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ASSEMBLING OF VIBRATING STRUCTURES WITH A NON-LINEAR INTERMEDIATE JUNCTION

J G Blomqvist

Dept of Applied Acoustics, Chalmers University of Technology, Gothenburg, Sweden

1. INTRODUCTION

Coupling of dynamic structures in order to obtain the dynamic behaviour of an assembly is a technique which is frequently used. This is a convenient approach if the assembly is to complicated as to allow a single grand model or if one wishes too combine different models or e.g. analytical calculations with experimental data. Usually the coupling is done in the frequency domain but can of course just as well be performed in the time domain. However, coupling structures in the frequency domain requires that the substructures to be assembled are linear and the same also yields for the intermediate structure (e.g. a rigid coupling). Far from all real constructions fulfils these requirement. Examples of non-linear systems are e.g. structures attached by friction couplings, snap-locks and systems with some gap. However, non-linear systems, can to a large extent, conceptually be divided into a linear part and a non-linear part. The assembling of non-linear systems can be addressed in the time domain and the method for doing this will here be demonstrated by a case study of two beams attached via a friction coupling. This type of problems has previously been used in calculating the velocity of a bowed string due to the contact between the string and a violin bow and in determining the oscillation of a clarinet and of a flute created by the flow through the mouthpiece [2], in analysing rattling noise in gear box of a car [3] and in describing the interaction between a tyre and the road surface [1].

2. CASE STUDY

The methodology of coupling structures with a non-linear element will here be demonstrated with a case study of two beams joined at one end with a friction type coupling and fixed at the other end as shown in fig. 1. Only longitudinal waves are considered in this non-stationary process were the excitation starts by applying a sinusoidal force at beam 1, away from the actual coupling point.

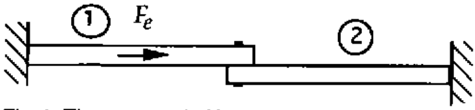

Fig. 1. The two coupled beams.

The characteristic function of the friction coupling is shown in fig. 2. The friction characteristics show that as long as the friction force is moderate the two beam ends stay rigidly connected but if the tangential force exceeds a critical value $f_t > F_{norm} \cdot \mu$ (F_{norm} is the normal force pressing the two beam ends together) then the coupling slips. A close up of the beam configuration at the coupling is shown in fig. 3 and gives the acting forces, the responses and the notation used.

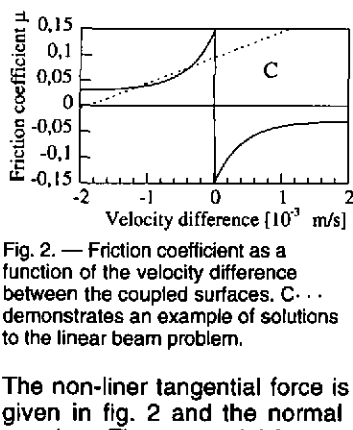

Fig. 2. — Friction coefficient as a function of the velocity difference between the coupled surfaces. C· · · demonstrates an example of solutions to the linear beam problem.

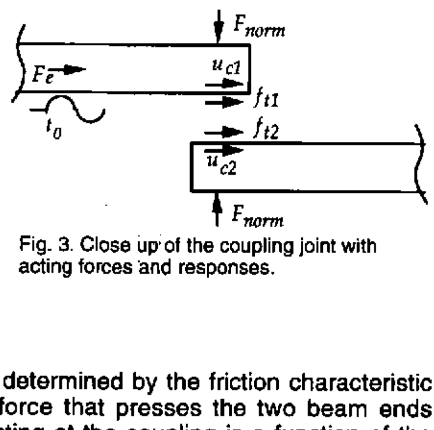

Fig. 3. Close up of the coupling joint with acting forces and responses.

The non-liner tangential force is determined by the friction characteristic given in fig. 2 and the normal force that presses the two beam ends together. The tangential force acting at the coupling is a function of the velocity difference between the two beams.

$$f_{t1}(t) = -F_{norm}\mu(u_{c1}(t) - u_{c2}(t)). \quad (1)$$

which can be described as a function of the type: $y = A\,f(x)$. The beam response at the coupling is determined by the forces acting on the system and the corresponding Green's functions. If the Green's functions for the beams are known, then the response at each side of the coupling can be expressed according to:

$$u_{c1}(t) = F_e(t) * g_{ue}(x_c, x_e, t) + f_{t1}(t) * g_{uf1}(x_c, t) \quad \text{and} \quad (2)$$

$$u_{c2}(t) = f_{f2}(t) * g_{uf2}(x_c, t). \quad (3)$$

where the asterix denotes convolution and $g_u(x_i, x_j, t)$ is the Green's function relating a velocity response to a force excitation. Eqs. 2 and 3 can be rearranged into expressing the velocity difference between the two beams at the connection point. Using the force equilibrium at the junction (no external forces applied at the junction) and expressing the velocity difference in discrete time, this becomes:

$$u_{c1}(N\Delta t) - u_{c2}(N\Delta t) = u_{c1}(N) - u_{c2}(N) = \sum_{n=0}^{N} F_e(n) g_{ue}(N-n)\Delta t$$
$$+ \sum_{n=0}^{N} f_{t1}(n) g_{uf1}(N-n)\Delta t + \sum_{n=0}^{N} f_{t1}(n) g_{uf2}(N-n)\Delta t. \tag{4}$$

It is now convenient to re-express the convolutions according to:

$$u(N) = F(N) g_u(0)\Delta t + \sum_{n=0}^{N-1} F(n) g_u(N-n)\Delta t. \tag{5}$$

In this expression the second part of the convolution represents the past or what has happened previously while the first part is the new information to be inserted in the system. At a particular time increment, the velocity difference of the beams at the coupling (eq. 4) can then be considered as a function of type: $y = Ax + B$ where B and A are constants. The solutions for the velocity difference of the linear problem can be demonstrated as a line with a constant slope which is shown in fig. 2 with a C. The displacement difference and the frictional force can then be obtained by solving eqs. 1 and 4 simultaneously (the intersections in fig. 2). The time increment is then advanced by a time step Δt and the process repeated.

3. RESULTS OF CASE STUDY

The investigated configuration is demonstrated with a sinusoidal excitation of 100 Hz and with two different normal forces. In the first case the normal force is chosen so that the two beams are effectively rigidly coupled while in the second case the normal force is decreased so that slipping occurs. The excitation starts at t_0=0 and the initial behaviour of the system is investigated. Note that all results in the time domain are normalised with the excitation frequency. In all time representations there is a small time lag in the beginning which corresponds to the time it takes for the waves to travel the distance between the excitation point and the coupling point.

Fig. 5 shows the tangential force over the first few periods. In case two one can see the initial effects due to the transient excitation (when the excitation begins) which cause the eigen frequencies of the system to oscillate. In case two the force shows the typical pattern which occurs when it "slides" back and forth on the friction characteristic curve. Fig. 6 points out the fact that no slipping occurs when the normal force is sufficiently high.

Fig. 5. Tangential force
for case 1 (······, F_{norm}=30 N)
for case 2 (——, F_{norm}=10 N).

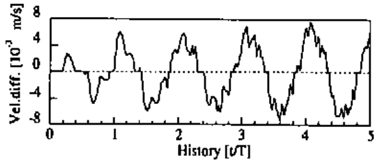

Fig. 6. Velocity difference between the beam ends at the coupling point
for case 1 (······, F_{norm}=30 N) and
for case 2 (——, F_{norm}=10 N).

Fig. 7 shows the velocity at the end of beam in the begining of the excitation. In fig. 8 the frequency spectra of the tangential force after approx. two seconds are shown. It can be seen here that in case one the initial effects have almost vanished (the peak at 250 Hz corresponds to the first natural frequency of the clamped system). In case two one can detect both harmonics and sub harmonics.

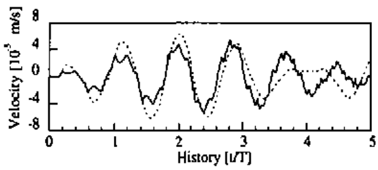

Fig. 7. Velocity on beam 2 at the coupling point
for case 1 (······, F_{norm}=30 N) and
for case 2 (——, F_{norm}=10 N).

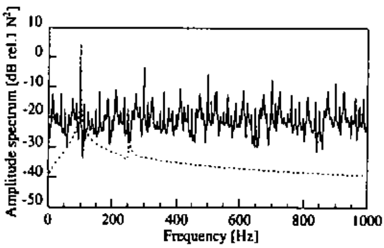

Fig. 8. Amplitude spectra of the tangential force, 2 seconds after the excitation has begun
for case 1 (······, F_{norm}=30 N) and
for case 2 (——, F_{norm}=10 N).

4. COMMENTS

Coupling structures in the time domain offers a possibility to solve many different types of non-linear problems. However, for real assemblies the most difficult task can often be to acquire the non-linear characteristics describing the coupling.

References

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