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THE IMPERFECTION OF IMAGE SOURCES MODELS OF THE FORCED RESPONSE OF FINITE PLATES

J L Horner & B A T Petersson

Dept of Aeronautical and Automotive Engineering, Loughborough University, Loughborough, Leicestershire, LE11 3TU, UK

1. INTRODUCTION

To obtain the forced vibration response of finite, thin plates with arbitrary boundary conditions it is usual to employ numerical methods. As eigen-functions can only be established explicitly for a thin plate with simply supported edges [1], the numerical approach involves for instance, a Rayleigh-Ritz formulation [2]. For the purposes of predicting responses, such a numerical approach may be time consuming, often related to slow convergence. If there are no critical requirements on the accuracy of the predicted response, rather say general trends are of interest, it is possible to consider other approximations. One such alternative approach is the use of image source models. The type of plate used for the initial investigations detailed below is a wholly simply supported one, as it is possible to compare the approximate image sources solutions with the closed form eigen-function expansion. The objective of the investigations is to determine the validity of the technique for transfer responses and number, position and grouping of image sources required to identify trends in the response. Previous investigations of image source models for plates [3], have concentrated on the point response. This investigation considers both point and transfer responses and is particularly concerned with the transition region between stiffness and resonance controlled behaviour.

2. GENERAL IMAGE SOURCE MODEL

A thin rectangular plate was modelled using up to four layers of image sources. In total eighty one individual sources could contribute to the forced plate response. By always keeping the forcing point of the plate in the bottom left hand quadrant of the plate, it was possible to observe that the image sources were always positioned in groups of four. These groups of four image sources were independent of the receiver location. The propagation from each image source was described by the solution to the wave equation for cylindrical waves in a thin, homogenous, linearly elastic plate. Thus, the transfer mobility for each source was described as follows:

$$\frac{V(r)}{F}=\left(\Gamma_{ff}^{m}+\Gamma_{nf}^{m}\right)\left(J_o(kr)-iN_o(kr)\right)+\left(\Gamma_{fn}^{m}+\Gamma_{nn}^{m}\right)\left(-\frac{2i}{\pi}K_o\,(kr)\right)$$

where r is the distance from the source to the receiver point, k the wave number, Γ_{ff} the far field reflection coefficient for far field impingement, Γ_{nf} the far field reflection coefficient for near field impingement, Γ_{fn} the near field reflection coefficient for far field impingement, Γ_{nn} the near field reflection coefficient for near field impingement, and m the number of image plate boundaries the waves traverses.

3. SIMPLY SUPPORTED PLATE

For the simply supported plated the cross-coupling between the travelling and evanescent waves at reflection vanishes and the remaining two reflection coefficients both equal -1, irrespective of angle of incidence.

The full eighty one image source model was run for Helmholtz numbers based on the distance in x-direction from the origin to the source position, in the range 0.01 to 10 to obtain the point response of a plate of aspect ratio 2:1. Source and receiver positions were given as co-ordinates, assuming the origin at the bottom left hand corner of the plate. The groups of four sources were then added together using the criterion that the groups with shortest radius from group centre to receiver position would be most dominant due to geometric attenuation effects. The results from the image source model, obtained by adding various images together, were compared with the normalised point mobility of a finite plate. This latter mobility, obtained using the eigen-function expansion, was normalised with respect to that of an infinite thin plate. It was found that a fit in the stiffness controlled region for the plate could be achieved with five groups using three layers of image sources. A good mean representation of the resonant region was obtained, as indicated in previous work [3], with especially good representation at high Helmholtz numbers.

4. TRANSFER MOBILITY OF PLATE

Various transfer mobilities were determined for the plate, with the receiver position being in a different quadrant from the forcing point. These calculations highlighted the difficulty in obtaining good approximate responses for both the stiffness controlled and resonant regions of the frequency response of the plate, once the receiver point was no longer in the same quadrant of the plate as the forcing point. For illustration consider the source position (0.33, 0.17) and the receiver position (0.67, 0.67). Figure 1 shows the transfer response obtained by adding the closest four groups using three layers. Although the mean response of the resonant region is predicted in sufficient detail, for many noise control purposes, it was impossible to obtain simultaneuosly an accurate representation of the stiffness region.

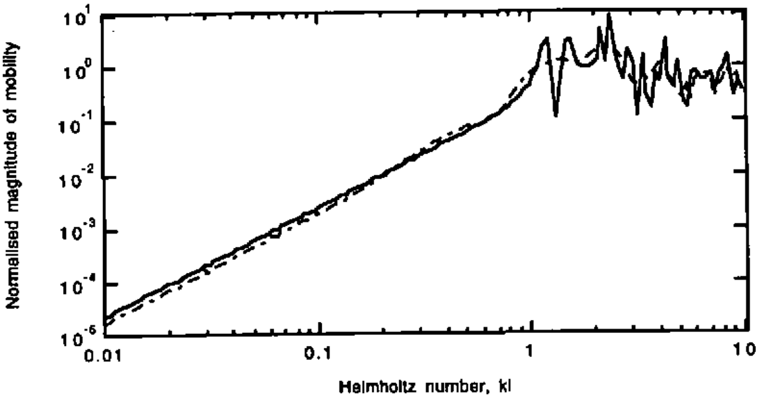

Fig 1: Transfer mobility between source (0.33, 0.17) and receiver (0.67, 0.67)

Figure 2 shows the reciprocal transfer response with the source at position (0.67, 0.67) and the receiver point at (0.33, 0.17). By adding the closest four groups for this position, requiring the use of four layers, it was possible to obtain a good fit for the stiffness line but little detail in the resonant region.

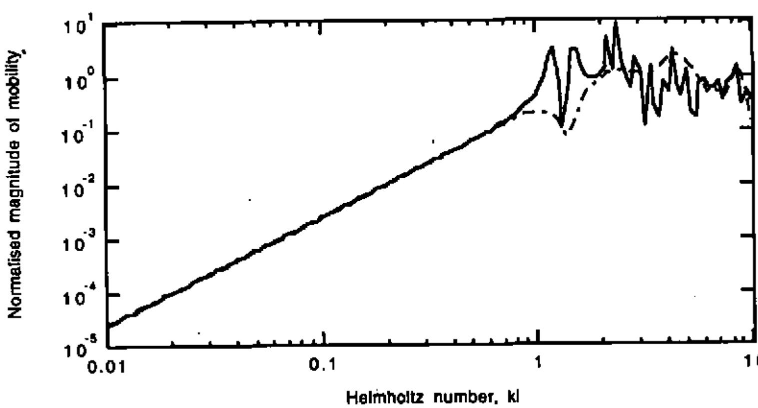

Fig 2: Transfer mobility between source (0.67, 0.67) and receiver (0.33, 0.17)

This indicated that the image source technique could only successfully describe both the stiffness and resonant regions if the source and receiver positions were geometrically close and located in one and the same quadrant of the plate. Once the receiver point moved away from that of the source only higher wavenumber data could be obtained successfully. This can be interpreted as the resonant region being predominantly controlled by the high wavenumber information which is essentially described by the corner reflection. A confirmation was obtained by predicting the transfer mobility for the reciprocal configuration, shown in Figure 2, where the receiver position was located to the corner, allowing the image sources to be at greater distances apart. This provided the long wavelength information necessary to correctly determine the stiffness line, but suppressed the spatially closest reflections required to construct the resonant region.

The most extreme case for a transfer mobility was run with the source at (0.33, 0.17) and the receiver at (1.67, 0.83) ie diagonally opposite. Regardless of the groups of sources added together it proved impossible to get either accuracy in the stiffness region, nor good average representation in the resonant region. Errors were particularly noticeable at the anti-resonances, sometimes underestimated by a factor of ten.

5. DISCUSSION

By grouping the image sources in groups of four, for balance in polarity, it was possible to obtain the average trends in the resonant region and good approximation to the stiffness line for the point response. The groups of sources were added using the criterion of shortest radius to point of interest. It proved more difficult to predict the trends of the transfer response when the source and receiver positions were in different quadrants of the plate. By employing reciprocity it was possible to improve the accuracy of either the stiffness or resonant region. When the source and receiver points were at extreme positions on the plate, it proved impossible to accurately predict average behaviour, with a limited number of image sources indicating that the method is only applicable for receiver positions either in the same quadrant of the plate as the source or in quadrants which are normal. This is particularly the case with respect to the transition region. Therefore it could be argued that the image source technique is not generally applicable for noise control work. Moreover to extend the technique to plates with non simply supported boundaries requires the calculation of all the reflection coefficients given in the general expression for a source. As each of the reflection coefficients would be unique to each source, the technique would be cumbersome to apply without further approximations.

References

[1] P.M. Morse, Vibration and Sound, 2nd Edition, (1948)
[2] S.P. Timoshenko and S. Woinowsky-Krieger, Theory of Plates and Shells, 2nd Edition (1959)
[3] E.J.Skudrzyk, The Mean-wave Method of Predicting the Dynamic Response of Dynamic Vibrators, Journal of the Acoustical Society of America, 67, 1105-1135 (1980)