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A COUPLED FEM/BEM MODEL FOR HANDLING THE VIBRO-ACOUSTIC RESPONSE OF STRUCTURES SUBJECTED TO RANDOM EXCITATIONS

J P Coyette & C Lecomte

Numerical Integration Technologies, Leuven, Belgium

1. INTRODUCTION

The characteristics of the dynamic response of a mechanical structure depend on one hand on the characteristics of the load (e.g. frequency content) and on the other hand on structural properties (stiffness, mass and damping). In general, time dependent loads show statistical variations and consequently the response is not deterministic. Since time dependency is involved, a simple characterization of the load as a random variable is not sufficient. For this reason, the concept of random process has been introduced[1,2]. At the same time, the studied mechanical structures can interact with the surrounding fluid in a way such that the dynamical response is perturbed. The handling of such kinematical and mechanical coupling effects requires the selection of appropriate numerical models. The unbounded character of the fluid domain calls also for the use of a boundary integral formulation which is able to handle exactly the Sommerfeld radiation condition at infinite distance. The coupled model presented here is precisely based on the selection of a displacement-based FEM model for the structure while a BEM model is selected for the acoustic fluid. This BEM model relies on an indirect boundary integral representation particularly well suited for thin structures (where the thickness is assumed to be small versus the acoustic wavelength). An efficient solution procedure is proposed in order to compute random mechanical and acoustical responses.

2. RANDOM EXCITATIONS

Characterization of random excitations

The considered excitations can be classified into two main categories: (1) mechanical and (2) acoustical. The mechanical excitations refer to prescribed mechanical loads (F) and/or prescribed displacements (u) or

accelerations (a) along supports. Acoustical excitations can be induced by sources with a random amplitude (A). It is assumed that each random excitation gives rise, in this discrete model context, to a load vector which appears as the product of a deterministic load vector (called a *load pattern*) by the related parametrized random variable (F, u, a or A). The particular case of a diffuse field (as produced in a reverberant room) deserves some attention since it implies (at least in principle) the consideration of a great number of acoustic sources. This could be impractical for some applications so that an alternative procedure based on the analytical investigation of an infinite number of plane waves could be used. This approach is addressed at the end of this section.
Whatever the key characteristic of the random excitation (F_i or u_i or a_i or A_i) is, his random nature could be defined in the same way. Let us denote by x_i such an excitation.
The random character of $x_i(t)$ can be described by referring to weakly stationary random processes. The key characteristics of any weakly stationary process $x_i(t)$ are the mean (which is constant) and the auto-correlation function $R_{x_i}(\tau)$ given by:

$$R_{x_i}(\tau) = E[x_i(t)x_i(t+\tau)] \tag{1}$$

Additionally the Fourier transform of the auto-correlation function plays an important role as a useful descriptor of the considered stationary process. This function is known as the power spectral density (PSD) function and is given as:

$$S_{x_i}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} R_{x_i}(\tau)e^{-i\omega\tau}d\tau \tag{2}$$

When a system is subjected to different random excitations, cross-correlation functions $R_{x_i x_j}(\tau)$ can be introduced in order to characterize the interdependency between load component values:

$$R_{x_i x_j}(\tau) = E[x_i(t)x_j(t+\tau)] \tag{3}$$

Again, the Fourier transform of these cross-correlation functions are the cross power spectral densities given by

$$S_{x_i x_j}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} R_{x_i x_j}(\tau)e^{-i\omega\tau}d\tau \tag{4}$$

Characterization of a diffuse acoustic field

A diffuse acoustic field is usually obtained by activating acoustic sources in a reverberant room (Figure 1). The multiple reflections along the boundary walls lead to the so-called 'diffuse' field. A formal way for getting such a diffuse acoustic field consists in superimposing an infinite number of plane waves having different propagation directions. This allows to set up the cross-PSD function for the acoustic pressure at two different locations along the considered structure (Figure 1). The diffuse field characteristics are obtained by averaging the above quantity for all wave directions so that the cross-PSD is given by:

$$\tilde{S}_{p_1p_2}(\omega)=\frac{\int \tilde{S}_{p_1p_2}(\omega)d\vec{k}/|\vec{k}|}{\int d\vec{k}/|\vec{k}|}=S_{p_0}(\omega)\frac{\int \exp\left(-i\vec{k}\cdot(\vec{r}_1-\vec{r}_2)\right)d\vec{k}/|\vec{k}|}{\int d\vec{k}/|\vec{k}|} \tag{5}$$

where $\vec{r}_1$ and $\vec{r}_2$ give the location of the two considered points while $\vec{k}$ is the wave vector.

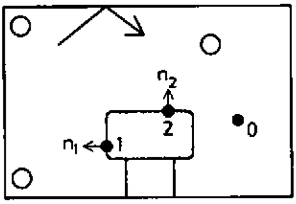

Figure 1: Reverberant chamber.

In a similar way, the cross-PSD function for the pressure normal gradients g_1 and g_2 at two arbitrary locations within the room can be obtained as

$$\tilde{S}_{g_1g_2}(\omega)=\frac{\int \tilde{S}_{g_1g_2}(\omega)d\vec{k}/|\vec{k}|}{\int d\vec{k}/|\vec{k}|}=S_{p_0}(\omega)\frac{\int \exp\left(-i\vec{k}\cdot(\vec{r}_1-\vec{r}_2)\right)(\vec{k}\cdot\vec{n}_1)(\vec{k}\cdot\vec{n}_2)d\vec{k}/|\vec{k}|}{\int d\vec{k}/|\vec{k}|} \tag{6}$$

As indicated by expressions (5) and (6), the diffuse acoustic field is simply characterized by the data related to the reference point (R_{p_0}, S_{p_0}) and the spatial correlation functions. These latter functions depend only on the frequency and the relative position of the considered points.

3. COUPLED FEM-BEM MODEL

The evaluation of the response of a mechanical elastic structure interacting with a surrounding acoustic fluid can be obtained in many different ways. In the present study, a displacement-based finite element model is selected for the structure while an acoustic boundary element model is used for the acoustic fluid. This choice is quite natural for handling the *low frequency* response of a conventional structure.

The boundary integral representation which sustains the development of the BEM model presents several advantages (the radiation boundary condition at infinity is automatically satisfied while the mesh requirements are reduced since only the boundary (i.e. radiating) surface has to be discretized. Several boundary integral representations are available for describing the acoustic field in the vicinity of a vibrating structure. The formulation selected here is based on an indirect representation in terms of single and double layer potential densities along the mean boundary surface. This leads to the following integral representation for the pressure at a point P outside from the boundary surface S:

$$p(P)=\int_S\left\{\mu(Q)\frac{\partial G(P,Q)}{\partial n_Q}-\sigma(Q)G(P,Q)\right\}dS(Q) \tag{7}$$

where σ and μ are the single and double layer potentials, respectively while G is the related Green's function.

This integral representation relies on the preliminary determination of layer potentials along the boundary surface S. This requires in turn to set up appropriate boundary conditions. In this context, a generalized approach has been selected such that both pressure, normal velocity, normal admittance and transfer admittance boundary conditions can be considered along the boundary surface. Additionally coupling effects with a mechanical structure can be accounted for.

The discrete coupled system appears as

$$\begin{bmatrix} Z^S & C & 0 \\ C^T & Z^F_{cc} & Z^F_{cu} \\ 0 & Z^F_{uc} & Z^F_{uu} \end{bmatrix}\begin{pmatrix} y^S \\ y^F_c \\ y^F_u \end{pmatrix}=\begin{pmatrix} x^S \\ x^F_c \\ x^F_u \end{pmatrix} \tag{8}$$

where Z^S is the structural impedance matrix, Z^F is the fluid impedance matrix, C is the geometrical coupling matrix, x^S is the structural load vector, x^F is the fluid load vector, y^S is the structural response vector (displacement or modal participation factors) and y^F is fluid response vector (single and double layer potentials). Subscripts c and u denote respectively coupled and uncoupled fluid degrees of freedom.

This formulation assumes implicitly that coupling effects occur only on one part of the boundary surface. The other part (which could be reduced to zero) involves purely acoustic boundary conditions. In a compact form, (13) could be rewritten as

$$Z(\omega)\cdot y = x \text{ or } y = H(\omega)\cdot x \tag{9}$$

where Z is the global impedance matrix of the coupled system, H is the global admittance matrix of the coupled system (also called the frequency domain transfer function matrix), x is the load vector and y is the response vector.

4. EVALUATION OF THE RANDOM RESPONSE

The random response can be evaluated quite efficiently provided the load vector can be expressed as a combination of load patterns:

$$x(t)=L\cdot f(t) \tag{10}$$

where L is a matrix of size ($n \times l_p$) and f a vector of size ($l_p \times 1$).

One assumes that functions f_k are stationary random processes characterized by the correlation matrix R_f (of size $l_p \times l_p$):

$$R_f(\tau)=E\left[f(t)\cdot f^T(t+\tau)\right] \tag{11}$$

The power spectral density matrix S_y related to the response can be obtained as:

$$S_y(\omega)=H(-\omega)\cdot L\cdot S_f(\omega)\cdot L^T\cdot H^T(\omega) \tag{12}$$

Defining by x_l and x_l' the matrices (n x l_p) of responses to each load pattern:

$$x_l = H(\omega) \cdot L \quad (13)$$

$$x_l' = H(-\omega) \cdot L \quad (14)$$

the power spectral density matrix S_y can be evaluated from

$$S_y(\omega) = x_l' \cdot S_l(\omega) \cdot x_l^T \quad (15)$$

The computational effort required is substantially reduced because the key operation is the evaluation (at each discrete frequency) of the responses associated to the l_p load patterns.

5. NUMERICAL EXAMPLE

The considered plate is represented at Figure 2. This square plate (side length a = 1m, thickness = 0.01 m) is made from steel (Young modulus = 2.1x10^{11} N/m2, Poisson ratio = 0.3, mass density = 7800 kg/m3). The plate is simply supported along the four edges. The excitation is a point load (F = 1000 N) at location (x_F = 0.6 m, y_F = 0.7 m). The load amplitude is considered as a random stationary process. The auto-correlation function is described by Dirac's function (Figure 3). The related PSD is thus constant on the whole frequency range (*white* noise).

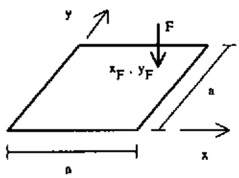

Figure 2: Plate problem.

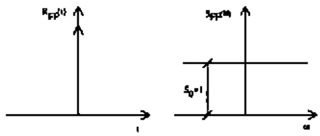
Figure 3: Auto-correlation and power spectral density functions.

A regular mesh of 100 QUAD4 elements has been selected for this calculation and the first 16 modes have been extracted. The related eigenfrequencies are summarized and compared to exact values in Table 1. The eigenvectors are normalized versus the structural mass matrix. In the present application, transversal displacements u_i at 2 points are considered. These points are the loaded point (labeled 1) and the plate's center (labeled 2). PSD and cross-PSD functions S_{11} and S_{12} related to these displacements are represented in Figures 4-1 and 4-2. The associated auto-correlation and cross-correlation functions R_{11} and R_{12} are represented in Figures 5-1 and 5-2. In these figures, the numerical solution obtained is compared to the exact solution.

As it can be seen from inspection of Figures 4-1 and 4-2, a good agreement is obtained for PSD's (except at frequencies higher than ... 150 Hz ... where the mesh should be refined and the modal basis extended).

Discrete auto-correlation and cross-correlation functions are also in a very good agreement with the analytical solution.

Mode	FEM (Hz)	Exact (Hz)	Mode	FEM (Hz)	Exact (Hz)
1 (1,1)	48.65	49.33	9 (3,1)	250.47	246.64
2 (1,2)	122.70	123.32	10 (3,2)	311.94	320.64
3 (1,3)	250.45	246.64	11 (3,3)	418.01	443.96
4 (1,4)	441.71	419.30	12 (3,4)	581.64	616.61
5 (2,1)	122.70	123.32	13 (4,1)	441.71	419.30
6 (2,2)	192.28	197.32	14 (4,2)	493.21	493.29
7 (2,3)	311.94	320.64	15 (4,3)	581.64	616.61
8 (2,4)	493.20	493.29	16 (4,4)	713.24	789.26

Table 1: Plate's eigenfrequencies

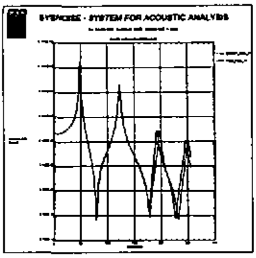

Figure 4-1: Power spectral density S_{11}.

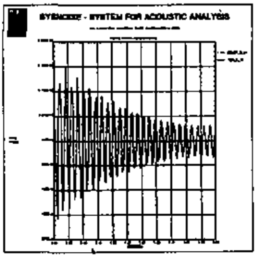

Figure 5-1: Auto-correlation function R_{11}.

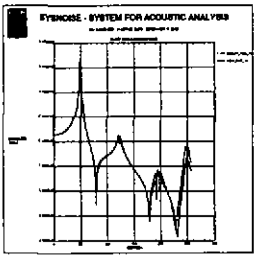

Figure 4-2: Cross PSD S_{12}.

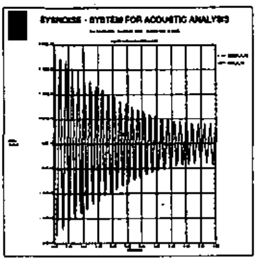

Figure 5-2: Cross-correlation function R_{12}.

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