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MODELLING THE FRICTION OF ROSIN

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1. INTRODUCTION

Rosin is well known for its ability to excite stick-slip vibration on a violin string when applied to the hairs of a bow. All theoretical work up to now on the bowed string has assumed a very simple model for rosin tribology, in which the coefficient of friction depends only on the instantaneous relative sliding speed between the two contacting surfaces. However, there is no obvious physical reason for relative sliding velocity to be the only state variable which influences the coefficient of friction. Other aspects of the motion and its history seem likely to play a role. Rosin is a brittle solid at normal room temperatures, but it softens at temperatures only slightly higher. Viscosity (or more general visco-elasticity or visco-plasticity), adhesion, and brittle or ductile fracture might all play some part in the micro-mechanics of stick-slip processes. These in turn might be influenced by, for example, the distance of sliding (perhaps relative to the contact size or to a necking length-scale), or the thermal history around the contact zone.

Measurements have been made of dynamic friction force during stick-slip oscillation mediated by a rosin layer. These measurements show that the traditional friction model is inadequate. Possible constitutive laws will be proposed to describe the friction of rosin as it excites stick-slip vibration. It will be suggested that contact temperature plays an important role. Friction laws are developed by considering that the friction arises primarily from the shear of a softened or molten layer of rosin, with a temperature-dependent viscosity or shear strength. The temperature of the rosin layer is calculated by modelling the heat flow around the sliding contact. Comparisons are made with the measurements of friction. One of the temperature-based models is shown to produce reasonable agreement with the observed behaviour.

2. MEASUREMENTS

Dynamic friction force is not easy to measure directly under the required conditions. An approach which may be adopted instead is to excite stick-slip oscillation in a system which is well understood, so that one may infer the waveform of force from observations of the motion. The most obvious system for this purpose would be a damped harmonic oscillator, with equation of motion

$$m\ddot{x} + R\dot{x} + Sx = f(t). \qquad (1)$$

If the motion is measured, the force can be calculated by simple substitution (provided the system mass, stiffness and damping coefficient are known). This method has been applied in previous investigations of friction [1], but not to rosin, and with contact conditions which are very far from those applicable to a bowed string. The chosen design employed here is a cantilever arrangement, with a lumped mass on the end of a thin strip. It was machined from Nylon 66. The contact region for frictional excitation was generally of a different material — a small wedge of the desired material was glued in place for this purpose. The apparatus is shown in Fig. 1.

The cantilever was designed in such a way as to minimise the amplitudes of vibrations other than the fundamental bending mode. The frequency separation between the first and second bending modes was maximised. Also, to suppress both the second bending mode and the first torsional mode, the excitation point was placed at the intersection of node lines of these modes (in a hole drilled through the cantilever). The measured frequencies of the first two bending modes were 108

Hz and 1625 Hz, and that of the first torsional mode was 602 Hz. The damping of the two bending modes was such as to produce Q factors of 20 for both. To excite the cantilever into stick-slip vibration, it was "bowed" with a cylindrical rod coated with rosin. This was loaded against the cantilever by an arrangement of wires and weights. The rod was attached to a large mass which could slide on an air bearing. Vibration of the cantilever was observed using a small accelerometer placed as close as possible to the driving point.

The steady-sliding characteristics of the same surfaces were also measured. The wedge was removed from the cantilever and clamped rigidly. In this case the frictional force was measured directly using a piezoelectric force transducer mounted in the rod. The steady-sliding results show a very marked decrease in friction coefficient with increasing sliding speed. The results are very similar to those obtained previously by Lazarus [2]. For use in the simulations to be described below, a curve-fit has been made which matches the measurements to a satisfactory approximation:

$$\mu = 0.4\exp(-V/0.01) + 0.45\exp(-V/0.1) + 0.35 \qquad (2)$$

where the sliding speed V is measured in m/s.

A typical set of results from the stick-slip apparatus is shown in Fig. 2. In this case a rosin-coated wooden rod was used, with normal load 6.3 N and rod velocity 0.042 m/s, against a perspex wedge. The output from the accelerometer (Fig. 2b) shows that during the sliding phase of the motion, the waveform is very roughly sinusoidal with a period corresponding to the first mode of vibration of the cantilever. During the sticking phase, the contact point on the cantilever moves at the same velocity as the rod. The abrupt transition to sticking excites the higher frequency modes of the cantilever, visible in the plot because the accelerometer could not be placed exactly at the contact point. The corresponding velocity waveform, shown in Fig. 2a, shows the stick-slip behaviour more clearly.

The friction-velocity characteristic calculated from these waveforms is plotted in Fig. 2c, with the steady-sliding characteristic superimposed as a dashed line. Figure 2c shows an anticlockwise hysteresis loop: at the end of the sticking phase, the coefficient of friction has reached a maximum static value of approximately 0.85. Once sliding begins the coefficient falls sharply, in a qualitatively similar manner to the steady sliding characteristic. However, a different path is traced out prior to returning to the sticking phase, with only a small increase in the friction coefficient being observed. During sticking, the plot does not show a single line, but instead has visible loops. This is an artefact of observing the acceleration a small distance away from the contact point — during sticking the cantilever can move in a vibration mode which involves rotation about the contact point, which is picked up by the accelerometer and produces the obvious decaying oscillations seen in Fig. 2b. This shortcoming in the instrumentation does not degrade the results in any serious way: the decaying oscillation picked up during the sticking phase is symmetrical about the correct sticking velocity, so the right result is obtained by imagining the loops collapsed horizontally to their mean position.

Figure 2c shows very clearly that instantaneous sliding speed does not determine the coefficient of friction during a stick-slip vibration. There is a large hysteresis loop, and no part of that loop follows the steady-sliding curve, even approximately. Similar stick-slip tests have been carried out over wide ranges of normal force and rod velocity.

2. CONSTITUTIVE MODELS

At room temperature rosin is a brittle solid which fractures in a glassy manner and powders easily. It is extracted from the resin of various species of coniferous tree. Its chief constituent is abietic acid, and it is soluble in organic solvents. Various grades exist according to the age of the tree from which the resin is drawn and the amount of heat applied in distillation. These range from a rather opaque black substance to a transparent colourless one. The possibility of transparency suggests that the material is amorphous rather than polycrystalline. Melting of rosin may be investigated qualitatively by slowly heating a small amount of rosin. By about 60°C it becomes possible to make a visible indent with the point of a knife. Rather than melting at a well-defined temperature the rosin then softens progressively, turning into a thick and sticky liquid by about 75°C. It then gradually became less viscous until it can be poured readily at 93°C.

Localised melting at a sliding contact causes a marked reduction in the coefficient of friction between two materials. This is caused by melt lubrication, in which sliding is lubricated by a layer of molten material which is produced at the interface by frictional heating [3]. For a material with a definite melting temperature an ingeniously simple argument can be used. As material melts, it tends to be squeezed out of the contact and melt wear occurs [4]. Thus, regardless of the amount of heat available for melting, the film of molten liquid at the contact remains thin. The presence of solid and liquid in close proximity at the contact suggests that the temperature at the contact must be very close to the melting point of the material.

Melt lubrication will now be considered as the basis of a constitutive frictional law for rosin. Since the melting, or softening, of rosin occurs over a range of temperature, some form of melt lubrication might be expected well before the rosin becomes fully liquid. Even for the simple configuration of these experiments, the full three-dimensional details of heat and material flow in the contact region are rather complicated. The geometry of the contact is that of two crossed cylinders loaded together to give an approximately circular contact area. One of the cylinders is stationary, while the other moves across it carrying a layer of rosin. This rosin will be at ambient temperature on entry to the contact zone. It will behave as a solid, and initially will slide over the surface of the fixed cylinder. As it moves further into the contact zone frictional energy will heat the surface of the rosin layer until it sticks to the fixed cylinder. There would then perforce be a layer of some thickness within which shearing motion is taking place. This might be a relatively thick layer within which the deformation is best described as viscous flow, or it might be a thin interfacial layer better described by plastic yielding of rosin in a "mushy solid" state. Once shearing motion is established, the volume flow rate of rosin through unit width of the layer will be reduced — to one-half the original rate, if the shear is uniform. To accommodate this, some rosin is probably squeezed out to the sides. By the exit of the contact region, the rosin in the active layer will be at its maximum temperature. If this reaches 75°C or so, the rosin will be sufficiently liquid that one would expect "whiskers" to be pulled out before the adhesive bond to the stationary cylinder was broken. These whiskers would cool rapidly, become brittle, and break up. This is a plausible explanation for the observed accumulation of finely-divided rosin dust in this region on the experimental rig, and indeed in normal playing of bowed-string instruments with a rosined bow.

The friction force will be the net effect of all processes producing tangential stress. This will include the shearing region, but there may also be contributions from rosin flow out to the sides of the contact zone, and from whisker formation and breakage. To model these processes in detail would require a very elaborate three-dimensional simulation, well beyond the scope of what is intended in this preliminary study. Instead, the assumption will be made that the dominant contribution to the friction force comes from the region of shearing flow. Further, we do not want to take detailed account of the variations of temperature, layer thickness and material properties within the contact region. The objective is to seek the simplest model which goes beyond the usual *ad hoc* one which allows coefficient of friction to be a function only of instantaneous sliding speed. We therefore work in terms of a single, averaged "contact temperature", T, and a uniform layer thickness δ.

The temperature at the contact can be calculated by modelling the heat generation and transfer in the material surrounding the interface. The cylinder of rosin in the deforming contact zone is assumed to be in a state of uniform shear, so that heat generation is uniformly distributed through the volume. The rate at which heat is generated at the contact is given by the product of the frictional force μN and the sliding velocity. This heat input is balanced by three effects: conduction into the two neighbouring solids, convection as cold material flows into the contact region while heated material flows out, and absorption in the contact volume, changing its temperature. The first of these effects can be represented in terms of a suitable Green's function for the heat-diffusion problem, while the other two have simple closed-form expressions within the approximations already stated. A simple time-marching algorithm allows the contact temperature to be calculated during a dynamic simulation.

We now seek simple constitutive laws for rosin friction n which T enters as a state variable. If a continuous film of viscous liquid is formed at the interface, the frictional force would arise entirely as a result of the viscous shearing of the layer. The simplest relevant constitutive law is that of a Newtonian fluid with a temperature-dependent viscosity $\eta(T)$. The shear strain rate is v/δ, so the friction force is then given by

$$\mu N = \eta A v / \delta \quad \text{(viscous model)} \qquad (3)$$

where A is the contact area. Alternatively, if the interfacial layer only heats up to a more modest level, the viscosity would be so high that one might expect the rosin to behave more or less as a solid. The friction force would then be governed by a different kind of interfacial process. The rosin near the sliding interface might be envisaged as a "mushy solid" and a rigid/plastic model might be used, in which the shear stress was a function of temperature only, independent of the shear strain (except that the sign of the strain governs the sign of the friction force). Then

$$\mu N = A k_y(T)\,\mathrm{sgn}(v) \quad \text{(plastic model)} \qquad (4)$$

where k_y is the shear yield stress. This model is equivalent to retaining the notion of a coefficient of friction, but treating it as a function of temperature rather than of sliding speed.

For comparison with results of simulation using the proposed constitutive laws, we first show results using the traditional model. For this, we may use the friction characteristic found by curve-fitting to the measurements obtained during steady sliding, Eq. (2). A simple time-stepping procedure is used to simulate stick-slip motion, similar to that used before in bowed-string study [5]. A particular typical case from the set of stick-slip measurements has been chosen. The observed behaviour was shown in Fig. 2. When this experiment is simulated using the constitutive law of Eq. (10), the result is as shown in Fig. 3. Stick-slip motion is indeed predicted, at the rather lower frequency of approximately 34 Hz. The velocity waveform is qualitatively similar to that in Fig. 2, except that to compensate for the longer sticking intervals the peak sliding speed is higher. The principal difference between the measured and simulated results is seen in the acceleration waveforms at the transition from slipping to sticking. The simulations show a rise to a very sharp peak immediately before sticking is initiated, in contrast to the measurements which show a much smoother variation and a lower peak value. This deviation is associated with the observed hysteresis in the friction/velocity plane.

The challenge for the thermally-based models is to produce simulated waveforms which more closely match the observations. Other comparisons, not reported here, suggested that the viscous constitutive law might be the most appropriate. Plausible values for the radius and thickness of the "melted" layer of rosin were determined by requiring that the contact temperature would be around 55° above ambient (i.e. 75°C) during steady sliding at the assumed rod speed. The first results of simulation with these values could only be described as disastrous. Although quasi-periodic motion was predicted, it had a very long time scale and waveforms with no resemblance to the observations. The velocity v of the oscillator was predicted to rise, on occasions, to a value approaching 1 m/s, faster than the rod. In frictional terms this would be described as "forward slipping" (although this model does not distinguish "sticking" and "slipping" states). There is no indication in any of the experimental results that forward slipping occurs in this apparatus. However, the values for the thermal properties of rosin used in this simulation were those given by Cobbold and Jackson [6]., who pointed out that their determination of the diffusivity, and hence the specific heat capacity, was not very accurate. Values listed for apparently comparable materials such as paraffin wax are an order of magnitude higher [7]. As an experiment, the simulation was re-run using a higher value for specific heat capacity.

The results are shown in Fig. 4. Something more like stick-slip motion is predicted. The frequency is very low, approximately 19 Hz. The acceleration waveform shows a very smooth form, certainly lacking the very sharp spike at the end of slipping seen in Fig. 3. The contact temperature varies between 40° and 60° above ambient, a plausible range for consistency with the assumptions of the viscous model. Although these results using the "wrong" specific heat are better than those with the "right" value, they are still clearly unsatisfactory. Similar simulations have been run with a wide variety of parameter values, and they all show the feature of "forward slipping". During an episode of sliding the rosin layer heats up, so that towards the end of the sliding episode the viscosity is low. There is then insufficient force available for the oscillator to be "recaptured" by the rod for a new sticking episode without some overshoot occurring. We conclude that the viscous constitutive model does not give a good representation of the physical behaviour of rosin.

Turning to the alternative constitutive model, Eq. (4), the result of a simulation is shown in Fig. 5. Now the velocity and acceleration waveforms show stick-slip motion at about 44 Hz, with details matching the measurement reasonably well. The contact temperature fluctuates in the range 20°–40° above ambient, low enough for self-consistency with the assumption of the plastic model. Based on this single case, the plastic-yield constitutive model performs rather well in comparison with the experiments. To test this more thoroughly, we explore the effect of changing parameters. The simulation program has been run for a set of six cases chosen to illustrate the range of observed behaviour. In Fig. 6 the friction/velocity curves for all six cases are overlaid on the measurements. Figure 6b shows the case of Fig.5. The comparison with measurements is by no means perfect, but some trends are reproduced fairly well. All cases show hysteresis in the correct sense. The loops tend to be more open with lower rod speed, turning to long and thin forms at high rod speed. The peak sliding speed is roughly correct (except in case d). Indeed, it should be noted that the peak sliding speed in the measurements often varied from cycle to cycle by amounts comparable with the discrepancies seen here between theory and experiment.

In conclusion, it appears that a model in which the coefficient of friction is regarded as a function of temperature, rather than of sliding velocity as has traditionally been assumed, gives predictions of stick-slip behaviour mediated by rosin which match observations fairly well. Work is now in progress using this model to simulate bowed-string motion and make similar comparisons with observation.

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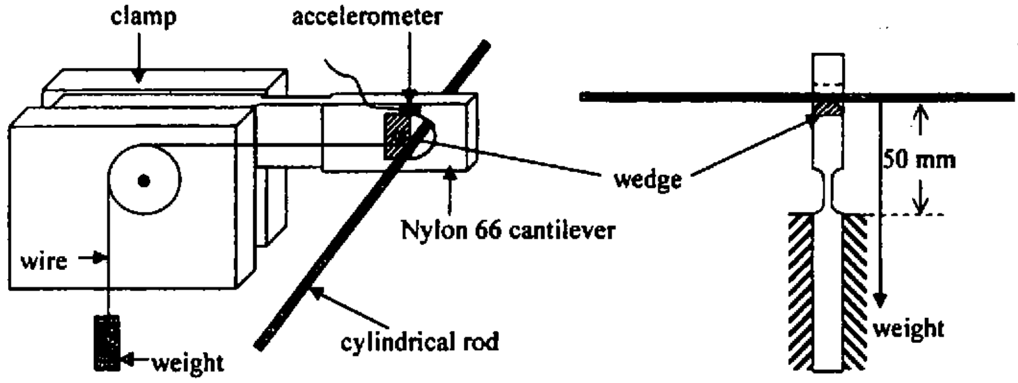

Figure 1: The experimental apparatus

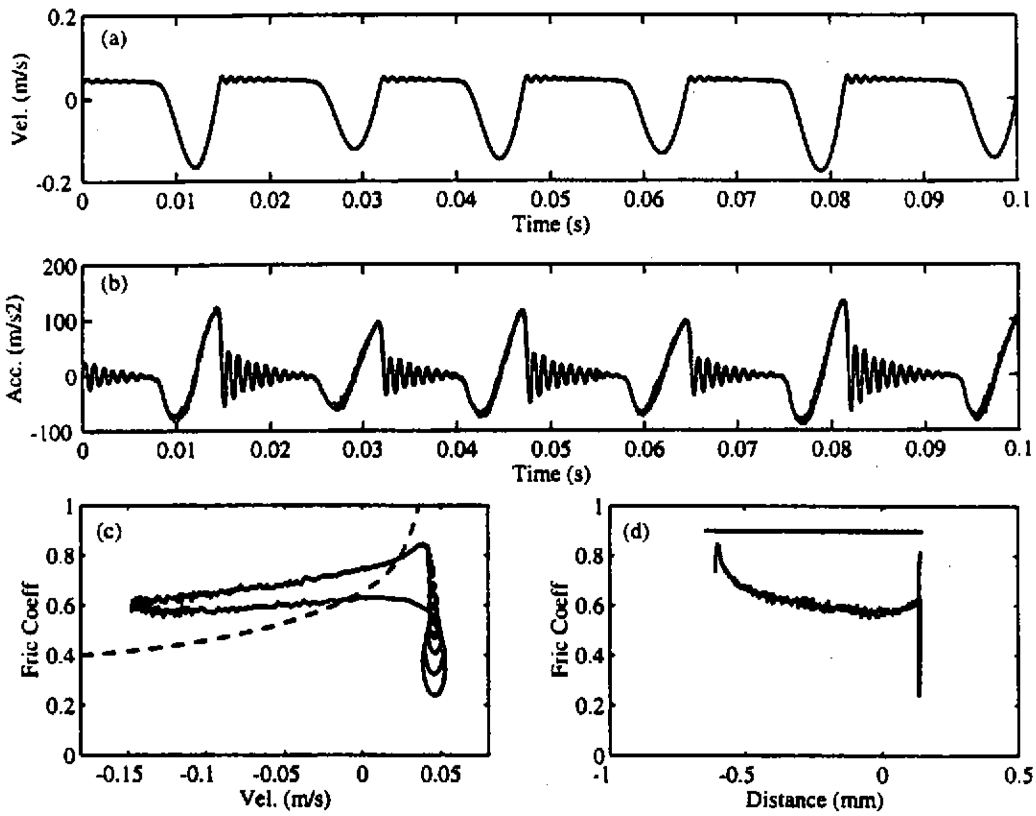

Figure 2: Stick-slip waveforms from the experimental apparatus

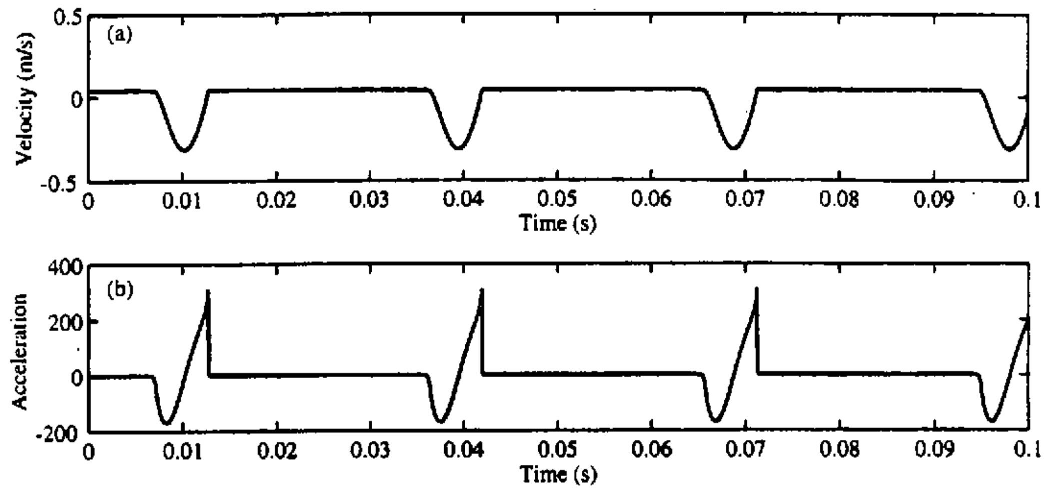

Figure 3: Simulated velocity and acceleration waveforms using the traditional model

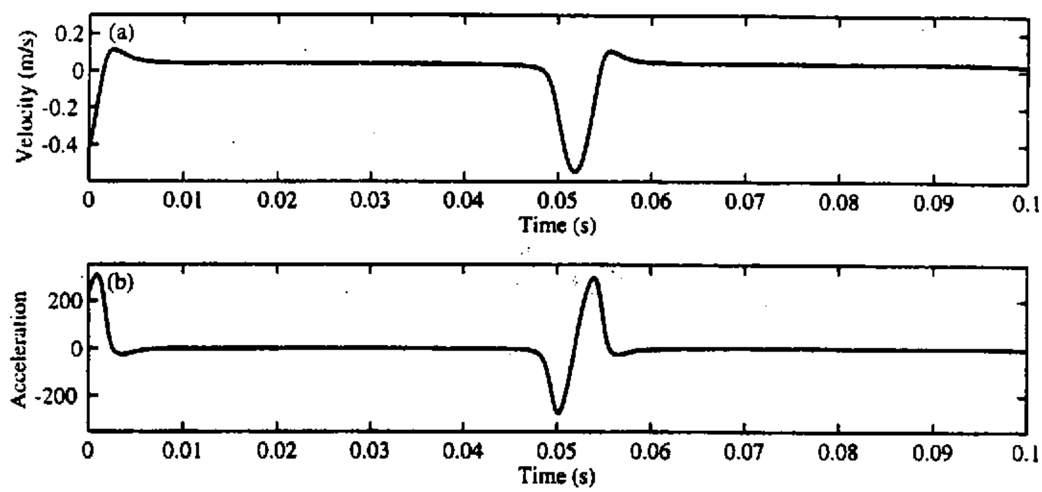

Figure 4: Simulated velocity and acceleration waveforms using the viscous model, eq. (3)

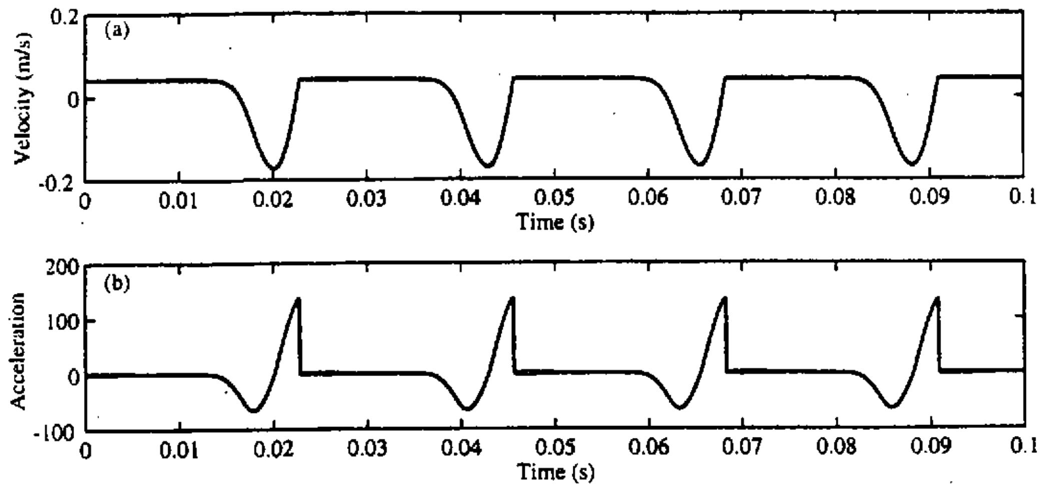

Figure 5: Simulated velocity and acceleration waveforms using the "plastic" model, eq. (4)

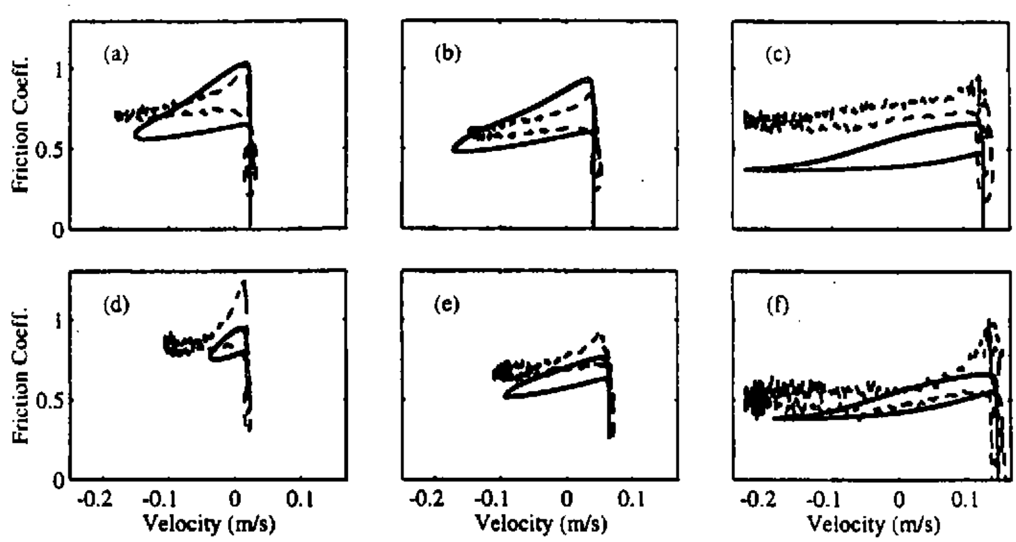

Figure 6: Simulated force/velocity plots for various cases, using the "plastic" model.