EFFECTS OF ASYMMETRY ON VIBRATION TRANSMISSION IN JOINTED BEAMS

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1 INTRODUCTION

Most built-up structures contain joints connecting together various beam-like members. The transmission of vibration through the junctions, and hence the whole structure, is affected by the material/geometric properties of the beams and the angle of the junction. By selecting a suitable connection angle for a non-collinear joint, it is possible for a junction to be designed, say, for reduced transmission of a particular wave type. Analysis has been undertaken on the effects of introducing asymmetry in both a branched joint in a beam and a simple framework containing branched and angled joints. The joint model used allows for the transmission of both compressive and bending waves between beam elements, allowing the effects of wave type conversion to be established. For example, a bending wave impinging on an angled junction [1] will result in both compressive and flexural waves being reflected and transmitted. To allow the quantitative comparison between the transmission of bending waves and compressive waves in the structure, vibrational power is determined for each arm of the structure [2]. This approach allows the individual wave components to be compared rather than determine overall displacement level [3]. Once the dominant wave type is established the appropriate control techniques may be applied if necessary. It is possible to extend the analysis to consider a series of repeated periodic junctions [4].

2 MODEL OF JOINT SYSTEM

The following model of a three-branch beam system represents the joint as a simple mass with inertia [1]. All notation is defined in Section 7. Regardless of angle the joint is considered to a quarter of a cylinder of the same width and thickness as beam 1. All three arms of the branch are considered as semi-infinite beams. In arm 1, always positioned at zero angle, either a flexural or longitudinal travelling wave impinges on the junction. This results in both reflected (and transmitted) flexural and compressive waves being created in all three arms. Euler-Bernoulli beam theory is used to represent the bending waves in the system, setting an upper frequency limit on the applicability of the analysis. By considering conditions of continuity and equilibrium at the junction, the following six equations are obtained. It is assumed that all three arms of the system have identical material and geometric properties.

$$U_1 = U_2 \cos\theta - W_2 \sin\theta + \frac{L}{2}\sin\theta \frac{\partial W_2}{\partial \psi}$$

$$U_1 = U_3 \cos\alpha - W_3 \sin\alpha + \frac{L}{2}\sin\alpha \frac{\partial W_3}{\partial \phi}$$

$$W_1 = U_2 \sin\theta + W_2 \cos\theta - \frac{L}{2}(1+\cos\theta)\frac{\partial W_2}{\partial \psi}$$

$$W_1 = U_3 \sin\alpha + W_3 \cos\alpha - \frac{L}{2}(1+\cos\alpha)\frac{\partial W_3}{\partial \phi}$$

$$\frac{\partial W_1}{\partial x} = \frac{\partial W_2}{\partial \psi}$$

$$\frac{\partial W_1}{\partial x} = \frac{\partial W_3}{\partial \psi}$$

$$EI\left(\frac{\partial^2 W_1}{\partial x^2} + \frac{L}{2}\frac{\partial^3 W_1}{\partial x^3}\right) = EI\left(\frac{\partial^2 W_2}{\partial \psi^2} + \frac{\partial^2 W_3}{\partial \phi^2}\right) - \frac{L}{2}\left(\frac{\partial^3 W_2}{\partial \psi^3} + \frac{\partial^3 W_3}{\partial \phi^3}\right) - I_j \frac{\partial}{\partial t^2}\left(\frac{\partial W_1}{\partial x}\right)$$

$$EA\,\frac{\partial U_1}{\partial x} = EA\left(\frac{\partial U_2}{\partial \psi}\cos\theta + \frac{\partial U_3}{\partial \phi}\cos\alpha\right) + EI\left(\frac{\partial^3 W_2}{\partial \psi^3}\sin\theta + \frac{\partial^3 W_3}{\partial \phi^3}\sin\alpha\right) - M_j\,\frac{\partial^2 U_1}{\partial t^2}$$

$$-EI\frac{\partial^3 W_1}{\partial x^3} = EA\left(\frac{\partial U_2}{\partial \psi}\sin\theta + \frac{\partial U_3}{\partial \phi}\sin\alpha\right) - EI\left(\frac{\partial^3 W_2}{\partial \psi^3}\cos\theta + \frac{\partial^3 W_3}{\partial \phi^3}\cos\alpha\right)$$
$$- M_j\,\frac{\partial}{\partial t^2}\left(W_1 - \frac{L}{2}\frac{\partial W_1}{\partial x}\right)$$

Solution of the above equations yields the amplitudes of the reflected and transmitted waves at the junction. By considering the amplitude of the travelling waves only, it is possible to determine the time-averaged vibrational power associated with each wave type [2].

For flexural waves

$$\langle P \rangle = EI\omega k_f^3\, {A_f}^2$$

For compressive waves

$$\langle P \rangle = \frac{1}{2} EA\,\omega k_L\, {A_L}^2$$

Percentage power may be determined by dividing the power associated with a particular wave type by the power associated with the impinging wave.

3 ANALYSIS OF THREE BEAM SYSTEM

The branched beam system shown in Figure 1 was considered for a range of values of the two angles θ and α. In the following results the material and geometric properties of all three beams are identical. Each branch was considered to be of square cross-section and a frequency was chosen to give a flexural wave number of 0.48m^-1 and a compressive wave number of 4.861m^-1 Figures 2 to 4 show the vibrational power in each arm of the system. In each figure the angle of arm 2 is varied from 0 to 90° for a range of set angles of arm 3. The set angles for arm 3 are 30° (solid line), 45° (dashed line) and 60° (dotted line). In the figures, the notation FF refers to a flexural wave generated by flexural wave impingement, FL refers to a flexural wave generated by longitudinal wave impingement, LL refers to a longitudinal wave generated by longitudinal wave impingement and LF to a longitudinal wave resulting from flexural wave impingement. Percentage power is defined as power reflected (or transmitted) divided by input power. Thus percentage power of 100% indicates total reflection (or transmission).

From Figure 2, the arm carrying the impinging wave, it may be observed that the proportion of reflected flexural power resulting from flexural wave impingement increases as both the angles of arms 2 and 3 increase. This is to be expected, as the arms move away from the zero degree position the bending wave would see an increasing impedance mismatch leading to increased reflected power. A similar trend was expected in the reflected longitudinal power resulting from longitudinal wave impingement. However, although there is a general increase in reflected longitudinal power with increasing angle α, at certain angles the reflected power was zero. These positions of zero reflected longitudinal power occurred when the sum of θ and α equaled 90°. For example, when the angle of beam 3 was 60° there was zero reflected power in beam one when beam 2 was at an angle of 30°. This implies to prevent maximum transmission of longitudinal power resulting from longitudinal wave impingement the angle between beams 2 and 3 should not equal 90°. It may also be noted that the reflected power resulting from wave conversion at the junction is always smaller than the reflected power directly resulting from the impinging wave regardless of the angles of arms 2 and 3.

Figures 3 and 4 show the transmitted power in arms 2 and 3 respectively. For the case of flexural power resulting from flexural wave impingement, both arms had approximately constant levels over the range of angles. In contrast the longitudinal power resulting from longitudinal wave impingement showed more variation with angle with values reducing as the angle of arm 2 approached 90°. Comparing both figures shows that at the point of symmetry for each configuration, as expected, the transmission in arms 2 and 3 is equal. It should be noted that significant longitudinal power resulting from flexural wave impingement on the junction is present in arm 3 for small angles. This power component reduces to below 10% once the angle of beam 2 increases above 20°.

By considering the above figures and similar results for other values of wave number, it may be shown that that the introduction of asymmetry by letting $\theta < \alpha$ causes a reduction in the flexural power both reflected and transmitted in the system. In addition, there is also an increase in the longitudinal power transmitted in arm 2 but a reduction in longitudinal power transmitted in arm 3. Considering asymmetry in the opposite direction with $\theta > \alpha$ gives opposite results with slight increase in both the flexural powers reflected and transmitted in the system. The longitudinal power transmitted is reduced in arm 2 and increased in arm 3. This implies that introducing certain asymmetry in the system results in reducing flexural wave levels.

4 ANALYSIS OF SIMPLE FRAMEWORK

The branched system discussed in Section 3 was extended to consider a simple framework consisting of five beams as shown in figure 5. As the framework contained finite sections, hysteric damping was introduced to the model in the form of a complex stiffness. From this model it is possible to consider a number of different asymmetric situations. Figures 6-11 are for a configuration where arms 4 and 5 are always positioned parallel to arm 1. The angle between arms 2 and 3 is varied with θ changing from 0 to 90° and α taking three set values of 30° (solid line), 45° (dashed line) and 60° (dotted line). All other parameters are identical to the branched system discussed in Section 3. Thus this framework configuration could be seen as an extension of the branched system.

Figures 6 and 7 show the percentage power reflected in arm 1. When θ equals zero, these figures show significant reflection of flexural power resulting from flexural wave impingement as opposed to minimal reflected longitudinal power from longitudinal wave impingement for all values of α. As the angle of arm 2 increases the proportions of reflected power increase for both impingement wave types. As with the branched system, it can be seen that reflected power that results from wave type conversion is significantly smaller than the power corresponding to direct reflection.

The power transmitted in arm 4 is shown in figures 8 and 9. As arm 4 is connected to arm 2, the angle of attachment of this arm must change with θ in order to stay parallel with arm 1. Thus the angle between arms 2 and 4 must always be 180°-θ. As three different values of α are considered each figures has three points of symmetry. Figure 8 shows that the flexural power resulting from flexural wave impingement is greatest when the angle of arm 2 is zero and there is little change with the angle α. The angle of arm 3 has the greatest effect on this power component in the range α =5°-30°. Conversely, the component of flexural power in arm 4 converted from longitudinal wave impingement reaches a maximum of approximately 36% at an arm 2 angle of 12° to 16°, depending on the position of arm 3. This component shows most variability with arm 3 position when θ equals 90°. Since at this angle the waves in arm 4 will have traveled through two right-angled junctions, it not surprising that the wave conversion component shows the largest differences. The transmitted longitudinal power in arm 4 is shown in figure 9. Again this shows maximum transmission of the direct wave type when arm 2 is at zero and decreases as the angle increases. The longitudinal power resulting from longitudinal wave impingement is also insensitive to the position of arm 3 with a small, no greater than 2%, variation being seen for values of θ between 0 and 10°. Unlike the converted bending wave, the longitudinal power converted from the impingement of a bending wave is small and shows little variation with the position of arm 3.

Figures 9 and 10 show the transmitted power in arm 5. This arm stays in a constant position to arm 3 while arm 2 changes angle. Again the three line types on the figures represent three different values of angle α and hence three different angles between arms 3 and 5. From these figures it may be seen that significantly more flexural power is transmitted than longitudinal power. Also the change in α produces a greater variation in transmitted flexural power rather than transmitted longitudinal. As the wavelength of the bending waves is significantly shorter than the compressive waves, the bending waves will be more sensitive to angle changes.

Asymmetry of the type where $\theta < \alpha$ results in a decrease in the power reflected in arm 1. This is a similar observation to that made for the branched structure. Introduction of this type of asymmetry resulted in increased flexural power transmission to arm 4, irrespective of the lower beam angle. The power transmitted to arm 5 was much more dependent on the lower beam angle showing significant differences in flexural components. When $\theta > \alpha$ there was an increase in the reflected power in arm 1. As seen before, the direct transmitted power components did not alter significantly with the position of arm 2 provided α was greater than 30°. The transmitted power components that were the result of wave type conversion decreased as the asymmetry was increased.

5 DISCUSSION

Two framework systems have been analysed for the effect of arm orientation on transmitted vibration. As the results were presented in the form of percentage transmitted vibrational power the results are independent of arm geometric and material properties. The framework configuration was chosen as a logical extension of the branched structure. The results presented considered a case where the compressive wavelength was long compared to the structural dimensions. From the two systems, it may be determined that introducing asymmetry can be used to control the reflected power from the junction. The transmitted power in the system is a more complex issue but generally the greater the asymmetry the smaller the components that resulted from wave type conversion

6 REFERENCES

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2. Horner, J. L, and White, R.G. 'Prediction of vibrational power transmission through bends and joints in beam-like structures', Journal of Sound and Vibration 147, 1991, pp 87-103.
3. Von Flotow, A.H. 'Disturbance propagation in structural networks', Journal of Sound and Vibration 106, 1986, pp433-450
4. Langley, R.S. 'Power transmission in a one-dimensional periodic structure subjected to single-point excitation', Journal of Sound and Vibration 185, 1995,pp552-558

7 NOTATION

A_f	flexural wave amplitude
A_l	compressive wave amplitude
A	cross-sectional area
E	modulus of elascity
I	second moment of area
I_j	interia of joint
k_f	flexural wave number
k_l	compressive wave number
L	length of joint
M_j	mass of joint
t	time
U	compressive displacement
W	bending displacement
x	position on arm 1
α	angle for arm 2
β	angle of arm 4
θ	angle for arm 3
τ	angle of arm 5
ϕ	position on arm 3
φ	position on arm 2
ω	frequency (radians per second)

FIGURES

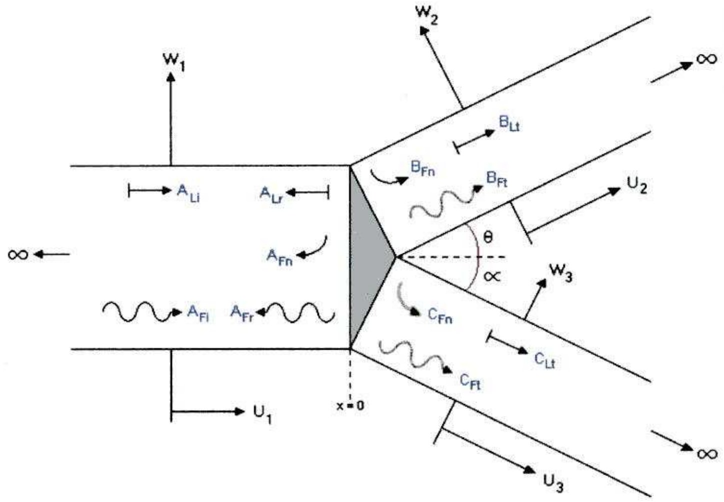

Fig 1: Branched Beam

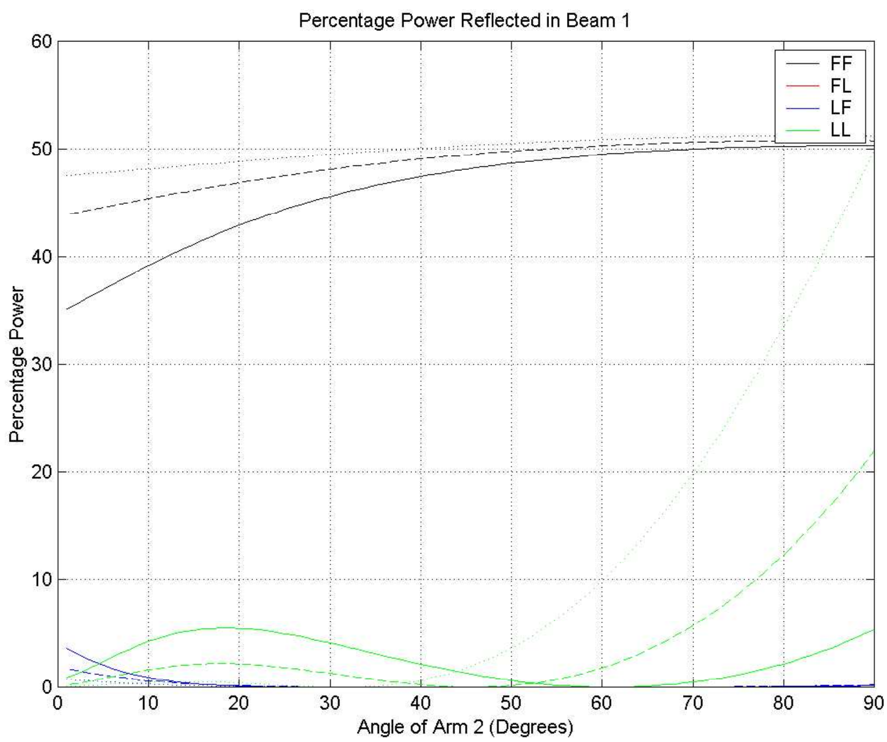

Fig 2: Percentage Power Reflected in Beam 1

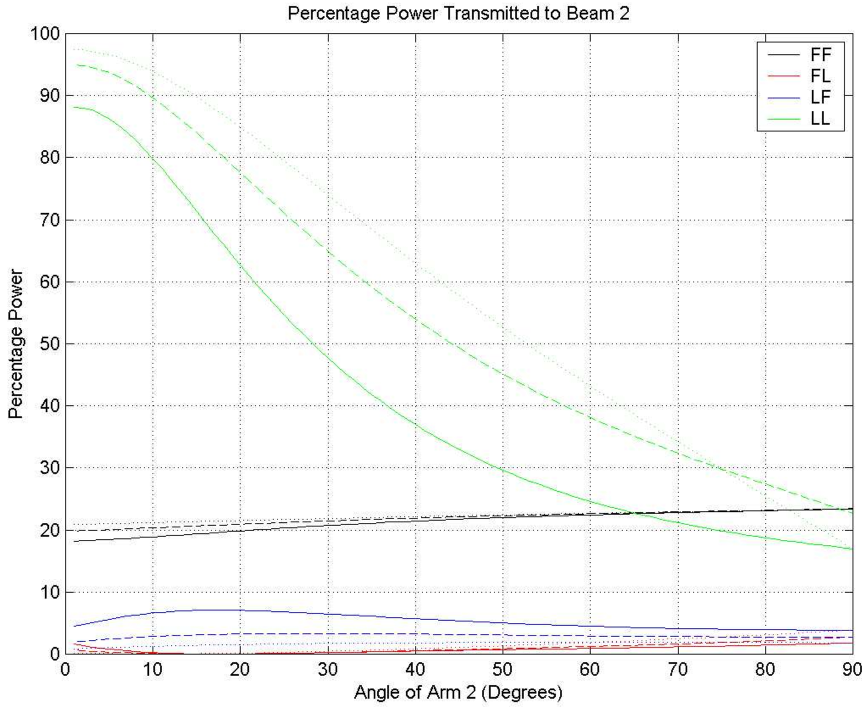

Fig 3: Percentage Power Transmitted to Beam 2

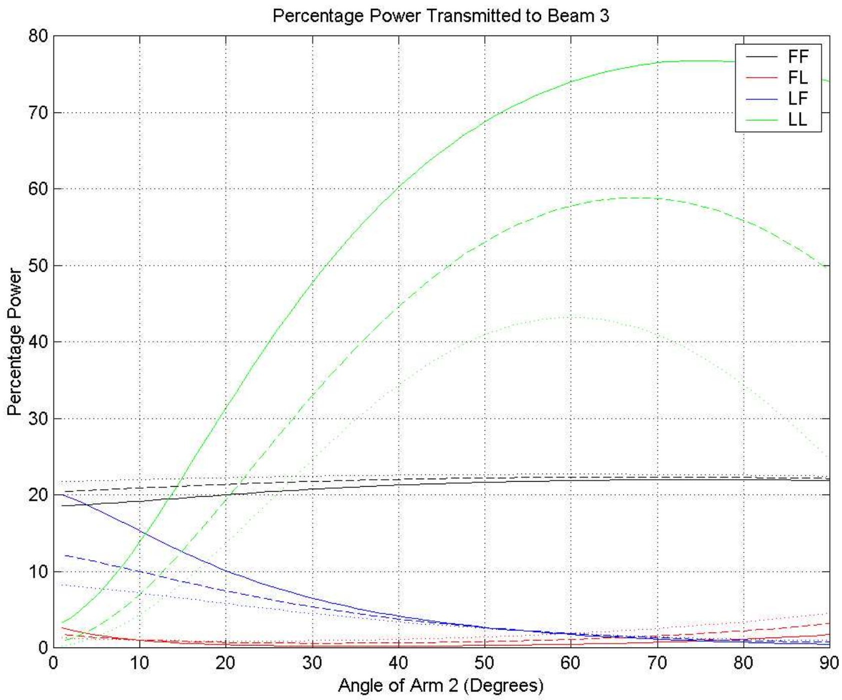

Fig 4: Percentage Power Transmitted to Beam 3

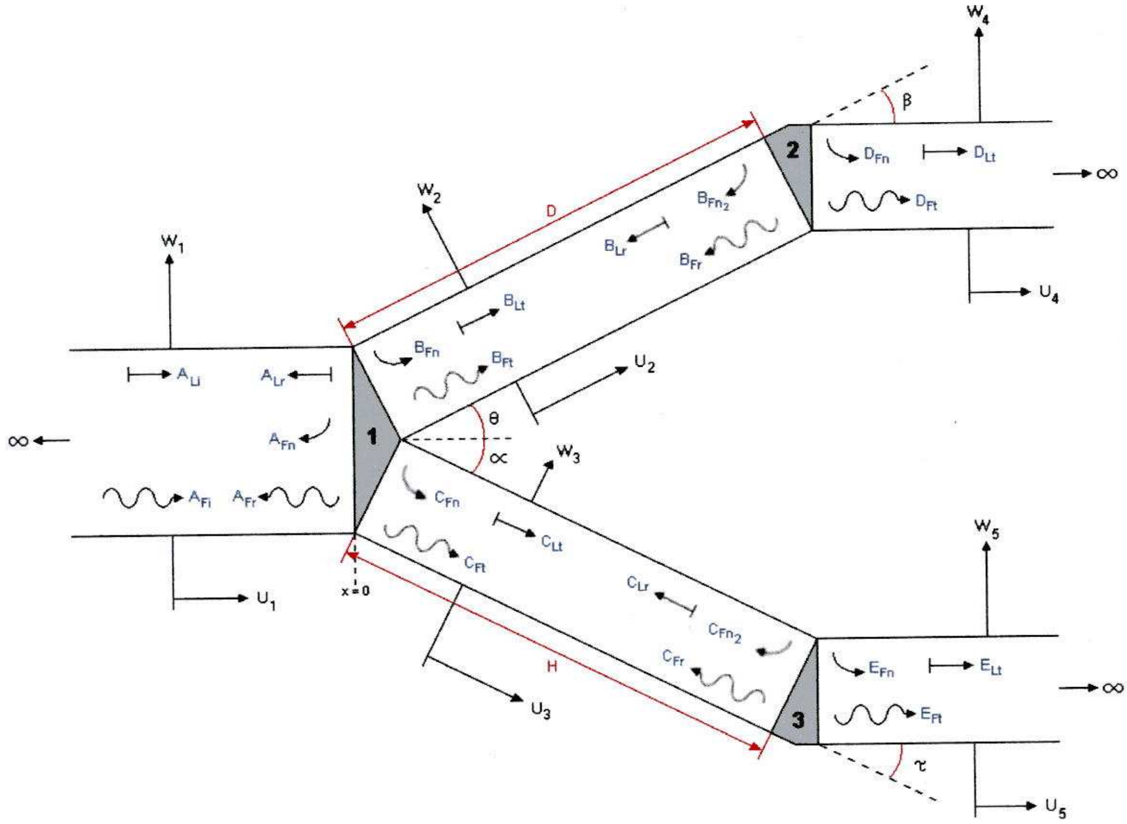

Fig 5: Multiple Linked Joints

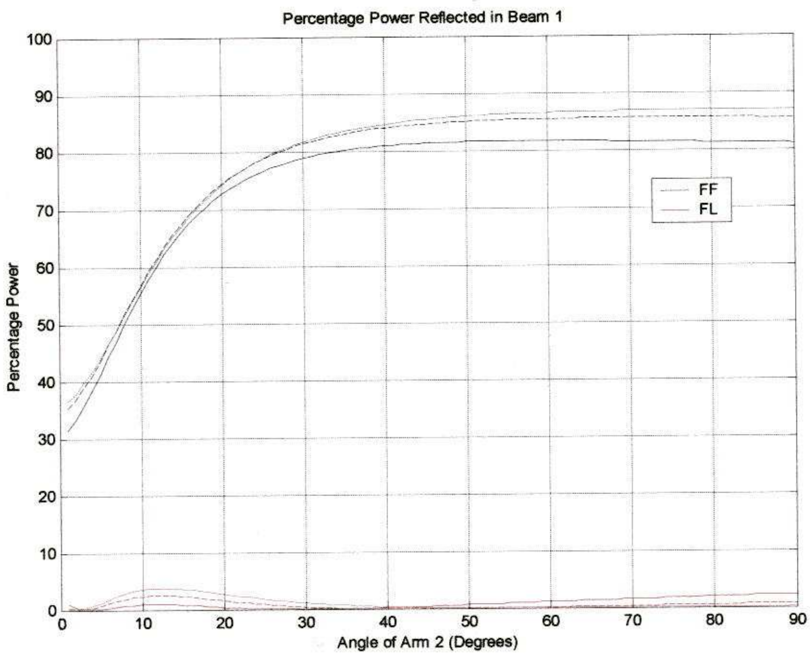

Fig 6: Percentage Power of Longitudinal Waves Reflected in Beam 1

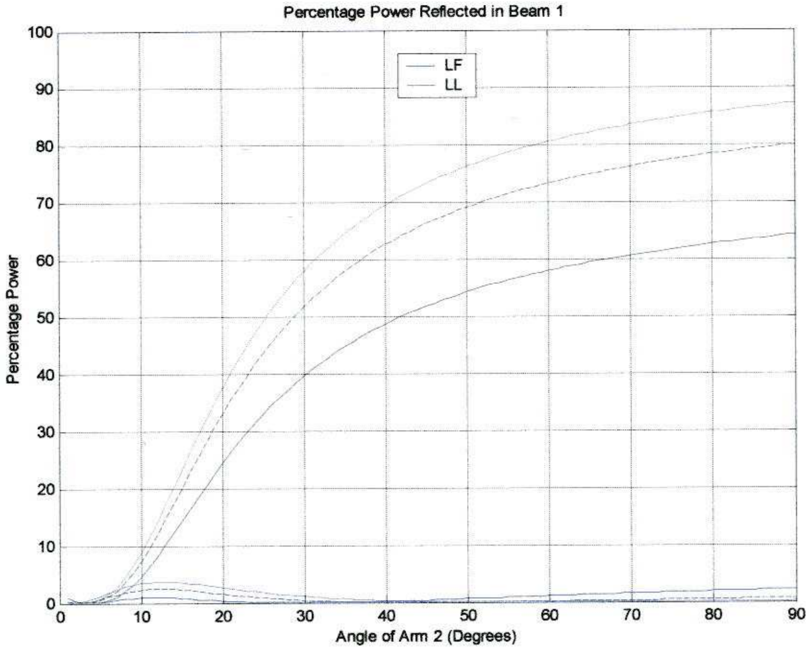

Fig 7: Percentage Power of Longitudinal Waves Reflected in Beam 1

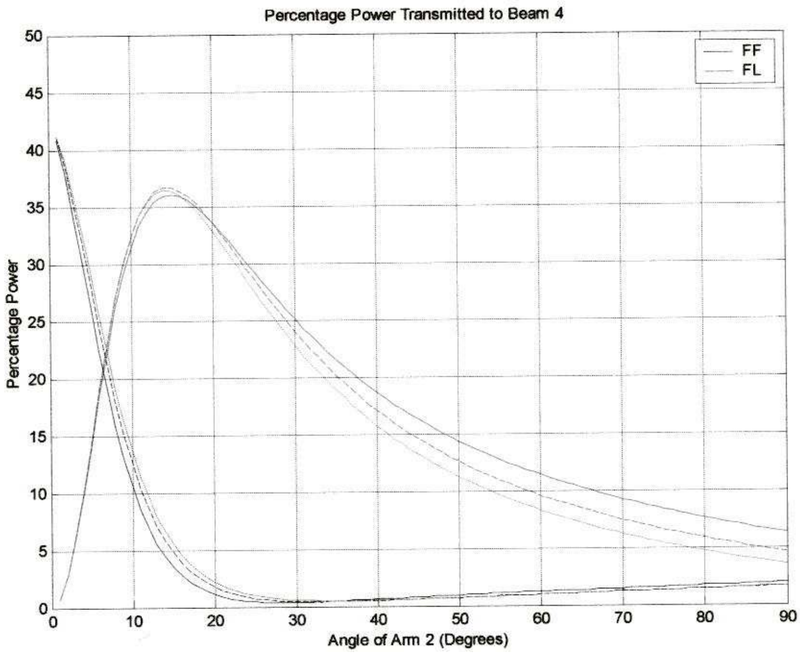

Fig 8: Percentage Power of Longitudinal Waves Transmitted to Beam 4

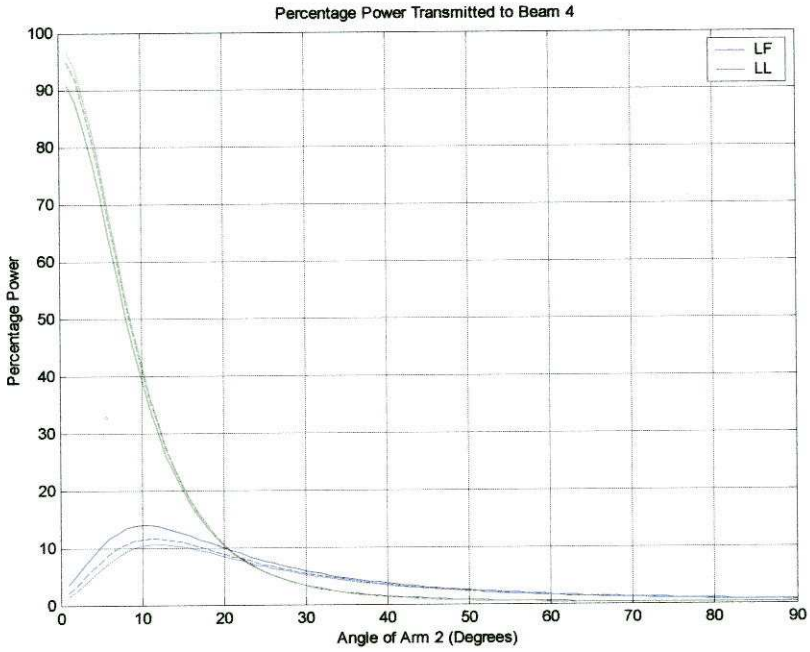

Fig 9: Percentage Power Longitudinal Waves Transmitted to Beam 4

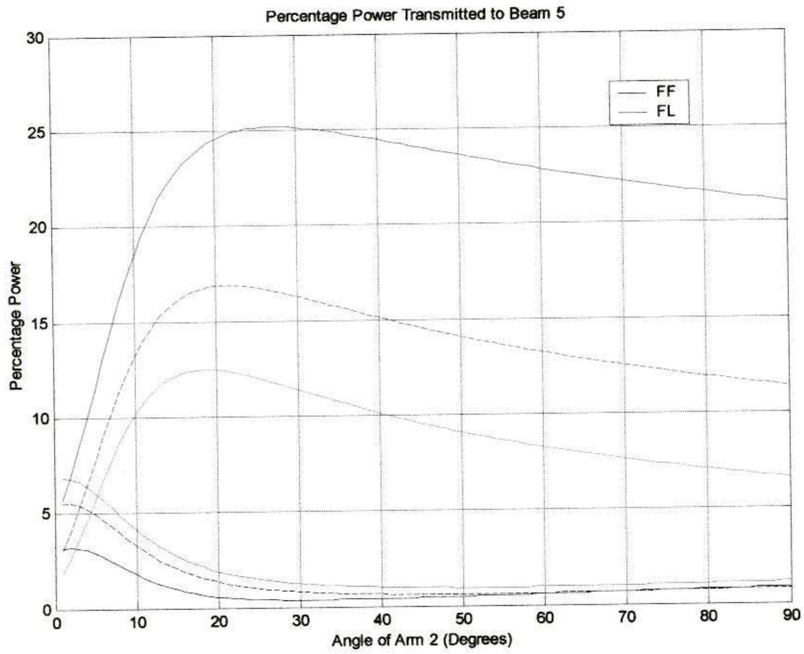

Fig 10: Percentage Power of Flexural Waves Transmitted to Beam 5

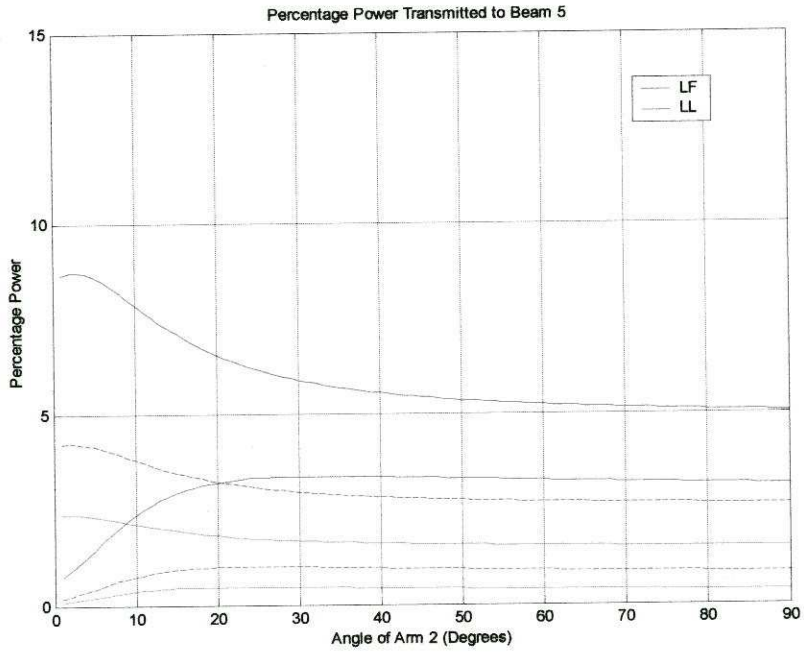

Fig 11: Percentage Power of Longitudinal Waves Transmitted to Beam 5