THE X-SPRING TRANSDUCER

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1. INTRODUCTION

The X-spring ("Transducer-Spring") is a flextensional transducer without the flexing. The transducer provides magnified output from the lever action of stiff arms connected to the ends of the electromechanical driver. Since the stiff arms exhibit only slight bending, there is little reduction in the coupling coefficient. Moreover, since the X-spring pistons are connected to the center point of a pair of straight lever arms, maximum magnified motion is obtained. The lever arm action produces a high impedance at the ends of the driver, a lower resonance and, possibly, a lower mechanical Q. The X-spring may be designed to perform as a lumped system and achieve a higher coupling coefficient and bandwidth than can be obtained from a conventional Class IV flextensional design. Additional (Tonpilz) pistons may be added to the ends of the drive stack for added high frequency output. We present a discussion of the transducer, a math model with an equivalent circuit, finite element results and measured results on a 1.2 to 10 kHz design.

2. THE X-SPRING TRANSDUCER MODEL

We show in Fig. 1 a sketch of the simplified model of the X-spring without the Tonpilz piston pair along with a photograph showing the additional Tonpilz pistons on the ends of the piezoelectric stack. The device yields amplified motion in the Y direction for a given piezoelectric drive stack motion in the X direction. If we assume the lever arm of Fig. 1 has a hypotenuse length H, then we may write $H^2 = Y^2 + X^2$ where Y is the half-height in the Y direction and X is the half-width in the X direction. With H a constant length we have $dH = 0$ in this simplified "stiff" model. If we let the differentials dX and dY be the displacements in the X and Y directions, we get

$$2XdX + 2YdY = 2HdH = 0 \quad \text{leading to} \quad dY/dX = -X/Y \qquad (1)$$

Thus, for $X > Y$ we get magnified motion of the attached X-spring pistons along the Y direction, as desired. The negative sign shows that the expansion in the X direction yields a contraction in the Y direction. The force amplification ratio is the reciprocal of the displacement amplification ratio (X/Y). We define the force amplification ratio as: $a = Y/X$.

Consider now the simplified mechanical model in Fig. 2a of an ideal X-spring transducer with a piezoelectric drive stack of compliance C_m of negligible mass and added masses M_1 and M_2 attached to opposite ends of the lever arms along with mechanical resistances R_1 and R_2 . The corresponding equivalent circuit is seen to be the representation of Fig. 2b which may be compacted to the form shown in Fig 2c. The electromechanical turns ratio is N and C_0 is the clamped capacity. The circuit may be improved to include a transmission line representation of the piezoelectric section by simply letting $C_m = (L/\rho c^2A)\tan(kL/2)/(kL/2)$ where L is the length of the ceramic stack, A is the cross sectional area, ρ is the density, c is the appropriate sound speed and the wave number $k = \omega/c$. This circuit represents the "stiff" ideal case for the X-spring where the lever arms and do not bend or extend or contract.

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In the extended model the finite extensional compliance of the lever arms is included along with a model for Tonpilz piston mass loading at the ends of the drive stack. The piston radiation from the ends of the drive stack becomes significant as the mass-loaded stack resonance is approached. This mode constitutes the upper resonance of the transducer while the mass-loaded X-spring resonance provides the lower resonance.

The extended model is shown in Fig. 3a where M_y is the mass of one of the X-spring pistons and M_x is the mass of one of the drive stack piston which we refer to as the Tonpilz piston. The stiffness of each X-spring arm is K and the stiffness of one half of the stack is K' and the angle between an arm and the drive stack is θ. From the geometry we see that $H^2 = X^2 + Y^2$ and $\cos(\theta) = X/H$ and $\sin(\theta) = Y/H$ and since $2HdH = 2XdX + 2YdY$ we get with $x = dX$ and $y = dY$

$$dH = (X/H)dX + (Y/H)dY = \cos(\theta)x + \sin(\theta)y. \tag{2}$$

Thus the change, dH, in the length of an arm is given by the normal displacements x and y of the Tonpilz and X-spring pistons respectively. (In the previous simplified model we assumed the arm was inextensible with dH = 0.)

It is convenient to consider one half of this symmetrical structure by dividing the X-spring masses by 2 as shown in Fig. 3b. The equation of motion of the Tonpilz piston may then be written as

$$M_x d^2x/dt^2 = F - K'x - 2K\cos(\theta)dH \tag{3}$$

and from the above equation for dH, Eq. (2), we may write Eq. (3) as

$$M_x d^2x/dt^2 = F - K'x - 2K\cos(\theta)^2 x - 2K\cos(\theta)\sin(\theta)y \tag{4}$$

As seen, this equation of motion depends on both the displacements x and y. Consequently, a second equation of motion is necessary for a simultaneous solution for x or y. This second equation comes from the equation of motion for the X-spring mass, M_y, which may be written as

$$(M_y/2)d^2y/dt^2 = -K\sin(\theta)dH = -K\sin^2(\theta)y - K\sin(\theta)\cos(\theta)x \tag{5}$$

with sinusoidal solution for y written as

$$y = 2K\sin(\theta)\cos(\theta)x/(\omega^2 M_y - 2K\sin^2(\theta)) \tag{6}$$

Equation (6) is then substituted into Eq.(4) with $V_x = dx/dt$ to yield the sinusoidal solution

$$i\omega M_x V_x + K'V_x/i\omega + 2KM_y\sin^2(\theta)V_x/a^2(i\omega M_y + 2K\sin^2(\theta)/i\omega) = F \tag{7}$$

with the solution for the Tonpilz piston velocity, V_x, written as

$$V_x = F/[i\omega M_x + K'/i\omega + 2KM_y\sin^2(\theta)/a^2(i\omega M_y + 2K\sin^2(\theta)/i\omega)] \tag{8}$$

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where X-spring leverage force amplification factor a = Y/X = tan(θ). The solution for the X-spring piston velocity, $V_y = dy/dt$, may then be obtained from Eq. (6) and written as

$$V_y = 2K\sin(\theta)\cos(\theta)V_x/(\omega^2 M_y - 2K\sin^2(\theta)) \quad (9)$$

Resistive mechanical damping or a radiation load of value R_y on an X-spring piston and of value R_x on a Tonpilz piston may be incorporated into the results by simply replacing the mass M_x by $M_x + R_x/i\omega$ and the mass M_y by $M_y + R_y/i\omega$.

The above analytical development for one-half of the transducer may be used to develop an equivalent circuit for the transducer. The force, F, in the above solution may be related to the impressed electrical voltage E by the equation F = NE where N is the electromechanical turns ratio. The piezoelectric clamped capacity in shunt with the voltage E would be C_0. We may then consider a stack of full length L=2X as a piezoelectric transmission line "T" network coupled to the Tonpilz masses, M_x, and, through the lever arms of stiffness K, to the X-spring masses, M_y.

In the transmission line representation the stiffness of the piezoelectric stack may be written with $K' = (\rho c^2 A/L)kL/\sin(kL)$ where ρ is the density of the material, c is the short circuit sound speed in the material, A is the cross-section area, L is the total length of the stack and the wave number k = ω/c. The mass for one-half of the stack, $M_s/2 = (\rho AL/2)\tan(kL/2)/(kL/2)$. We presume this piezoelectric stack to be composed of n segments, of a small thickness t = L/n, all wired in parallel for additive output. In this case the electro-mechanical turns ratio is $N = (nA/L)Y_{33}d_{33}$ where Y_{33} is the 33 mode short circuit Young's modulus for the material and d_{33} is the 33 mode piezoelectric "d" constant. The clamped shunt capacity $C_0 = n^2A\varepsilon^S/L$ where the clamped dielectric constant $\varepsilon^S = \varepsilon^T(1 - k_{33}^2)$, ε^T is the free dielectric constant and k_{33} is the 33 coupling coefficient.

The equivalent circuit for the above extended transducer may accordingly be represent by the circuit of Fig. 4a where $Z_a = i\rho cA\tan(kL/2)$ and $Z_b = -i\rho cA/\sin(kL)$ and the force amplification factor a = Y/X = tan(θ). The displacement amplification factor is given by 1/a = cot(θ). The above circuit may be simplified by folding it over resulting in the configuration shown in Fig. 4b. Further simplification may be obtained by translating the factor "a" into the electromechanical turns transformer N as we did in Fig. 2c.

3. RESULTS

The transducer design which incorporates both the X-spring and end radiation features is shown in the photos of Figs. 1 and 5. The lever arms of the X-spring are steel and extend 5 inches in depth. The two ceramic stacks are each 1.6 inches in depth and each are composed of twenty pieces of Navy Type III piezoelectric material. The aluminum X-spring pistons have been shaped for minimum flexure and are seven inches in depth. The aluminum Tonpilz pistons are also seven inches in depth. These Tonpilz pistons move without the amplified motion and provide added output above the X-spring resonance. The transducer is fitted with two 2 inch thick aluminum end plates separated by four stiff rods. The overall size of the transducer shown in Fig. 5 is approximately 9 inches long by 5.5 inches wide by 8 inches high.

A finite element quadrant model for the transducer shown in the photograph of Fig. 1 is given in Fig. 6 with a fundamental modal resonance at 2.27 kHz. The displacement amplification factor is seen to be approximately 3:1. Also, as seen, the lever arm shows only very slight bending. The effective coupling coefficient for this X-spring mode is 0.46 based on an ideal piezoelectric Navy Type III material coupling coefficient of 0.64. The Tonpilz mode yields a coupling coefficient of 0.29 and occurs at 8.02 kHz as show in Fig. 7. Here we see the aluminum end (Tonpilz) pistons are not entirely stiff and show considerable bending. Although not shown here, the bending modes of the lever arms were evaluated and the first was found to occur at 12.22 kHz. Two dimensional water loaded finite element models were also evaluated and used to estimate the transmitting response of the transducer before construction and testing was undertaken.

The X-spring transducer was measured under both air-loaded and water-loaded conditions. The measured coupling coefficient of the cemented piezoelectric drive stack was found to be 0.58 (instead of the ideal 0.64 value) yielding a finite element X-spring predicted coupling coefficient value of 0.42. The measured X-spring effective coupling coefficient was found to be 0.38. In-water response measurements were made on the unit at Seneca Lake with both the X-spring and Tonpilz pistons active. The measured TVR resonant frequency and Q were found to be 1.54 kHz and 7.4 respectively. We show the TVR response measured off one of the Tonpilz pistons (——) and off one of the X-spring pistons (- - -) in Fig. 8. The TVR response off one of the Tonpilz pistons (——) is compared in Fig. 9 with the response off one of the endplates (- - -). Here we see the responses are very similar up through 7.5 kHz. Beam patterns in this plane were found to be nearly omnidirectional up through 5.2 kHz. There were no sharp nulls revealed in the measurements taken in this plane. We believe the nulls shown in Fig. 8 are a result of bending resonance of the piezoelectric stack creating a dipole (body) motion of the transducer in a direction perpendicular to the stack length.

4. SUMMARY AND CONCLUSIONS

We have presented a new broad band transducer based on the X-spring design. An analytical model along with an equivalent circuit were presented along with a set of finite element results for the case of steel lever arms, aluminum X-spring and Tonpilz pistons. The finite element and measured results show an effective coupling coefficient of approximately 0.42 and 0.38 respectively based on a cemented drive stack effective material coupling coefficient of 0.58. Seneca Lake measurements show a smooth TVR response off the endplate and Tonpilz pistons and reveal a series of nulls off the X-spring pistons, which we believe are a result of a piezoelectric stack bending mode. This bending (sometimes resulting in a so-called "banana mode") has also been detected, measured, analyzed and mitigated in a number of Class IV flextensional transducers. We believe the same mitigation procedures should also be possible for the X-spring design.

5. ACKNOWLEDGEMENTS

The X-spring transducer is based on U.S. Patent "Electro-Mechanical Transduction Apparatus," 4,845,688, July 4, 1989 issued to J. L. Butler, Image Acoustics, Inc., USA. Fabrication and initial testing of the transducer was performed at Massa Products Corporation, Hingham, MA, USA. We would like to thank Jan Lindberg of ONR, for his encouragement, support and the measurements made at Seneca Lake.

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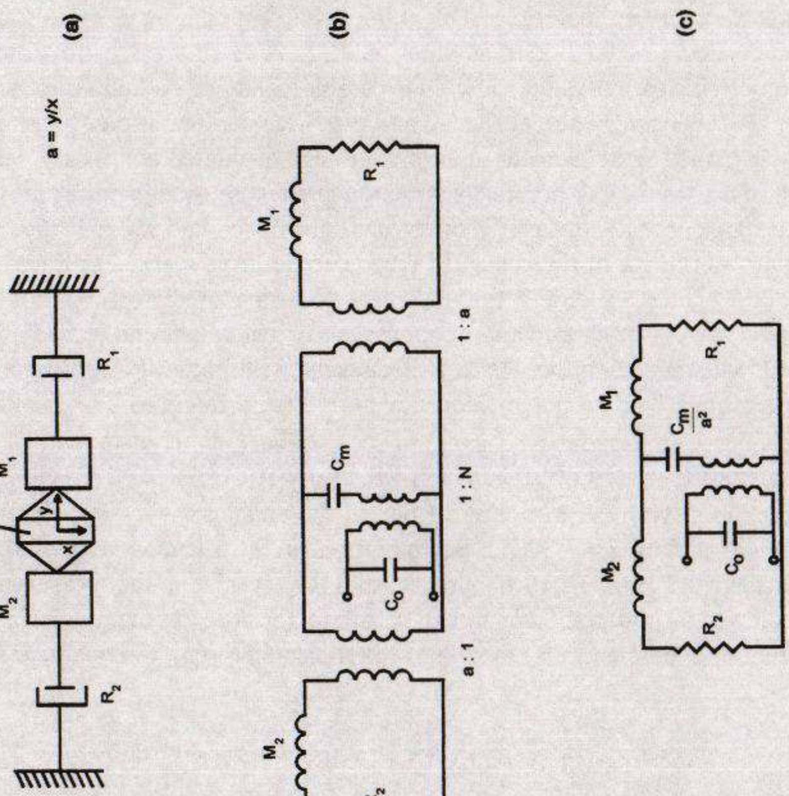


Fig. 2 Ideal X-spring mechanical model (a) equivalent circuit (b) and compacted equivalent circuit (c).

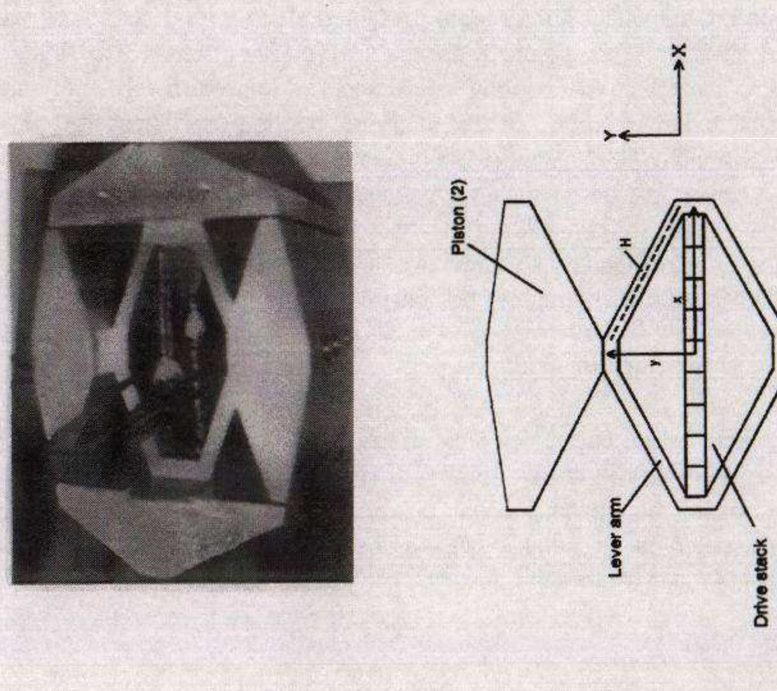


Fig. 1 X-spring transducer.

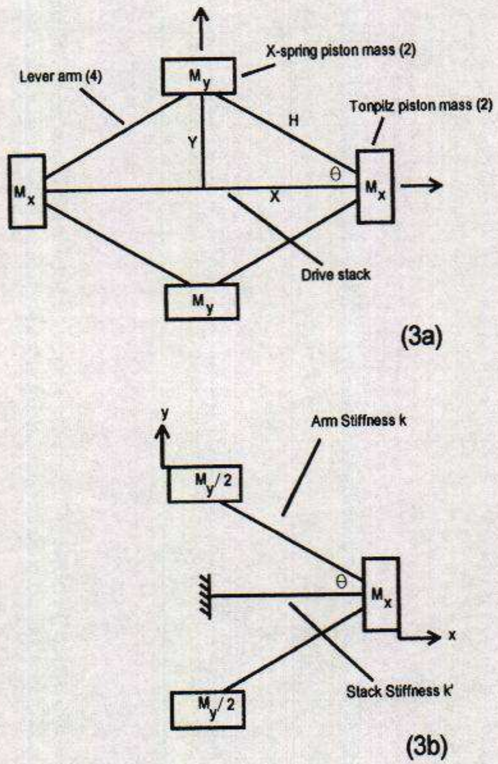
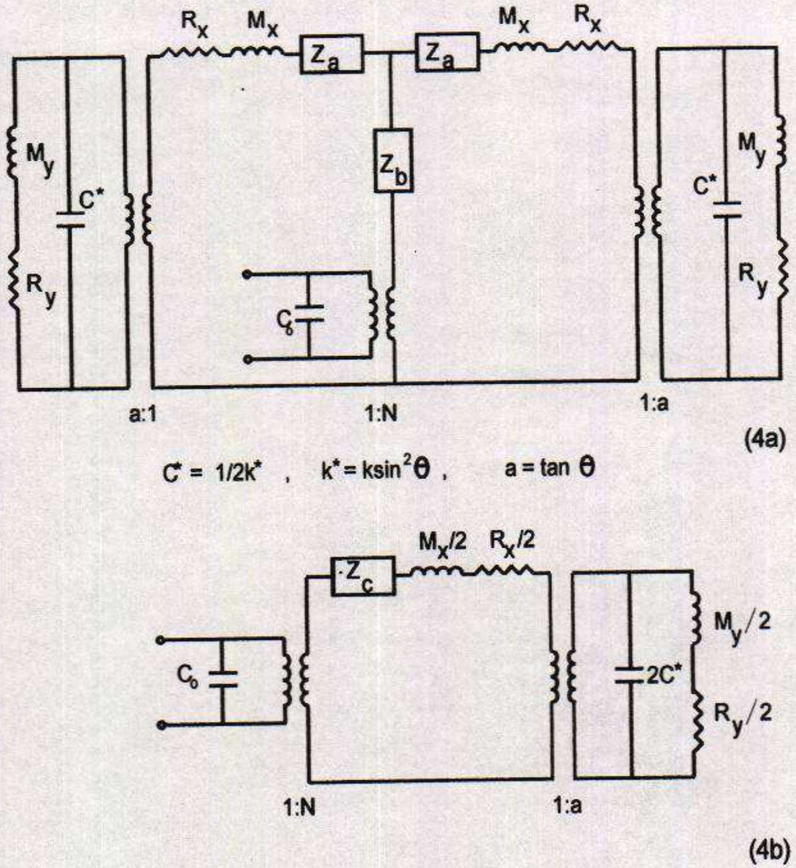


Fig. 3 X-spring model with tonpilz pistons: (a) full model (b) 1/2 model.



$C^* = 1/2k^*$, $k^* = k\sin^2\theta$, $a = \tan\theta$

$$Z_c = \frac{-i\rho cA}{2}\cot\left(\frac{kL}{2}\right)$$

Fig. 4 Equivalent circuit of extended model: (a) full circuit (b) folded circuit.

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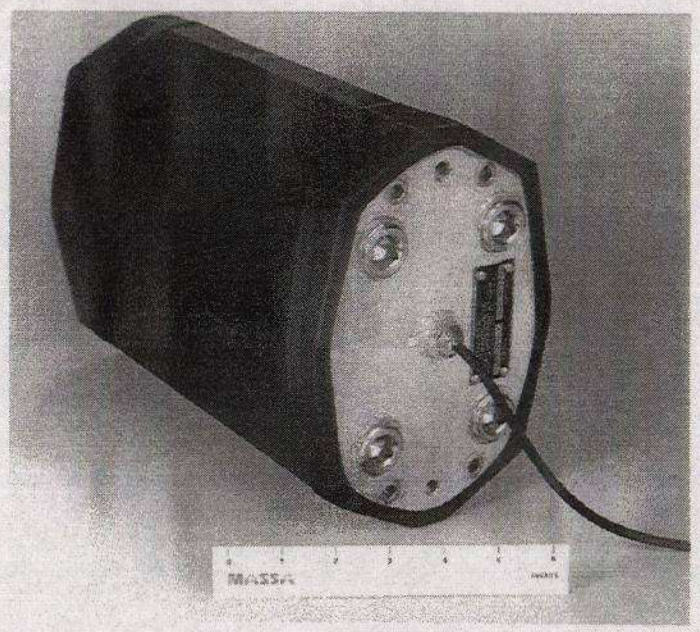

Fig. 5 Photograph of the X-Spring transducer.

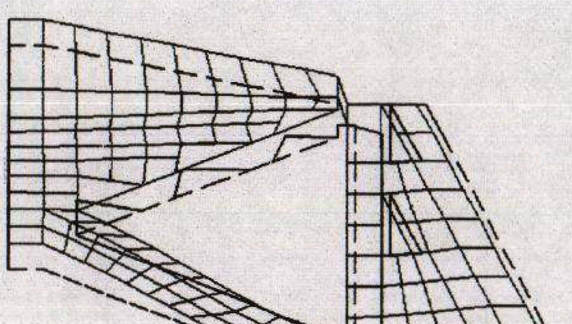

Fig. 6 Fundamental in-air x-spring mode at 2.27 kHz.

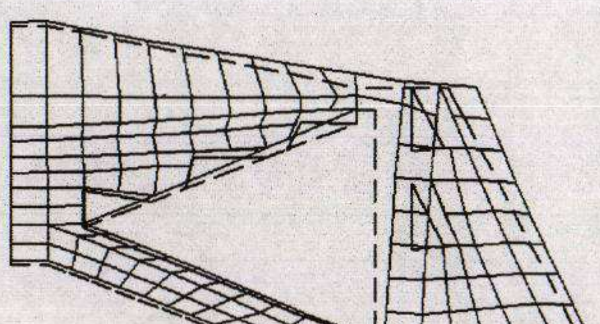

Fig. 7 Fundamental in-air tonpilz mode at 8.02 kHz.

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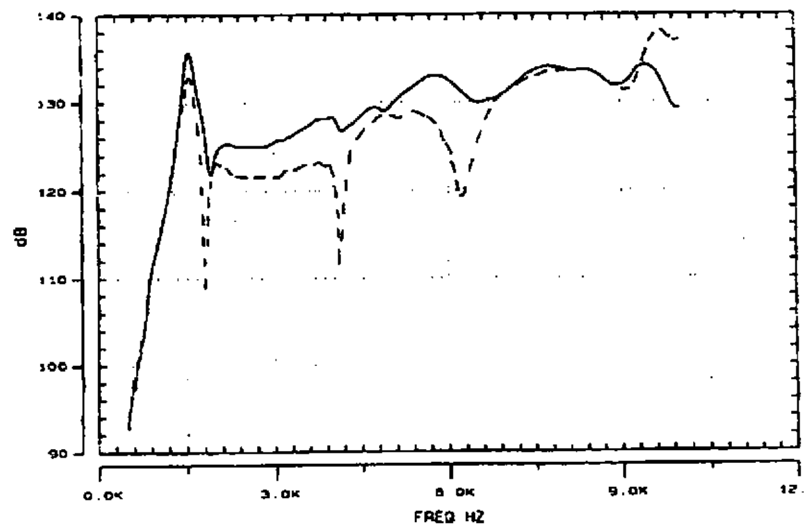

Fig. 8 TVR measurements at Seneca Lake. Off tonpilz (—) and off X-spring (- - -).

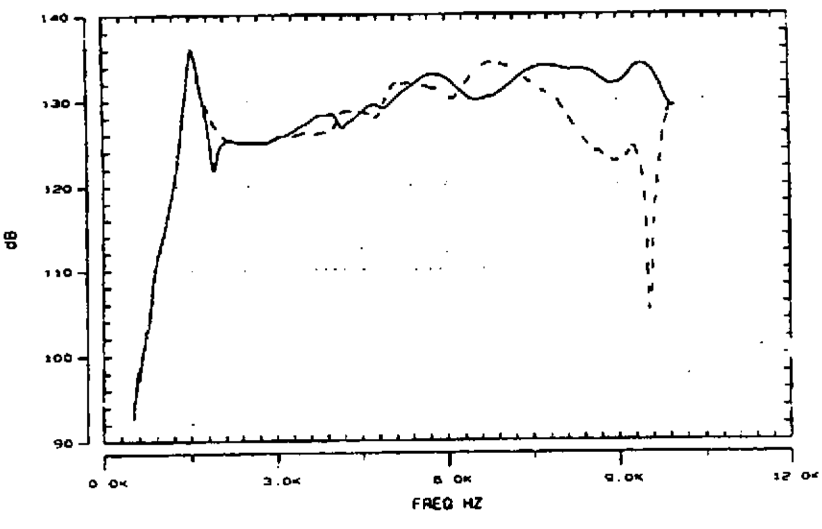

Fig. 9 TVR measurements at Seneca Lake. Off tonpilz (—) and off end plate (- - -).