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TRANSMISSION OF VIBRATIONAL POWER IN A FRAMEWORK STRUCTURE

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1. INTRODUCTION

When attempting to control vibration levels transmitted from a machine through the various connections to the structure upon which it is mounted, it is desirable to be able to identify and quantify the vibration paths in the structure. Often large machinery installations are installed on frameworks consisting of beam like members. These frameworks are then isolated from the main structure. If the dominant transmission path in the framework is identified it is possible to reduce vibration levels by absorbing the mechanical energy along the propagation path in some convenient manner. By utilising the concept of vibrational power it is possible to quantitatively compare the wave type contributions to each transmission path.

In order to predict vibrational power transmission in a framework, it is necessary to identify the wave amplitude reflection and transmission coefficients for each joint in the structure. Lee and Kolsky [1] investigated the effects of longitudinal wave impingement on a junction of arbitrary angle between two rods. Similarly Doyle and Kamle [2] examined the wave amplitudes resulting from a flexural wave impinging on the junction between two beams. By using the reflection and transmission coefficients for different joints, it is possible to predict the vibrational power associated with flexural and longitudinal waves in each section of the framework. Previous investigations [3, 4] have considered the effects of bends and junctions in infinite beams. This work has been extended to consider the finite members which constitute frameworks. The investigations [5] concentrated on multi-wave type (flexural, longitudinal and torsional) descriptions of transmitted vibrational power in a system of two finite beams joined at an arbitary angle. Detailed here are the results of an investigation into a closed framework consisting of four beams of equal cross-section and length. Only flexural and longitudinal wave motion is considered in the solution. Unlike other techniques [6] utilising structural intensity to analyse frame-works, the technique proposed in this paper produces power distributions for each wave type present in the structure. By comparing the results for each wave type, it is possible to apply the correct methods of vibration control.

2. TRANSMITTED POWER IN A UNIFORM BEAM

For flexural wave motion, consider a section of a uniform beam carrying a propagating flexural wave. Two loads act on this beam element, the shear force and the bending moment. It is assumed that the flexural wave can be described by using Euler-Bernoulli beam theory, so that the displacement can be expressed as

$$W(x,t) = A_f \sin(\omega t - k_f x),$$

the shear force acting on a section as

$$S = EI\, \partial^3 W/\partial x^3,$$

and the bending moment on the section as

$$B = EI\,\partial^2 W/\partial x^2,$$

Then the instantaneous rate of working X at the cross-section is given by the sum of two terms (negative sign merely due to sign convention).

$$X = S\frac{\partial W}{\partial t} - B\frac{\partial^2 W}{\partial x\,\partial t} = EI\frac{\partial^3 W}{\partial x^3}\;\frac{\partial W}{\partial t} - EI\frac{\partial^2}{\partial x^2}\;\frac{\partial^2 W}{\partial x\,\partial t}$$

The time averaged power

$$\langle P\rangle_f = (1/T)\int_0^T X\,dt \text{ then is given by } \langle P\rangle_f = EIK_f^3\omega A_f^2 \qquad (1)$$

For longitudinal wave motion consider a section of a uniform beam with a longitudinal wave propagating through the beam $U(x, t) = A_l \sin(\omega t - k_l x)$. The instantaneous rate of working X is then $X = -EA(\partial u/\partial x)\,\dot{u}$ and the time averaged power is

$$\langle P\rangle_l = \frac{1}{T}\int_0^T X\,dt = \frac{1}{2}EA\omega k_l A_l^2 \qquad (2)$$

If dissipation is present in the structure, the modulus of elasticity may be considered to be a complex quantity $E^* = E(1 + i\eta)$ where η represents the loss factor of the material, present due to inherent material damping.

The displacement of a beam at a distance x from the source, due to flexural wave motion may now be considered to be, assuming that material damping is small.

$$W = A_f\, e^{-k\eta\frac{x}{4}} \sin(\omega t - k_f x)$$

and the resulting time averaged power is given by

$$\langle P\rangle_f = EI\omega k_f^3\; e^{-k_f\eta\frac{x}{2}}\, A_f^2 \qquad (3)$$

The above reduces to equation (1) at the source. Similarly, the displacement of beam, due to longitudinal wave motion may be considered to be

$$U = A_L\, e^{-k\eta\frac{x}{2}} \sin(\omega t - k_L\, x)$$

and the resulting time averaged longitudinal power may be rewritten as

$$\langle P\rangle_L = \frac{1}{2} EA\omega k_L \; e^{-k_L \eta x} A_L^2 \tag{4}$$

Again the above reduces to equation (2) at the source.

3. WAVE TRANSMISSION IN A FRAMEWORK

In the above section it is shown that time averaged transmitted vibrational power is proportional to the square of the travelling wave amplitude, regardless of wave type. Thus to determine the power distribution in a framework it is necessary to determine the wave amplitude reflection and transmission coefficients for each discontinuity in the system. Figure 1, shows the framework under consideration, four beams of equal length excited simultaneously by a harmonic compressive force and a harmonic bending force. The angle θ may vary between 0 and 90°. Two different types of discontinuity occur in the structure.

3.1 Transmission Through a Bend. Consider an infinite beam bent through an angle θ (Figure 2). A flexural wave and a longitudinal wave impinge on the bend. Assuming only flexural and longitudinal waves propagating in the structure. Then the displacements of Arm 1 will be

$$W_a(x,t) = (A_1 e^{k_{f1} x} + A_3 e^{ik_{f1} x} + A_4 e^{-ik_{f1} x}) \; e^{i\omega t} \tag{5}$$

$$U_a\,(x,t) = (A_b \; e^{ik_L x} + A_a \; e^{-ik_L x}) \; e^{i\omega t} \tag{6}$$

Similarly for Arm 2 the displacement will be,

where $\psi = x\cos\theta$

$$W_b\,(\psi,t) = (B_2\, e^{-k_{f2}\psi} + B_4\, e^{-ik_{f2}\psi})\, e^{i\omega t} \tag{7}$$

$$U_b\,(\psi,t) = B_a \; e^{i(\omega t - k_L \psi)} \tag{8}$$

Here A_3, A_4, B_3 and B_4 are travelling flexural wave amplitudes, A_1, A_2, B_1 and B_2 are near field wave amplitudes and A_a, A_b, B_a and B_b are travelling longitudinal wave amplitudes.

In previous work [2] in this field a theoretical model was used in which it was assumed that the joint between the two parts of the beam was a rigid mass. The mass or joint is modelled here as a section of a cylinder. This represents the physical shape of most joints in practical systems. It has been shown shown [4] that the joint mass has an insignificant effect on the reflected and transmitted power for the range of values used in this work.

The joint mass $M_j = p_j \pi L^2 J_w / 4$, and the moment of inertia of the joint is $I_j = M_j L^2 / 8$.

By considering the conditions for continuity and equilibrium for the system, the following expressions may be written for the junction at $x = j$.

$$U_a = U_b \cos\theta - W_b \sin\theta + \frac{L}{2}\,\frac{\partial W_b}{\partial \psi}\sin\theta$$

$$W_a = U_b \sin\theta + W_b \cos\theta - \frac{L}{2}\,\frac{\partial W_b}{\partial \psi}(1+\cos\theta)$$

$$\frac{\partial W_a}{\partial x} = \frac{\partial W_b}{\partial \psi}$$

$$E_1\ I_1\ \frac{\partial^2 W_a}{\partial x^2} + \frac{L}{2}\ E_1\ I_1\ \frac{\partial_3 W_a}{\partial x^3} = E_2\, I_2\left(\frac{\partial^2 W_b}{\partial \psi^2} - \frac{L}{2}\frac{\partial^3 W_b}{\partial \psi^3}\right) - I_j\,\frac{\partial W_a}{\partial x}$$

$$E_1\ A_1\,\frac{\partial U_a}{\partial x} = E_2\ A_2\,\frac{\partial U_b}{\partial \psi}\cos\theta + E_2\, I_2\,\frac{\partial^3 W_b}{\partial \psi^3}\sin\theta - M_j\,\frac{\partial^2 U_a}{\partial t^2}$$

$$-E_1\, I_1\,\frac{\partial^3 W_a}{\partial x^3} = E_2\ A_2\,\frac{\partial U_b}{\partial \psi}\sin\theta - E_2\, I_2\,\frac{\partial^3 W_b}{\partial \psi^3}\cos\theta - M_j\,\frac{\partial}{\partial t^2}\left[W_1 - \frac{L}{2}\ \frac{\partial W_a}{\partial x}\right]$$

By substituting the wave equations in to the above six equations, it is possible to obtain a set of simultaneous equations. The six unknown wave amplitudes can be obtained by solving the set of equations. Also the above six equations are applicable for any of the four bends in the system shown in Figure 1. For each bend the correct expressions for U_a, U_b, W_a and W_b must be substituted.

3.2 Wave Transmission From Excitation Point. Consider on infinite beam excited at a point, f, by a harmonic bending force $F\,e^{i\omega t}$ and a harmonic compressive force $Q\,e^{i\omega t}$ (Figure 3). At position f, six waves are generated and the wave motion can be described by

$$W_-(x,t) = (C_1\, e^{k_f x} + C_3\, e^{ik_f x})\, e^{i\omega t}$$
$$U_-(x,t) = C_b\, e^{i(\omega t + k_L x)}$$

and

$$W_+(x,t) = (D_2\, e^{-k_f x} + D_4\, e^{-ik_f x})\, e^{i\omega t}$$
$$U_+(x,t) = D_a\, e^{i(\omega t - k_L x)}$$

at $x = f$, the following boundary conditions apply

$$W_- = W_+$$

$$EI\frac{\partial^2 W_-}{\partial x^2} = EI\frac{\partial^2 W_+}{\partial x^2}$$

$$U_- = U_+$$

$$\frac{\partial W_+}{\partial x} = 0$$

$$EI\frac{\partial^3 W_+}{\partial x^2} = \frac{F}{2}$$

$$EA\frac{\partial U_+}{\partial x} = -\frac{Q}{2}$$

Again substitution of the wave equations in the above six equations yields a set of simultaneous equations which may be solved to find the unknown wave amplitudes.

Thus the sets of equations shown in sections 3.1 and 3.2 may be used to describe the wave motion of the structure shown in Figure 1. Due to the five discontinuties in the structure thirty wave amplitudes must be determined. Equations (3) and (4) can then be used to calculate time averaged transmitted power by substituting each of the travelling wave amplitudes in to the necessary expression.

4. POWER TRANSMISSION IN RHOMBUS FRAMEWORK

Figures 4 - 9 show the normalised nett vibrational power for each wave type propagating in beams 2 - 4 of the framework. The angle θ between beams 1 and 2 varies between 0 and 90° and the frequency of excitation varies between 100 - 200Hz. Appendix 2 contains details of the beam model. Normalised nett vibrational power is calculated at the centre of each arm constituting the structure. Nett vibrational power may be considered to be the difference between power following in the positive direction and power following in the negative direction for each wave type. Normalised nett power is considered to be nett power divided by total input power. The input power to a structure may be calculated from the following expression [7],

$$\text{Input Power} = \frac{1}{2}|F||V|\cos\varnothing$$

where ø is the phase angle between the applied force and the velocity of the structure at the forcing position. Positive values indicate power flowing away from the excitation point.

From Figures 4-9, it may be noted that generally the proportions of flexural power decrease with increasing angle θ and the proportions of longitudinal power increase with increasing angle θ. This is to be expected as flexural waves do not propagate effectively through bends approaching 90°. Similarly, longitudinal wave propagate very effectively through bends of approximately 90° [2, 4]. Maximum values for flexural power are observed in beam 4 (Figure 6), whilst maximum values for longitudinal power are observed in beam 2 (Figure 7). The maximum values for longitudinal power occur at 90° when the structure becomes a square. This is when flexural waves convert to longitudinal waves most effeciently at the junctions. The negative value for longitudinal power observed in Figure 9 indicates that at one frequency and angle dominant power transmission in arm 4 was in the direction of arm 3 to arm 1 ie. towards the source.

5. DISCUSSION

Results are presented for normalised nett time averaged vibrational power for a closed framework of arbitrary angle under harmonic excitation. From the results of the analysis it is possible to determine frequencies and angles for maximum and minimum power on the structure. Although a simple type of structure is illustrated in this paper, there is no limit to the number of junctions or excitation points a framework may contain. The main limitation on the size of model is ill conditioning of the matrices inverted to obtain wave amplitude coefficients.

6. REFERENCES

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APPENDIX 1 - NOTATION

A	- Cross sectional area	Q	- Axial force
A_f	- Amplitude of flexural wave	S	- Shear force
A_L	- Amplitude of longitudinal wave	T	- Time period
B	- Bending moment	t	- Time
E	- Young's modulus	U	- Displacement due to longitudinal wave motion
E^*	- Complex Young's modulus	V	- Velocity
F	- Excitation force	W	- Displacement due to flexural wave motion
I	- Moment of inertia	X	- Instantaneous rate of working
I_j	- Moment of inertia of joint	x	- Distance
J_w	- Joint width		
k_f	- Flexural wave number	η	- Loss factor
k_L	- Longitudinal wave number	θ	- Angle of Arm
L	- Joint length	ρ_j	- Joint density
M_j	- Joint mass	ϕ	- Phase angle
P	- Transverse force	ψ	- Distance along Arm 2
$\langle P \rangle_f$	- Time averaged flexural power	ω	- Frequency (rad/s)
$\langle P \rangle_L$	- Time averaged longitudinal power		

APPENDIX 2 - MODEL PROPERTIES

Axial Force	=	10N
Bending Force	=	10N
Beam Breadth	=	50mm
Beam Depth	=	6mm
Length of Each Arm	=	1m
Youngs Modulus	=	207GN/m2
Density	=	7800 kg/m3
Loss factor	=	0.001

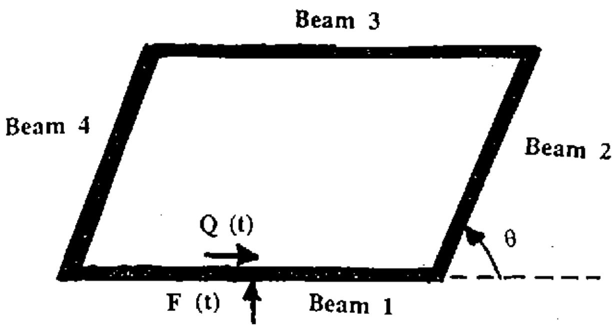

Figure 1: Rhombus Framework System

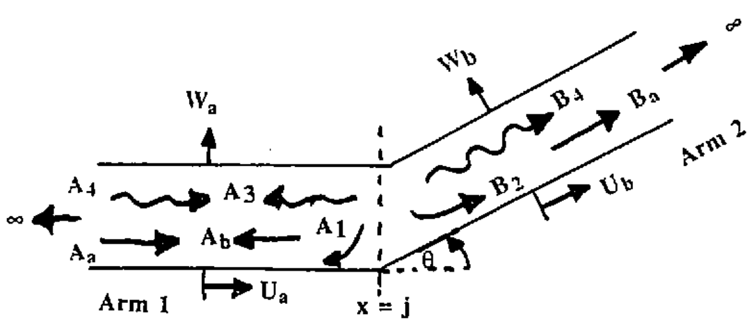

Figure 2: Wave Motion at a Bend

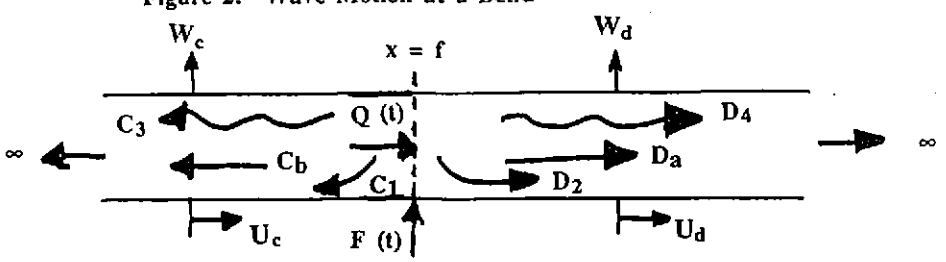

Figure 3: Wave Motion at Excitation Point

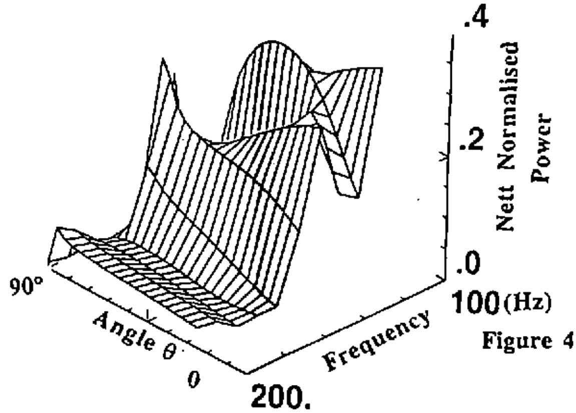

Figure 4: Beam 2 - Flexural

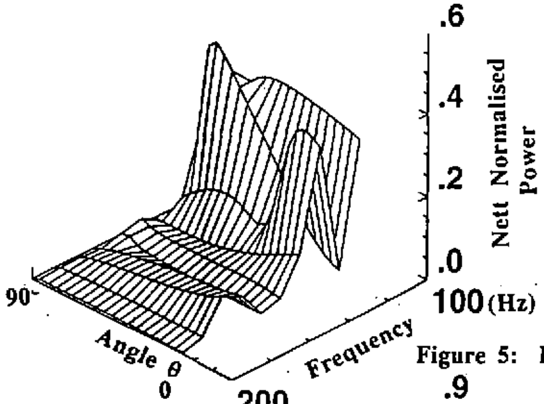

Figure 5: Beam 3 - Flexural

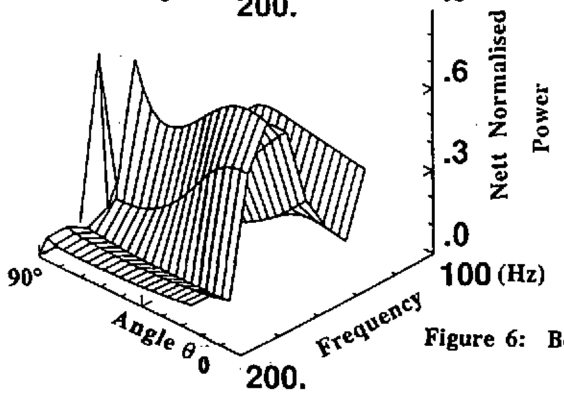

Figure 6: Beam 4 - Flexural

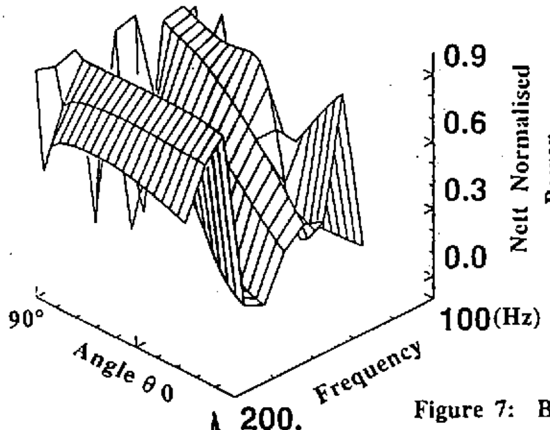

Figure 7: Beam 2 - Longitudinal

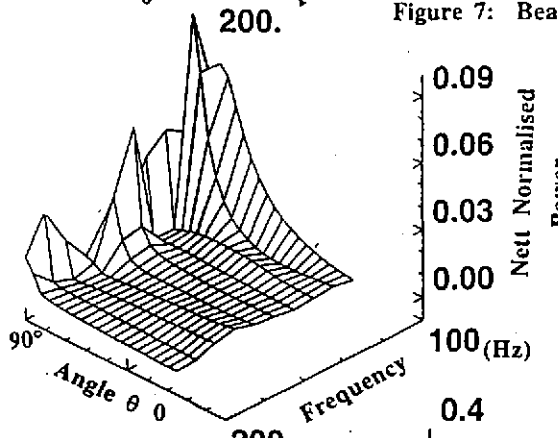

Figure 8: Beam 3 - Longitudinal

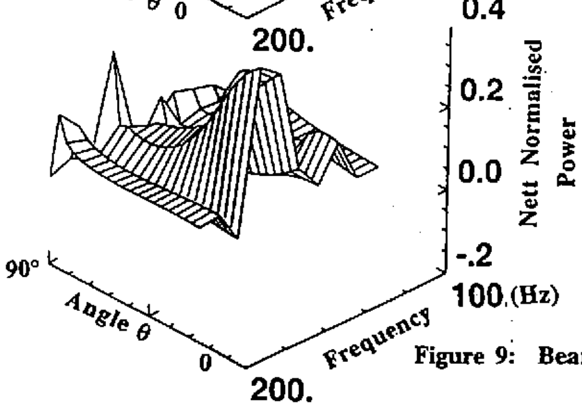

Figure 9: Beam 4 - Longitudinal