

## **HYPERSENSITIVITY OF VIBROACOUSTICS BEHAVIOUR OF PLATE LATTICES**

J L Guyader (1) & E Rebillard (2)

(1) Institut National des Sciences Appliquées, Laboratoire Vibration Acoustique, 69621 Villeurbanne Cedex, France, (2) Dept of Applied Acoustics, Chalmers University of Technology, S 412 96 Göteborg, Sweden

### **SUMMARY**

The vibroacoustic behaviour of lattices of plates is investigated. The vibrational formulation using a semi-modal decomposition describes in-plane and flexural motions. A boundary integral formulation calculates the acoustical pressure radiated in light fluid.

The phenomenon of hypersensitivity as observed when dealing with two coupled plates [1] (a slight modification of connexion angle can yield a large modification on the vibrational and acoustical behaviour) is again observed when the number of plates is largely increased.

For the case of non-periodic lattices close to the reality (for example, a machine tool hood), a numerical approach shows results which could be issued from an experimental tool detecting hypersensitive connections.

### **1. INTRODUCTION, MODEL UNDER STUDY**

Structures constructed from coupled plates are numerous in the reality. Often analytical models dealing with coupled plates only consider the bending motion; the structure is limited to the T, H, L or X shape [2,3,4]. The analytical model considers structures constructed from different length but same width plates connected at any angle. Coupling between bending and in plane motions is due to the connection angle. The behaviour of structures with numerous of plates is observed: lattices of eighteen identical plates and a lattice of nine different plates of fig. 1.

### **2. ANALYTICAL FORMULATION: VIBRATIONAL AND ACOUSTICAL (IN LIGHT FLUID) BEHAVIOURS**

The vibrational formulation is based on three basic points presented in detail in [1], nevertheless we can summarize it:

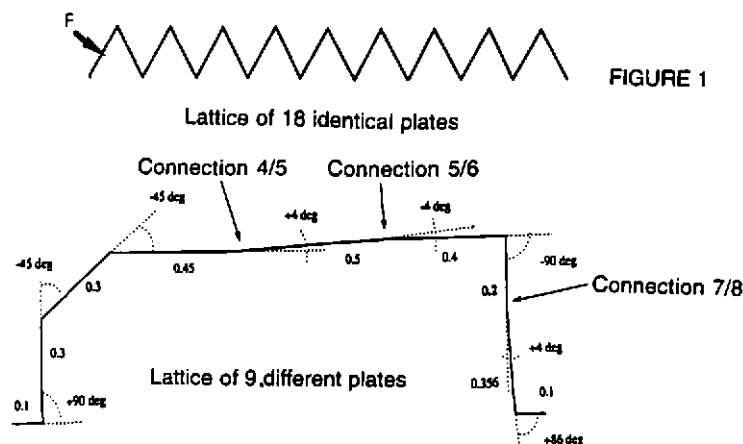


FIGURE 1

(1) The motion of a plate is expressed with the Donnell operator for a shell of infinite radius, (2) It is then developed with a semi-modal decomposition combined to a wave formulation and (3) the expression of the continuity of motions and forces at the connection between two plates gives rise to a matricial equation whose the unknowns are the coefficients of the semi-modal decomposition.

Here is briefly presented acoustical formulation one can find detailed expressions in [5].

The acoustical behaviour of the baffled coupled plates is calculated by the integral formulation:

$$P(M_0) = \varepsilon(M_0) \int_{S_v} \left( G(M, M_0) \rho \omega^2 W(M) - P(M) \frac{\partial G(M, M_0)}{\partial n} \right) dM \quad (1)$$

where:

$M_0$  is a point of the half-space  $V$  limited by the baffle, where is calculated the pressure  $P(M_0)$ ,  $S_v$  the surface of the structure,  $\omega$  the pulsation,  $\rho$  the fluid density,  $W(M)$  the plate radial displacement,  $\varepsilon(M_0)$  a factor such as  $\varepsilon=1$  if  $M_0 \in V$ ,  $\varepsilon=2$  if  $M_0 \in S_v$  and  $G(M, M_0)$  the Green function defined by:

$$G(M, M_0) = \left( \frac{e^{-jkR}}{4\pi R} + \frac{e^{-jkR'}}{4\pi R'} \right)$$

$R$  (respectively  $R'$ ) is the distance between  $M_0$  and  $M$  (respectively  $M'$ , the symmetrical point of  $M$  in the relation to the baffle).

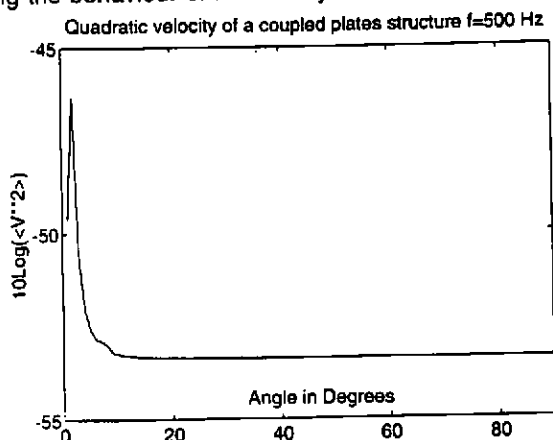
The solution of equation (1) is made discretizing the integral equation with a collocation scheme. Then it gives rise to a matricial equation whose the unknown vector is the parietal pressure at discretization points.

The radiated power is then calculated by equation (3):

$$\Pi = \frac{1}{2} \sum_{i=1}^N P(M_i) W^*(M_i) S_i \quad (3)$$

### 3. HYPERSENSITIVITY: BASIC RESULT WITH TWO PLATES

Let's remind notion of hypersensitivity observed with two plates of [1]: when dealing with two coupled plates, for a low connection angle a light modification of it can lead to a large modification of the vibrational and acoustical (in a weaker way) behaviour, but when the connection angle is large a modification of it produces no differences on the behaviour, see fig.2. This phenomenon is related to the coupling between flexural and in-plane motions which depends greatly on the value of the angle for small connection angle. For larger angle, it is less important and slowly variable. This phenomenon can explain, by a certain way, fluctuations noted when considering the behaviour of industrially identical structures.



### 4. LATTICE OF PLATES: PERIODIC STRUCTURE

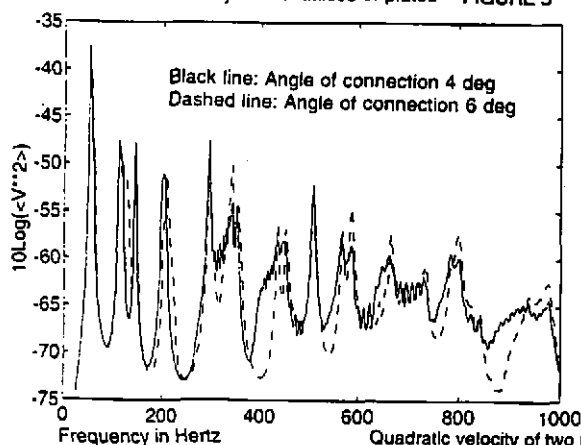
In this section we deal with four lattices of eighteen plates. The steel plates are such as: length: 0.5m, width: 0.4m, thickness: 2mm. For the first structure, all connexion angles are 4 degrees, for the second 6 degrees, for the third 25 degrees and for the last one 90 degrees. We will compare the two similar lattices, the first with the second, and the two different lattices, the third with the fourth.

In figure 3 are presented the quadratic velocity spectra of each structure when the connexion angle is respectively 4 and 6 degrees. For the both it is easy to identify differences. This sensitivity to the connexion angle between plates is explained by the strong variation of the coupling between bending and in plane motions effects.

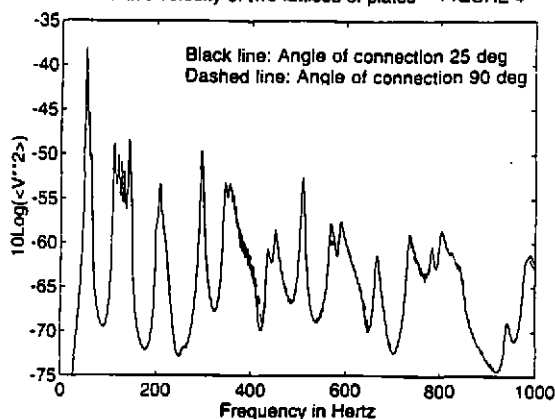
On the other side, when the angle is large, this coupling is no more sensitive even to a large variation of the angle. For example for a 25 or a 90 degrees connexion angle, the two lattices which are physically very different present close vibrational behaviour as presented in figure 4. Physically, at this angle, the effects of in plane motion are negligible.

Obviously the properties of hypersensitivity as exposed in [1] when dealing with two plates are still existing when dealing with a lattice of plates. The coupling between in plane motion and bending is strongly variable when the angle of connexion is small; a weak modification on it can bring strong variations on the vibrational behaviour: this property is not dependant on the number of plates.

Quadratic velocity of two lattices of plates **FIGURE 3**



Quadratic velocity of two lattices of plates **FIGURE 4**



## 5. LATTICE OF PLATES: NON PERIODIC STRUCTURE

Let us consider the structure proposed in figure 1b. This kind of shape could be an industrial structure as the hood of a machine. From the knowledge about the hypersensitivity we can guess that the three connexions noted 4/5, 5/6 and 7/8 are hypersensitive.

To verify it we put an angular defect of one degree successively on each connection. The comparison between the perfect and the altered structure

is down by the following way:

Let us consider two structures. The first one is the lattice without defect and the second one has a defect. For each plate of the altered structure is defined the relative error on the transfer mobility modulus at a fixed frequency.

$$Er_i(\omega) = \sqrt{\frac{(|\bar{Y}_{ij}(\omega)| - |Y_{ij}(\omega)|)^2}{|\bar{Y}_{ij}(\omega)|}}$$

where  $i$  is the receiving plate index and  $j$  the excited plate index,  $\bar{Y}_{ij}(\omega)$  is the transfer mobility modulus for the reference structure without defect and  $Y_{ij}(\omega)$  the transfer mobility modulus for the altered one. To make trends to appear, the observation at a particular frequency is not convenient, then we propose an average over frequency to smooth the phenomenon. We define a mean value all over the spectrum:

$$\langle Er_i \rangle_{\Delta} = \frac{1}{\Delta} \int_{\omega_c - \frac{\Delta}{2}}^{\omega_c + \frac{\Delta}{2}} Er_i(\omega) d\omega$$

Where  $\Delta$  is the length and  $\omega_c$  the centre of the frequency range.

Results of the angular defect successively put on each connection angle are proposed on figure 5. Through this figure it is possible to verify that the three connexions 4/5, 5/6 and 7/8 are hypersensitive.

An other kind of defect is simulated: a mass defect. In the reality it is really easy to add this kind of defect by a non-destructive way. Then, a light mass (comparatively to the mass of the plates) is successively added on each connexion. Comparisons with the perfect structure, are proposed in figure 6. One observes that when the defect is located on one of the hypersensitive connexion, differences between the reference structure and the defective one are important.

Then, here we simulate a non destructive experimental approach which could identify by comparison of the same structure successively and temporarily changed by a defect, where are the hypersensitive connexion angles.

## 6. CONCLUSION

We reminded an analytical formulation which is able to define the vibrational and acoustical behaviours of connected plates at different angle. The hypersensitivity phenomena observed on the vibrational and acoustical behaviour of two coupled plates is also observed when dealing with plate lattices. We presented the case of periodic lattice. When dealing with non periodic lattice, this effect is still obvious, we proposed a

numerical simulation of a non destructive way to identify the hypersensitive connexions of a structure.

# ACKNOWLEDGEMENT

A DRET-CNRS grant supported this work to complete a PhD thesis.

# REFERENCES

- [1] Rebillard E., Guyader J.L., 1995 Journal of Sound and Vibration 188(3), 435-454. Vibrational behaviour of a population of coupled plates: hypersensitivity to the connexion angle.
- [2] Shen Y., Gibbs B.M., 1987 J. of Sound and Vibration 112(3), 469-485. The predicted and measured bending vibration of an L-combination of rectangular thin plates.
- [3] Cushman J.M., 1990 J. Acoust. Soc. Am. 87(3), 1159-1165. Structural power flow analysis using a mobility approach of a L-shaped plate.
- [4] Kim H.S., Kang H.J., Kim J.S., 1994 J. Acoust. Soc. Am. 96(3), 1557-1562. Transmission of bending waves in interconnected rectangular plates.
- [5] Rebillard E., 1995 PhD Thesis: Insa-Lyon, France. Vibro-acoustique des reseaux de plaques: modelisation, hypersensibilite et populations de structures

Effect of an angular defect put successively on each connection angle

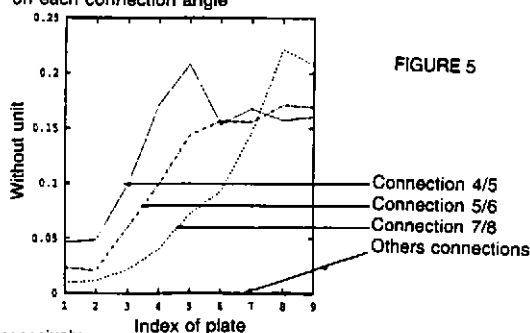


FIGURE 5

Effect of a mass defect put successively on each connection angle

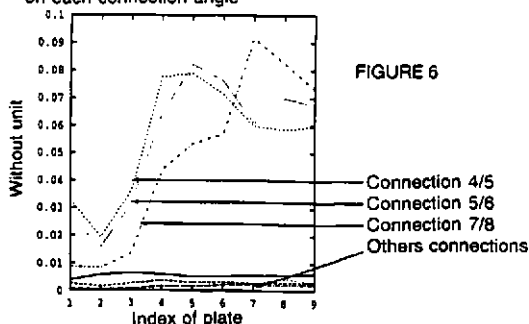


FIGURE 6