AXISYMMETRIC WAVE PROPAGATION IN BURIED, FLUID-FILLED PIPES: EFFECTS OF THE SURROUNDING MEDIUM

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1. INTRODUCTION

Water leakage from buried pipes is a subject of great concern in Britain, and across the world because of decreasing water supplies due to changing rainfall patterns, deterioration or damage to the distribution system, and an ever increasing population. A significant amount of water can be lost due to leakage, and over the past few years much attention has been focused on trying to reduce this wastage of resources. Correlation techniques are widely used in leak detection [1], but for these to be effective, the propagation wavespeeds and wave attenuation must be known. Relatively predictable for metal pipes, these are largely unknown for the newer plastic pipes, being highly dependent on the pipe wall properties and the surrounding medium.

In a previous paper [2], pipe equations for n=0 axisymmetric wave motion were derived for a fluid filled pipe, surrounded by an infinite elastic medium which could sustain both longitudinal and shear waves. These equations were solved for two wave types, s=1,2, corresponding to a fluid dominated wave and an axial shell dominated wave. Both of these wave types involve motion of the shell and the fluid. Solutions were expressed in terms of a complex wavenumber for each wave, the real part of which gives the wavespeed, and the imaginary part of which gives the wave attenuation.

In this paper, the focus of attention is the fluid dominated wave, and the effect different surrounding media have on its wavespeed and attenuation. Two distinct regimes are considered for the case when only one wave type is present in the surrounding medium: high wavespeed in the surrounding medium, and low wavespeed in the surrounding medium.

2. THE WAVENUMBER EXPRESSIONS

In previous work [2] the following expression for the wavenumber, k_1, for the axisymmetric (n=0) fluid-dominated (s=1) wave in a buried, fluid-filled pipe was derived:

$$k_1^2 = k_f^2\left(1+\frac{\dfrac{2B_f}{a}}{\dfrac{Eh}{a^2}-\omega^2\left(\rho h+M_{rad}\right)+i\left(\omega R_{rad}+\dfrac{\eta Eh}{a^2}\right)}\right) \qquad (1)$$

where k_f is the contained fluid wavenumber; ω is the angular frequency; B_f is the bulk modulus of the contained fluid; a and h are the radius and thickness of the shell wall respectively (h<<a); E and η are the shell material Young's modulus and loss factor respectively; ρ is the density of the shell material; and M_{rad} and R_{rad} are the mass and resistance components of the radiation impedance, z_{rad}, of the surrounding medium at the pipe wall, such that

$$z_{rad} = R_{rad} + i\omega M_{rad} = \sum_{m} \frac{-i\rho_m c_m k_m}{k_{m1}^r} \frac{H_0\left(k_{m1}^r a\right)}{H_0'\left(k_{m1}^r a\right)} \quad (2)$$

ρ_m, c_m, and k_m are the density, wavespeed and wavenumber respectively of each wavetype present in the surrounding medium, and the summation is performed over all wavetypes present. k_{m1}^r, is the radial component of the wavenumber in the surrounding medium, given by

$$\left(k_{m1}^r\right)^2 = k_m^2 - k_1^2 \quad (3)$$

H_0 is a Hankel function of the second kind, representing outgoing waves, and the prime denotes differentiation with respect to the argument. It is assumed that the surrounding medium is of infinite extent, so that no incoming waves are present. When the argument of the Hankel function is purely (or predominantly) real, it is found that the radiation impedance is complex, with positive real and imaginary components [2,3]. Conversely, when the argument of the Hankel function is purely (or predominantly) imaginary, the resulting radiation impedance is purely (or predominantly) imaginary and mass-like (i.e. positive) [2,3].

Expressing k_1 in the form of equation (1) allows the individual terms to be readily identified as stiffness components of the contained fluid ($2B_f/a$) and the pipe wall (Eh/a^2), a pipe wall mass component ($\rho h\omega^2$), and the radiation mass and resistance of the surrounding medium (M_{rad} and R_{rad}). Alternatively, equation (1) may be re-expressed in terms of the impedances of the fluid, pipe wall and surrounding medium as

$$k_1^2 = k_f^2\left(1 + \frac{z_{fluid}}{z_{pipe} + z_{rad}}\right) \quad (4)$$

where $z_{fluid} = \dfrac{-2iB_f}{a\omega}$ and $z_{pipe} = i\left(\rho h\omega - \dfrac{Eh}{a^2\omega}\right)$

3. EFFECTS OF THE SURROUNDING MEDIUM

From equations (1)-(3), it can be seen that the radiation impedance of each wave in the surrounding medium is dependent on the radial component, k_{m1}^r, of its wavenumber k_m, where the axial component is k_1. k_1 must therefore strictly be found recursively. However, an overall understanding of the factors controlling k_1 may be gained by examining certain extreme conditions, in particular very high or very low wavespeeds in the surrounding medium.

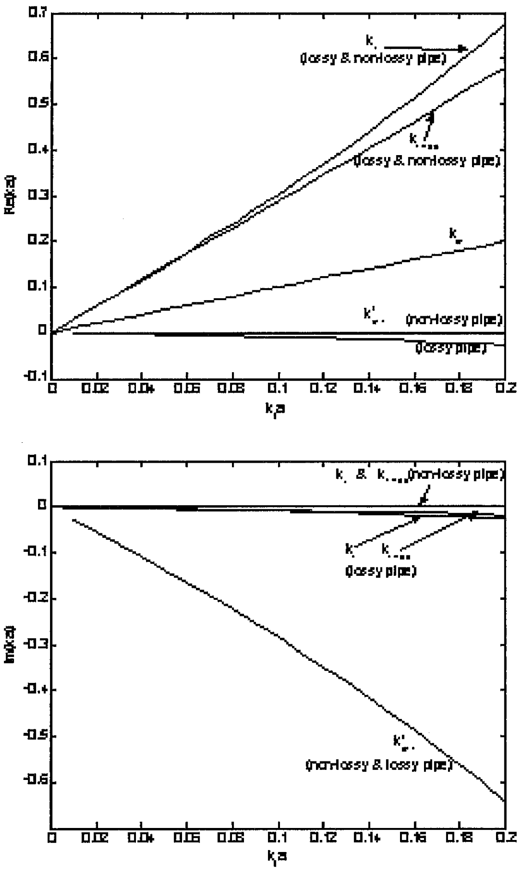
3.1 HIGH WAVESPEED IN THE SURROUNDING MEDIUM

Consider firstly the case when the wavespeed in the surrounding medium is very large. As a result, its corresponding wavenumber, k_m, will be very small. From equation (3), the radial component of this wavenumber in the surrounding medium then becomes

$$\left(k_{m1}^{r}\right)^2 \approx -k_1^2 \qquad (5)$$

At this stage, if it is assumed that k_1 will either be purely or predominantly real, the radial wavenumber will be purely or predominantly imaginary and of approximately the same magnitude as Re{k_1}. The resultant radiation impedance, z_{rad}, will then be wholly or predominantly imaginary, and mass-like.

Figures 1a and 1b depict the real and imaginary components of k_1, k_{m1}^{r}, k_m, and k_{1vac} (the pipe axial wavenumber for the case of no surrounding medium) calculated from equations (1)-(3) for a typical fluid-filled plastic pipe surrounded by an elastic medium supporting only one wavetype with a high wavespeed. The wavenumbers are calculated for both a lossy and non-lossy pipe wall material. The surrounding medium is considered to be non-lossy. The pipe and media properties are shown in table 1. Figure 2 depicts the real and imaginary components of the radiation impedance along with the fluid and pipe wall impedance components.



(a) (b)

Figure1
Component Wavenumbers
(wavespeed in the surrounding medium is large compared with that of the pipe)
(a) real part; (b) imaginary part

Thickness/radius ratio	0.125

Young's modulus (N/m^2)	5.0x10^9
Density (kg/m^3)	2000
Poisson's ratio	0.4
Material loss factor	0.065

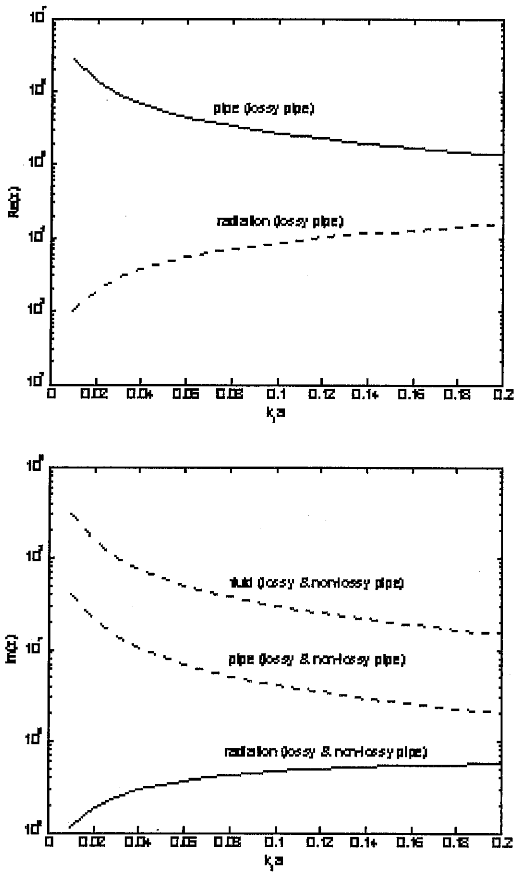
(a)

	Surrounding medium (high wavespeed)	Surrounding medium (low wavespeed)	Contained fluid
Density (kg/m^3)	2000 (1500,2500)	2000	1000
wavespeed (m/s)	1500	200	1500

(b)

Table 1
(a) Pipe Properties; (b) Media Properties
figures in brackets indicate values used in additional parameter study

With reference to figure 1a, it can be seen that for both the lossy and non-lossy case, the effect of the surrounding medium is to increase the real part of the wavenumber of the s_1 wave from the *in-vacuo* case. This is as expected given that the presence of the surrounding medium effectively mass loads the pipe. For the non-lossy pipe, it can be seen that k_1 is purely real, k^r_{m1} is purely imaginary, and z_{rad} is purely imaginary, so the s_1 wave does not radiate into the surrounding medium, as, expected. For the lossy pipe (figure 1b), the imaginary part of k_1 is much greater than for the equivalent *in-vacuo* case. This might suggest that, even though the wavespeed in the surrounding medium is high, the s_1 wave radiates into the surrounding medium. However, examination of the component impedances in figure 2 shows that this is not, in fact, the case. The real part of the radiation impedance (radiation resistance) is extremely small compared with the pipe wall loss term (and in fact negative), and the cause of the increase in wave attenuation is the slowing down of the wave and concomitant decrease in wavelength, rendering the pipe wall loss mechanism more effective.



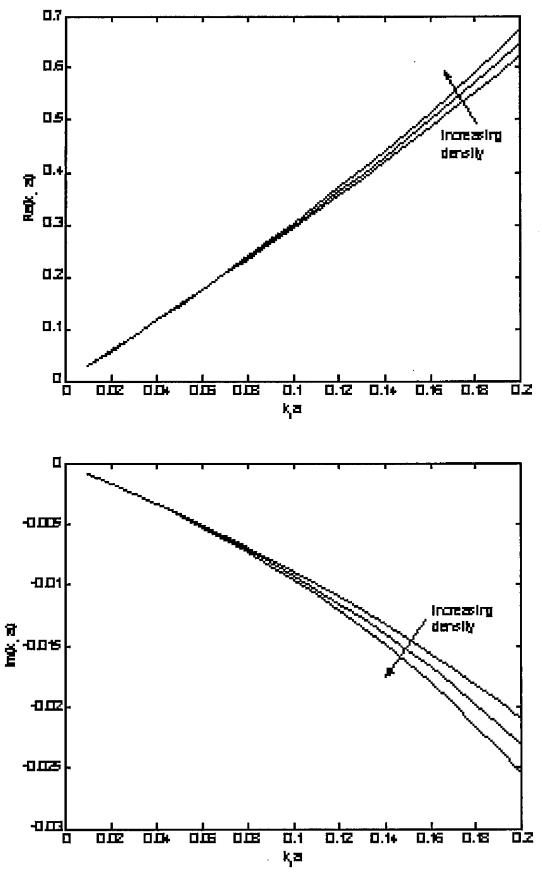
(a) (b)

Figure 2
Component Impedances
(wavespeed in the surrounding medium is large compared with that of the pipe)
(a) real part (pipe & radiation components are zero for non-lossy pipe); (b) imaginary part
(dotted lines indicate negative values)

The negative radiation resistance suggests that energy is being transferred from the surrounding medium to the pipe, which, at first sight, might seem surprising. However, the predominantly imaginary radiation impedance means that the disturbance in the surrounding medium is a nearfield one, decaying radially; but this wavefield must also decay along the pipe at the same rate as the s_1 wave. The only loss mechanism available is the pipe wall loss, so energy must be entrained from the surrounding medium into the pipe wall in order for the required decay to occur. Furthermore, what energy remains in the surrounding medium suffers no loss as this medium is assumed to be non-lossy.

3.1.1 Effects of Changes in Density

Figures 3a and 3b show the effect of changes in the density of the surrounding medium whilst keeping the wavespeed the same, for a lossy pipe. As expected, it is found that decreasing the medium density decreases the magnitude of both the real and imaginary components of the radiation impedance, and increasing the density increases the radiation impedance. The increase in radiation mass loading with increased density results in a greater reduction in the s_1 wavespeed compared with the *in-vacuo* case. Correspondingly, decreases in density result in a decrease in the mass loading and hence a lesser reduction in sound speed, compared with the *in-vacuo* case (figure 3a). Two opposing mechanisms, however, control the attenuation of the s_1 wave (imaginary part of the wavenumber, as shown in figure 3b). Considering the case of increased density in the surrounding medium, the reduction of the wavespeed of the s_1 wave renders the pipe wall loss mechanism more effective, which tends to increase the wave attenuation; however, the increase in the radiation resistance (which becomes increasingly negative) counteracts the pipe loss terms, as discussed previously, and tends therefore to decrease the wave attenuation. As can be seen from the graphs, the effect due to the change in wavespeed dominates, and increases in density results in greater wave attenuation. Similarly, decreases in the surrounding medium density result in reduced wave attenuation.



(a) (b)

Figure 3
Effect of changes in density of surrounding medium on s_1 wavenumbers
(wavespeed in the surrounding medium is large compared with that of the pipe)
(a) real part; (b) imaginary part

3.1.3 Effect of decreasing the Wavespeed

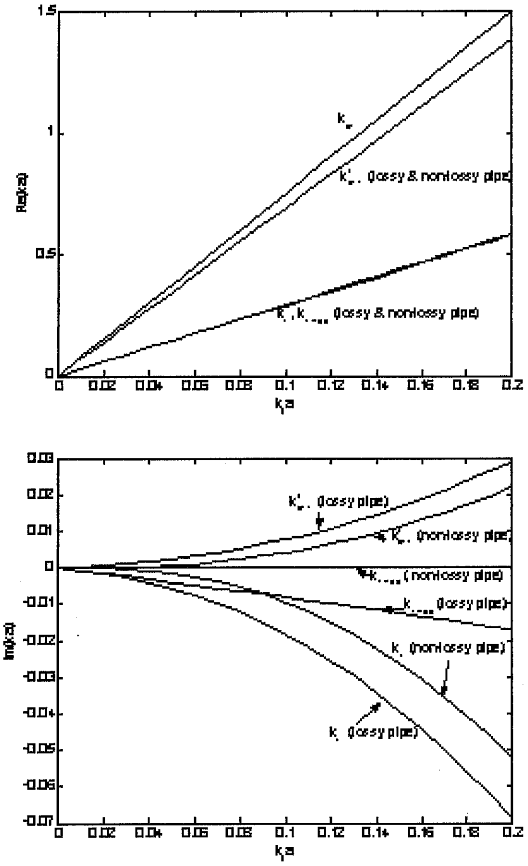
As the wavespeed in the surrounding medium decreases (for the same medium density), the wavenumber, k_s, becomes larger so, with reference to equation (3), the magnitude of the radial component, k_{s1}^{r} starts to decrease whilst remaining predominantly (or wholly) imaginary. The radiation impedance remains predominantly (or wholly) imaginary, but its magnitude is increased relative to the very high wavespeed case. However, the increase in mass loading (even with halving the wavespeed) is found to be small, and as a result the corresponding change in wavespeed (not shown) is small. As for changes in density, discussed in the previous sub-section, two opposing mechanisms control the wave attenuation. However, in this case, the relative change in the radiation resistance compared with the reactance is increased. The pipe wall loss mechanism is found not to become much more effective, so these opposing mechanisms roughly balance so that the wave attenuation is more or less unaltered, or with a sufficient wavespeed decrease the dominant mechanism changes so the wave attenuation can actually decrease.

3.2 LOW WAVESPEED IN THE SURROUNDING MEDIUM

Consider now the case when the wavespeed in the surrounding medium is very small. As a result, its corresponding wavenumber, k_m, will be very large. From equation (3), the radial component of this wavenumber in the surrounding medium then becomes

$$\left(k_{m1}^r\right)^2 \approx k^2 \tag{6}$$

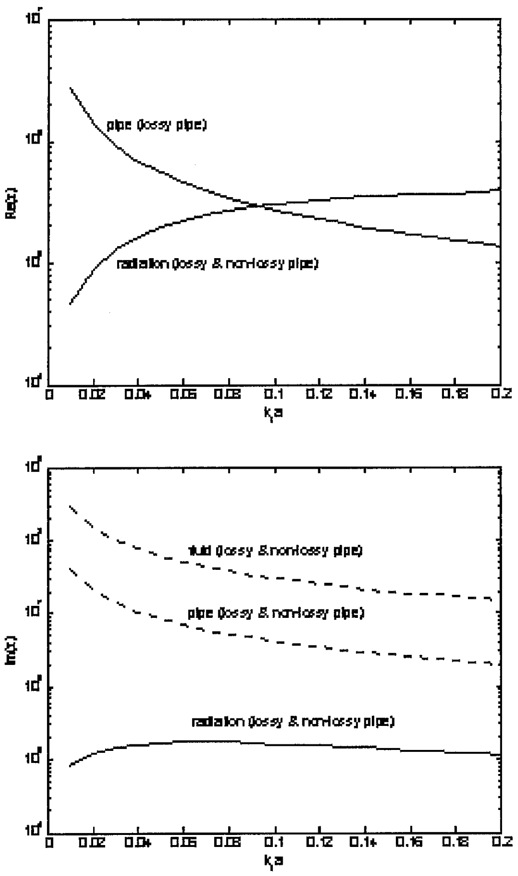
suggesting that the radial wavenumber will be purely real and large or, if k_1 has an imaginary component, then predominantly real and large. The resultant radiation impedance, z, will then have positive real and imaginary components, indicating that the surrounding medium mass loads the pipe and that energy radiates from the pipe into the medium.



(a) (b)

Figure 4
Component Wavenumbers
(wavespeed in the surrounding medium is small compared with that of the pipe)
(a) real part; (b) imaginary part

Figures 4a and 4b depict the real and imaginary components of k_1, k^r_{w1}, k_w, and k_{1vac} (the pipe axial wavenumber for the case of no surrounding medium) and figures 5a and 5b depicts the real and imaginary components of the radiation impedance along with the pipe wall and fluid impedances, for both a lossy and non-lossy pipe wall material. The pipe and media properties can be found in table 1 and again it is assumed that the surrounding medium is not lossy. With reference to figure 4a, it can be seen that for both the lossy and non-lossy case, the effect of the surrounding medium on the real part of the wavenumber, i.e. the wavespeed, is negligible (although, in fact the wavespeed is slightly reduced). This is as expected given that (see figure 5b) the imaginary part of the radiation impedance is very small compared with the pipe wall terms. From figure 4b it can be see that for both the non-lossy and lossy pipe, the imaginary part of the s_1 wavenumber increases in the presence of the surrounding medium. For the lossy pipe, the real part of the radiation impedance (figure 5a) is comparable to the pipe wall loss term except at very low frequencies, and so it has an observable effect. In this case, therefore, the increased attenuation is due to radiation into the surrounding medium, the increase for the lossy and non-lossy pipe being similar. The effect of pipe wall loss does not change in the presence of the surrounding medium as the wavespeed is largely unaffected.

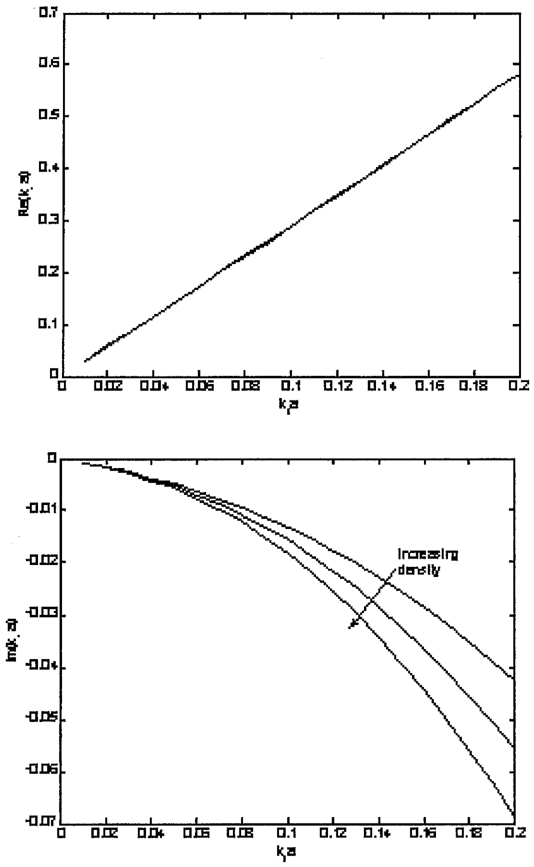


(a) (b)

Figure 5
Component Impedances
(wavespeed in the surrounding medium is small compared with that of the pipe)
(a) real part (pipe component is zero for non-lossy case); (b) imaginary part
(dotted lines indicate negative values)

3.2.1 Effect of Changes in Density

Figures 6a and 6b show the effect of changes in the density of the surrounding medium whilst keeping the wavespeed the same, for a lossy pipe. As for the high wavespeed case, decreasing the medium density decreases the magnitude both the real and imaginary components of the radiation impedance, and increasing the density increases the radiation impedance. However, unlike the high wavespeed case, as the radiation resistance is positive and the wave radiates, increasing the radiation resistance results in increased wave attenuation, shown in figure 6b. However, it is found that, even with large increases in density, the imaginary part of the radiation impedance is negligible compared with the pipe terms, so no marked change in wavespeed would be expected. This is borne out by the wavenumber lines depicted in figure 6a, showing that the changes in density have no noticeable effect on the wavespeed.



(a) (b)

Figure 6

Effect of changes in density of surrounding medium on s_1 wavenumbers

(wavespeed in the surrounding medium is small compared with that of the pipe)

(a) real part; (b) imaginary part

3.2.2 Effect of increasing the Wavespeed

As the surrounding medium wavespeed increases (for the same medium density), the wavenumber, k_m, becomes smaller so, with reference to equation (3), the magnitude of the radial component, k_{1m}^r, starts to decrease whilst remaining predominantly real. The radiation impedance remains complex, but its magnitude is increased relative to the very low wavespeed case. The surrounding medium starts to have a small observable effect on the s_1 wavespeed, which is further reduced (not shown). The effect on the imaginary part of the wavenumber also becomes more pronounced, and the wave attenuation increases. This increase is largely due to increased radiation into the surrounding medium, but, in addition, the wave slowing will also render the pipe wall loss mechanism slightly more effective.

4. CONCLUSIONS

In this paper, the effects of the surrounding medium on the wavespeed and attenuation of the 'fluid-borne' (s=1), axisymmetric (n=0) wave in a fluid-filled buried pipe have been investigated.

Two regimes, in particular, have been studied: high wavespeed in the surrounding medium; and low wavespeed in the surrounding medium.

For the high wavespeed case, it has been found that the presence of the surrounding medium slows the s=1 wave; the greater the medium density, the greater the reduction in wavespeed, and the lower the medium wavespeed, the greater the reduction in wavespeed (although this effect is small). The wave does not radiate into the surrounding medium, but if the pipe wall material is lossy, the wave attenuates as it propagates. In general, it has been found that the greater the medium density, the greater the wave attenuation; decreasing the medium wavespeed may either increase or decrease the wave attenuation, depending on the relative effects of pipe wall loss and entrainment of energy from the surrounding medium.

For the low wavespeed case, the s=1 wave is slightly slowed by the presence of the surrounding medium, but under most circumstances this slowing is negligible; however, the greater the medium density, the greater the reduction in wavespeed, and the higher the medium wavespeed, the greater the reduction in wavespeed. The wave does radiate into the surrounding medium; the greater the medium density, the greater the wave radiation and wave attenuation, and the higher the medium wavespeed, the greater wave radiation and wave attenuation.

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