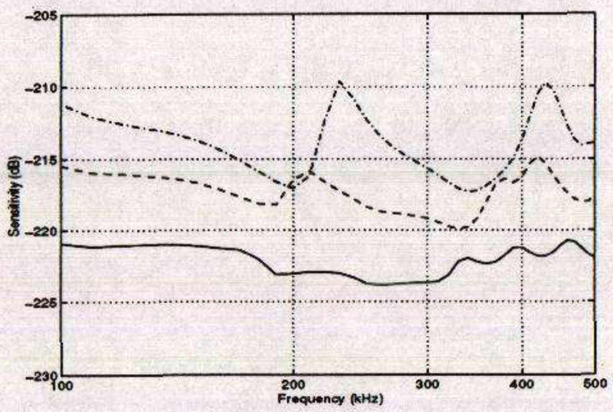

Figure 15. Comparison of the sensitivity of the hydrophone model for different thicknesses of the piezoelectric slab; solid line: 0.4 mm; dashed line: 0.7 mm; and dot-dashed line: 1.0 mm.

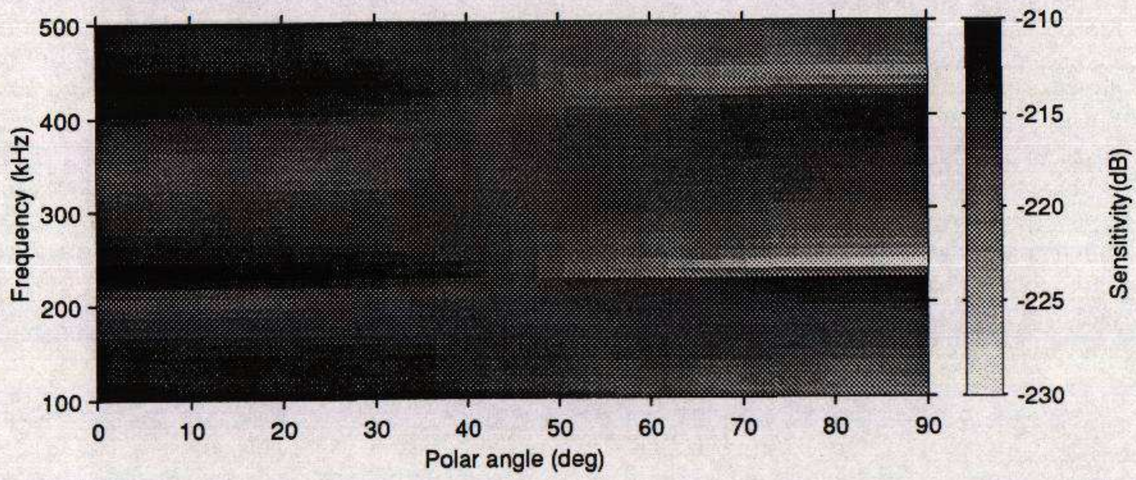

Figure 14. Directionality of the hydrophone model when the thickness of the slab is 1.0 mm.