

# THE EFFECT OF SIDE-RESTRAINT BEARINGS ON THE PERFORMANCE OF BASE-ISOLATED BUILDINGS

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## 1. INTRODUCTION

Ground-borne vibration levels are often large enough to justify the base isolation of a building, that is, the building is designed with vibration isolation bearings between the building and its foundation. Either steel springs or rubber bearings are employed and both have been used with success since the introduction of rubber bearings in the 1960s.

Conventional designs locate the bearings at the base of a building, aligned in the vertical direction so as to isolate the building from vertical motion of its foundation. In some cases, in order to accommodate horizontal loads, additional bearings are required aligned in the horizontal direction. The requirement for these *side-restraint* bearings varies and may involve the restraint of small building elements, such as lift shafts, or, in some cases, the retention of neighbouring structures such as the retaining walls of a basement cavity.

This paper describes a theoretical investigation into the effect of side-restraint bearings on the performance of base-isolated buildings. Three generic models, based on a modern concrete-framed building, are used to demonstrate that a building's flexibility, the nature of the vibration input and the presence of a flexible foundation are all important in determining isolation performance. The models indicate that side-restraint bearings reduce isolation performance, primarily due to the transmission of additional vibration through bending of the structure against which they bear. It is concluded that, for maximum performance, the stiffness of any side-restraint bearings should be minimised, although further work is required before more specific statements may be made.

## 2. MODELS OF A BASE-ISOLATED BUILDING

This section presents three generic models of a building design that includes side-restraint bearings between the primary structure and the walls of the basement cavity. The main physical properties used in the models are given in the appendix.

### 2.1 Rigid-Body Model

A two-dimensional rigid body is the simplest model of a base-isolated building that enables the effect of side-restraint bearings to be investigated. A rigid body of uniform density represents the building and damped linear springs,  $k_b$  and  $k_s$ , represent the base and side bearings respectively.

In the vertical direction, the model behaves as a mass on a spring. The stiffness of the base bearings  $k_b$  may therefore be chosen to give a particular vertical isolation frequency  $f_v$ , as given by Equation 1. Only the longitudinal stiffness of each bearing is modelled and therefore the side-restraint bearings, which are aligned in the horizontal direction, have no effect on the vertical isolation frequency.

$$k_b = 2\pi^2 f_v^2 M \quad (1)$$

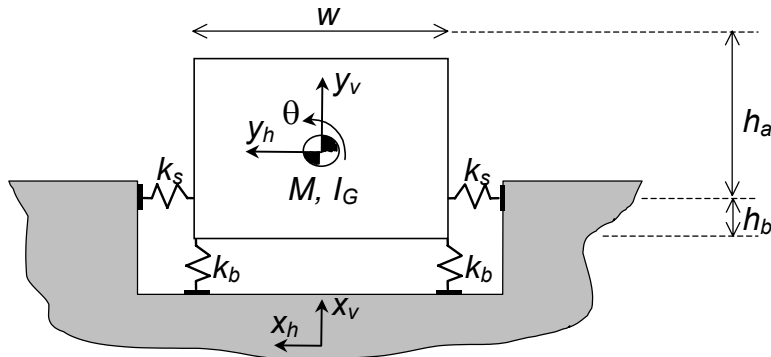


Figure 1: A two-dimensional rigid-body model of a base-isolated building. The building is assumed to be of uniform density and isolated from a rigid foundation by linear springs.

The foundation is assumed to be rigid, with an input motion described by the harmonic displacement amplitudes  $x_h$  and  $x_v$  in the horizontal and vertical directions respectively. The resultant motion of the building is described by three linear equations giving the rotation amplitude  $\theta$  and the displacement amplitudes  $y_h$  and  $y_v$ .

## 2.2 Flexible-Column Model

The rigid-body model accounts for the global behaviour of a building but is unable to model vibration propagation within a building's flexible structure. The flexible-column model provides the simplest way to account for the latter by using Euler beam theory and the solution for an elastic bar [1]. Either an entire building or a single isolated structural element may be represented, depending on the building's height/width aspect ratio.

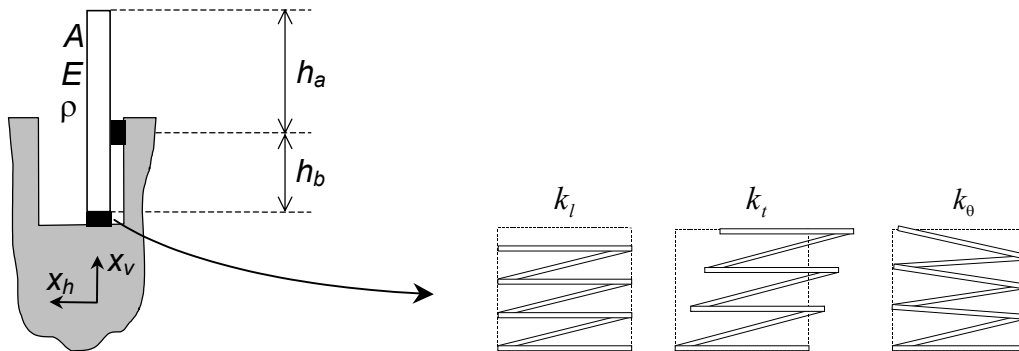


Figure 2: A flexible-column model of a base-isolated building. Euler beam theory and the solution for an elastic bar describe the building, which is isolated from a rigid foundation by linear springs representing the three stiffnesses associated with each isolation bearing.

The representation of the bearings is improved by accounting for the transverse  $k_t$  and rotational  $k_\theta$  stiffness, as well as the longitudinal stiffness  $k_l$ . For the base bearing, the value of  $k_l$  is determined in the conventional way from the desired vertical isolation frequency by assuming the column behaves as a rigid mass on a spring. However, the transverse stiffness of the side bearing also contributes to the vertical stiffness, with the result that the isolation frequency is higher than intended. In practice, the values of  $k_t$  and  $k_\theta$  depend heavily on the design details of the particular application and no general relationship exists between them and the value of  $k_l$ .

Motion of the rigid foundation results in longitudinal, transverse and rotational deformation of the bearings and the isolation performance predicted by the flexible-column model cannot be described

simply in terms of displacement amplitudes. Instead, a power-flow approach is used, as discussed in Section 3.2.

### 2.3 Portal-Frame Model

The portal-frame model is the most comprehensive of the three models presented here. This uses a flexible two-dimensional portal frame to represent the primary structure of a modern concrete-framed building, thereby accounting for the longitudinal and transverse behaviour of the individual floors and columns, and the dynamic coupling between them.

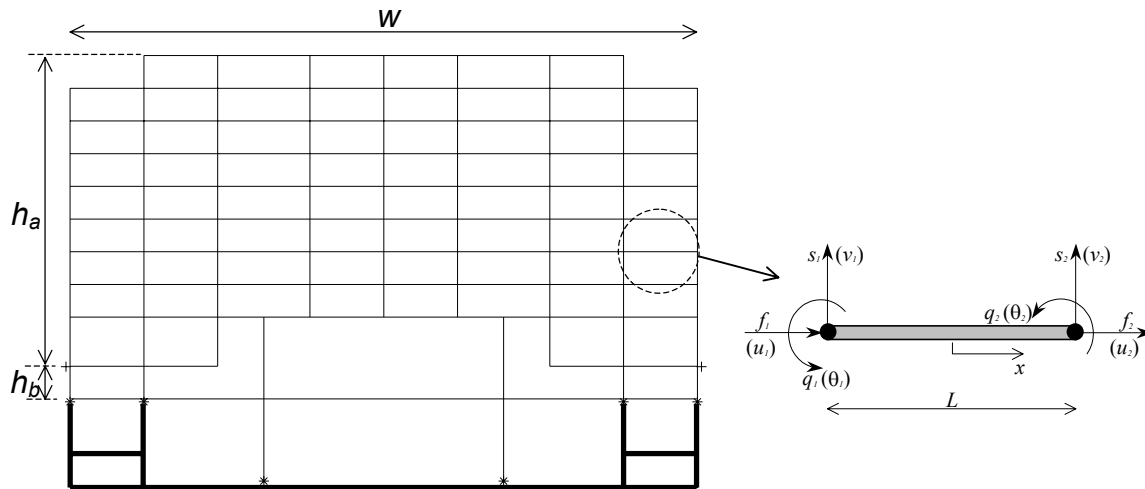


Figure 3: A flexible two-dimensional portal-frame model of a base-isolated building. The isolation bearings consist of (\*) 6 base bearings and (+) 2 side-restraint bearings. The unisolated structure of the portal frame is shown bold.

The dynamic behaviour of each element of the frame is modelled using Euler beam theory and the solution for an elastic bar – an approach known as the dynamic stiffness method [2]. The isolation bearings are modelled in the same way as those of the flexible-column model. The base bearings are assumed to be identical with a longitudinal stiffness determined from the desired vertical isolation frequency and the total isolated mass of the building.

Section 3.3 presents results from both a rigid and a flexible foundation model. With the former, the unisolated structure of the portal frame is coupled directly to the foundation and the input motion is again described by the displacement amplitudes  $x_h$  and  $x_v$ . In the case of the flexible foundation, the boundary-element method [3] is used to model the ground as a uniform elastic half-space containing the basement cavity. The foundation model is also two-dimensional and behaves as though it were part of a three-dimensional foundation that is invariant in the third dimension.

## 3. RESULTS & DISCUSSION

A common measure of isolation performance is *insertion gain* (IG), which represents the benefit of inserting isolation bearings beneath a building. This is the ratio, usually expressed in decibels, of

the vibration response of the building with the bearings in position  $x_{isol}$  to that with no bearings at all  $x_{unisol}$ . For example, the vertical IG of the rigid-body model described in Section 2.1 is given by:

$$IG = 20\log_{10}\left(\frac{x_{isol}}{x_{unisol}}\right) = 20\log_{10}\left(\frac{y_v}{x_v}\right) \quad (2)$$

### 3.1 Rigid-Body Model

Figure 4 shows the IG predictions for a building with a 3.5 Hz isolation frequency and identical side and base bearings, that is  $k_s = k_b$ .

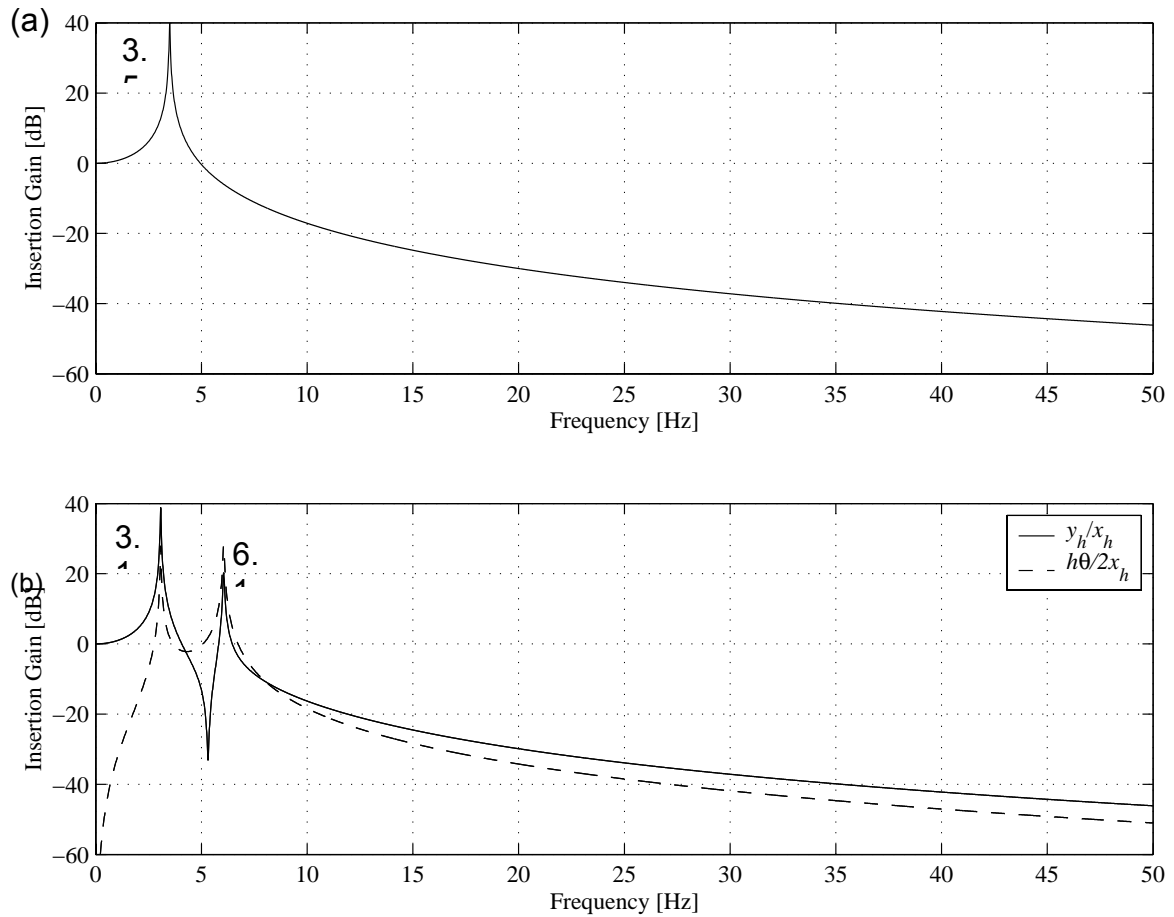


Figure 4: The insertion gains predicted by the rigid-body model when subject to (a) vertical and (b) horizontal motion of the foundation.

Under vertical motion, the IG curve takes the form of the familiar resonant response curve of a single-degree-of-freedom oscillator. Since only the longitudinal stiffness of each bearing is

modelled, the introduction of side-restraint bearings has no effect on the vertical isolation performance predicted by this model.

Horizontal motion of the foundation leads to both horizontal displacement and rotation of the building, due to its centre of mass lying above the level of the side-restraint bearings. Figure 4(b) therefore shows two curves:  $y_h/x_h$  corresponds to horizontal displacement of the building's centre of mass and  $h\theta/2x_h$  corresponds to rotation (IG is not strictly an appropriate term for  $h\theta/2x_h$  since there is no rotation of the unisolated building). Despite the fact that the side and base bearings are identical, neither of the two resonance peaks in Figure 4(b) occurs at 3.5 Hz; the first occurs at 3.1 Hz, the second at 6.1 Hz. This is because coupling between  $y_h$  and  $\theta$  ensures that both modes involve a combination of translation and rotation.

The conclusion from these results is that, if significant horizontal motion exists, the stiffness of the side-restraint bearings should be minimised to keep the resonance frequencies as low as possible.

### 3.2 Flexible-Column Model

Figure 5 shows the vertical IG predicted by the flexible-column model when subject to vertical motion of the foundation. The values of  $x_{isol}$  and  $x_{unisol}$  in Equation 2 are taken to be the displacement amplitudes at the base of the isolated and unisolated column. The steady improvement in performance with frequency predicted by the rigid-body model is now disrupted by the longitudinal vibration modes of the column. Here, only the longitudinal behaviour of a single column has been accounted for and it is to be expected that accounting fully for the flexibility of the whole building will lead to poorer performance.

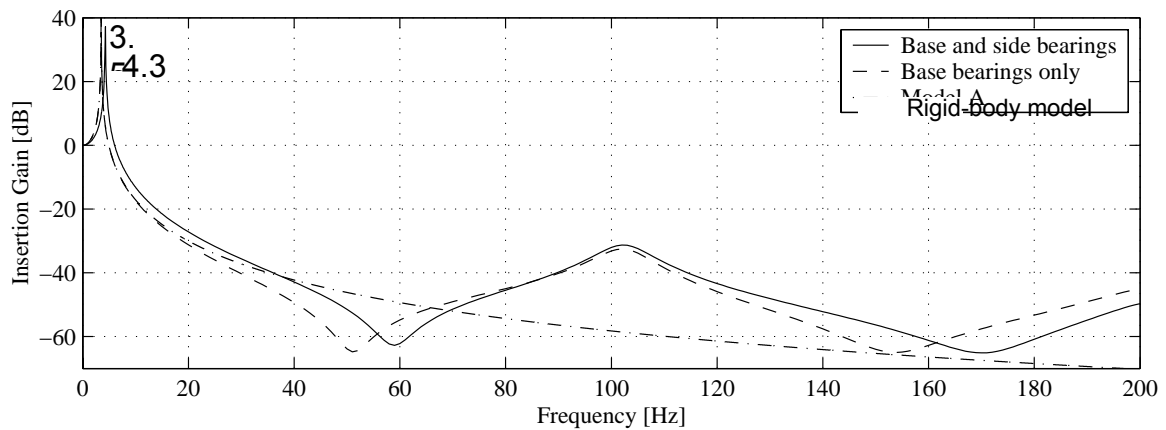


Figure 5: The vertical insertion gain at the base of the flexible-column model when subject to vertical motion of the foundation. The corresponding prediction from the rigid-body model is shown for comparison.

These results also illustrate how the addition of the side-restraint bearing shifts the IG curve to higher frequencies; the resonance peak shifting from 3.5 to 4.3 Hz. This is due to the additional vertical stiffness provided by the transverse stiffness of the side-restraint bearing. Clearly, if a particular vertical isolation frequency is required, the total vertical stiffness must be accounted for and not just that associated with the longitudinal stiffness of the base bearings.

### Power-Flow Insertion Gain

Figure 5 shows only one of many possible, different IG curves that may be calculated with the flexible-column model. This is because the column has 6 'inputs' – through longitudinal, transverse and rotational deformation of each bearing – and any number of 'outputs', since the response may be calculated at any point along the column and in either the vertical or horizontal direction. While IG remains useful if a particular location and direction is of concern, it no longer provides an overall measure of isolation performance.

As a more appropriate measure, *power-flow insertion gain* (PFIG) is now considered. This is based on the total mean vibrational power entering the column; the underlying principle being that the vibrational energy entering a building drives all internal structural vibration and re-radiated noise, assuming there are no internal sources of either. PFIG enables a single measure of performance to be defined for a given vibration source:

$$\text{PFIG} = 10 \log_{10} \left( \frac{\bar{P}_{isol}}{\bar{P}_{unisol}} \right) \quad (3)$$

where  $\bar{P}_{isol}$  and  $\bar{P}_{unisol}$  are the total mean power flows entering the column in the isolated and unisolated conditions, as calculated from the axial and bending stresses and strains [2, 4].

Figure 6(a) shows the PFIG calculated for the flexible-column model when subject to vertical motion of the foundation. This is very similar to Figure 5 because the response of the column to vertical motion is dominated by the column's longitudinal vibration modes. Both figures indicate that, in this case, the effect of the side-restraint bearing on the isolation performance is negligible. There are two reasons for this. Firstly, the transverse stiffness of the side-restraint bearing is small compared to the longitudinal stiffness of the column. Secondly, at the frequencies considered, the transverse deformation of the side-restraint bearing is small due to the rigid foundation and the low relative displacement between the base of the column and the location of the side-restraint bearing.

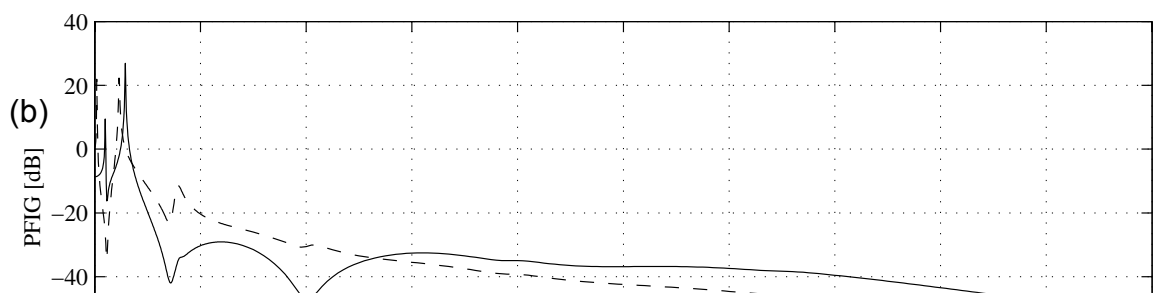
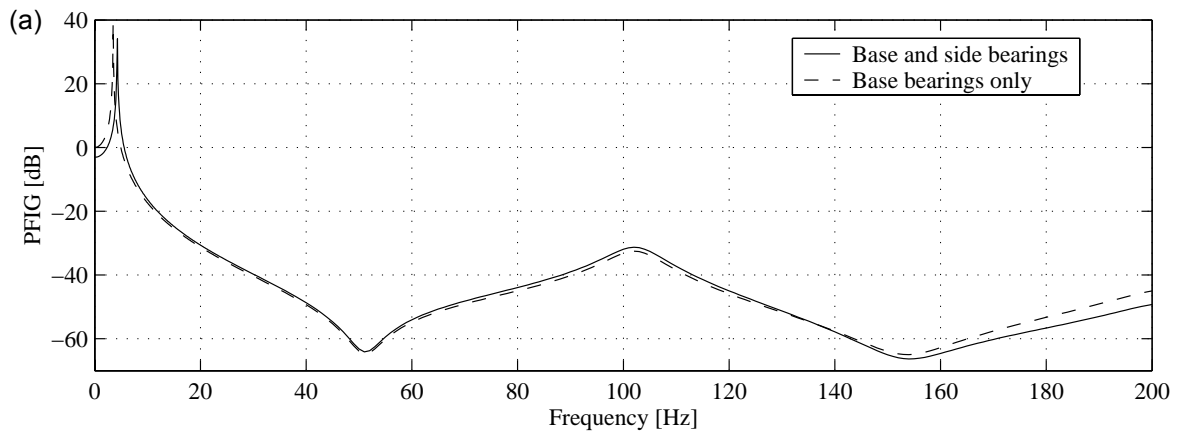


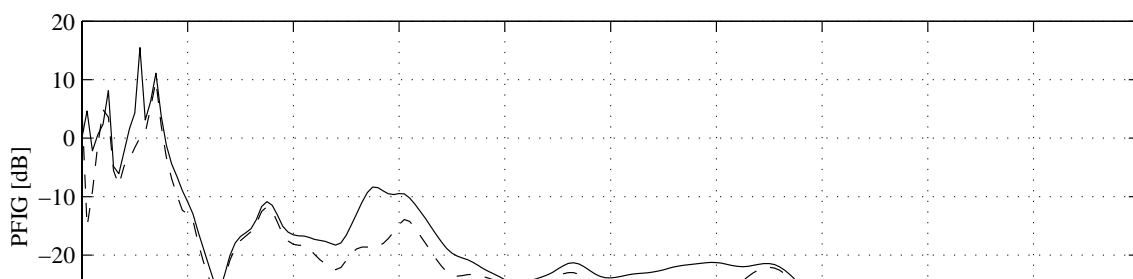
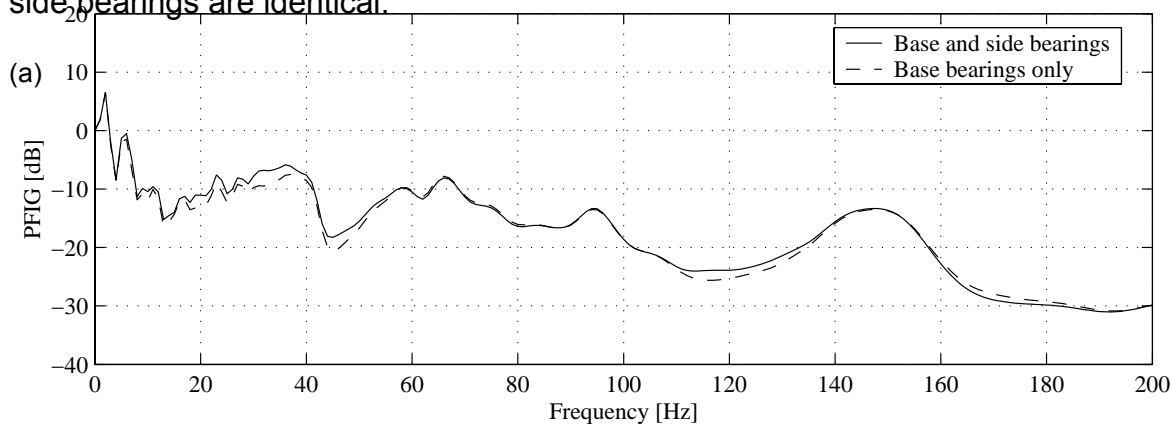
Figure 6: The power-flow insertion gains predicted by the flexible-column model when subject to (a) vertical and (b) horizontal motion of the foundation.

Under horizontal motion of the foundation, the bending modes of the column dominate and the effect of the side-restraint bearing is more significant, as evident in Figure 6(b). This is because the side-restraint bearing is ideally placed to apply a transverse force to the column that is efficient at exciting bending vibration.

The conclusion from this model is that the stiffness of the side-restraint bearings should be minimised, not only to keep the global resonance frequencies of the building as low as possible, but also to minimise the excitation of bending vibration in the building structure.

### 3.3 Portal-Frame Model

The primary reason for considering the portal-frame model is the flexible nature of a building. Figure 7 illustrates the effect this has on the predictions of PFIG under vertical and horizontal motion of the rigid foundation. In this case, the base and side bearings are identical.





(b)

Figure 7: Power-flow insertion gains predicted by the portal-frame model when subject to (a) vertical and (b) horizontal motion of the rigid foundation. Identical stiffnesses are assigned to the base and side bearings.

It is clear that the various vibration modes of the building significantly reduce the isolation performance. As with the flexible-column model, the addition of the side-restraint bearings has a greater effect on the performance under horizontal motion of the foundation due to the transmission of additional vibration through bending of the structure.

Accounting for the flexibility of a foundation is just as important as accounting for that of a building. As well as accounting for the interaction between the building and the ground, the flexible foundation model enables more realistic vibration fields to be investigated. The field due to a point harmonic force, applied at a depth of 7m and 62 m off the centre-line of the building, is considered here. This generates pressure and shear waves that radiate outward to strike the base of the portal frame, in a similar way to the excitation from an underground railway tunnel. Some example plots of the resulting vibration of the isolated building are shown in Figure 8.

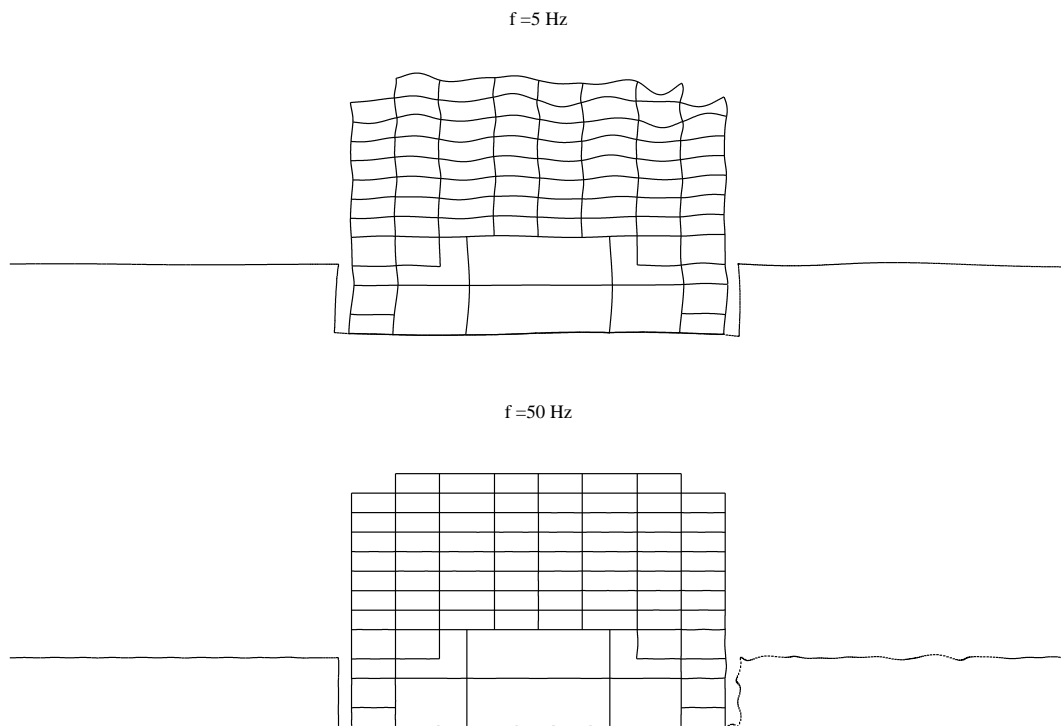
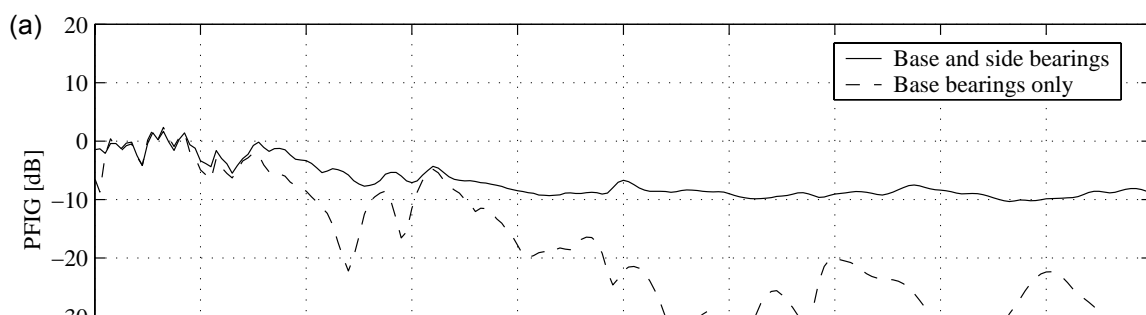


Figure 8: Vibration predicted by the portal-frame model when subject to a ground vibration field due to a buried unit-amplitude point force applied in the horizontal direction at a depth of 7m and 62 m off the centre-line of the building. Identical stiffnesses are assigned to the base and side bearings. All displacements are magnified by  $2 \times 10^9$ .

The effect of the flexible foundation on the predictions of isolation performance may be observed by comparing Figures 7 and 9. Firstly, the peaks occurring in the frequency range from 0 to 20 Hz are reduced in amplitude – a direct result of coupling the building to a flexible foundation, which acts as a vibration sink as well as a source. Secondly, the addition of the side-restraint bearings has a significant effect under excitation from both a horizontally and vertically applied force. This is because both forces result in horizontal ground motion at the locations of the side bearings.

The results of Figure 9 indicate that the vibration field due to the vertically applied force represents the worst-case excitation. Figure 10 reproduces these results together with additional results obtained by varying the stiffness of the side-restraint bearings. Side-bearing stiffnesses of 1.00, 0.30 and 0.10 times the stiffness of the base-bearings are considered. These cases correspond to 'horizontal isolation frequencies', as would be observed with a single-degree-of-freedom model, of 0.58, 0.32 and 0.18 times the vertical isolation frequency of 3.5 Hz. It is clear that minimising the stiffness of the side-restraint bearings maximises the isolation performance.



(b)

Figure 9: Power-flow insertion gains predicted by the portal-frame model when subject to ground vibration fields due to a buried point force applied in the (a) vertical and (b) horizontal direction at a depth of 7m and 62 m off the centre-line of the building. Identical stiffnesses are assigned to the base and side bearings.

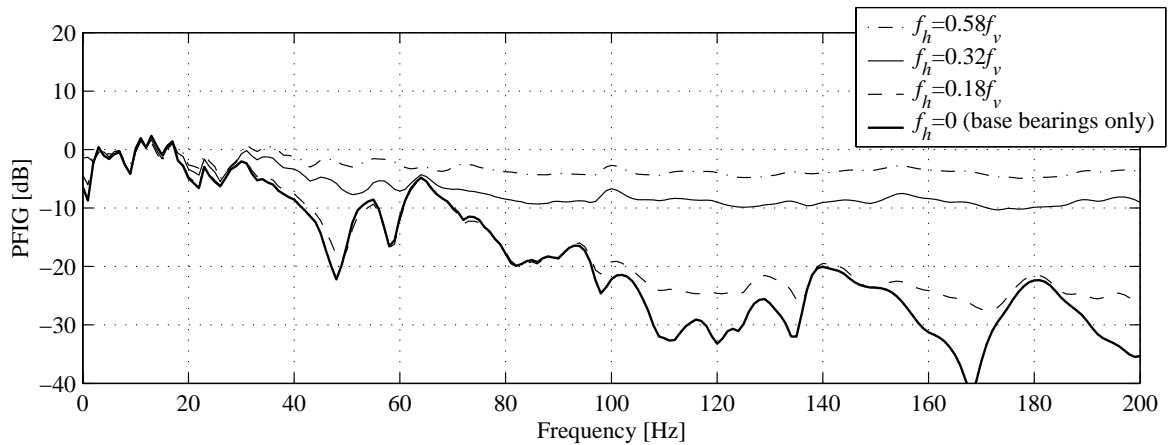


Figure 10: The effect of side-bearing stiffness on the power-flow insertion gain predicted by the portal-frame model. The results correspond to the 'worst-case' ground vibration field due to a vertically-applied point force at a depth of 7m and 62 m off the centre-line of the building.

#### **4. CONCLUSIONS**

This paper has reported on an investigation into the effect of side-restraint bearings on the performance of base-isolated buildings. A building design has been considered that includes side-restraint bearings between the primary structure and the walls of the basement cavity. The results from three mathematical models allow the following conclusions to be drawn:

- the longitudinal, transverse and rotational stiffnesses of the isolation bearings all influence isolation performance, including those associated with side-restraint bearings;
- the stiffness of side-restraint bearings should be minimised to keep the global resonance frequencies of the building as low as possible and to minimise the transmission of additional vibration through bending of the structure against which they bear.

It is noted that the scope of this investigation has been limited, particularly concerning the location of the isolation bearings and the nature of the ground vibration field. Further work is required before more specific statements may be made.

#### **REFERENCES**

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4. Talbot, J.P. and Hunt, H.E.M., *On the performance of base-isolated buildings*. Building Acoustics, 2000. 7(3).

#### **APPENDIX**

The main physical properties used in the model are as follows:

## The Effect Of Side-Restraint Bearings - J.P. Talbot & H.E.M. Hunt

### Dimensions

Rigid-body and portal-frame models (full building):  $h_a = 38\text{m}$ ,  $h_b = 4\text{m}$ ,  $w = 76.5\text{m}$

Flexible-column model (single column):  $h_a = 6\text{m}$ ,  $h_b = 4\text{m}$

### Material Properties

	Concrete	Soil (typical values for clay)
Density:	2400 kg/m <sup>3</sup>	2000 kg/m <sup>3</sup>
Young's Modulus:	10 kN/mm <sup>2</sup>	0.56 kN/mm <sup>2</sup>
Poisson's ratio:	0.2	0.4
Damping loss factor:	0.1	0.02

Isolation Bearings

Isolation frequency: 3.5Hz

Damping loss factor (undamped steel spring): 0.01