Proceedings of the Institute of Acoustics

FINITE ELEMENT MODELLING OF UNDERWATER ACOUSTIC TRANSDUCERS

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1. INTRODUCTION

The Finite Element Method was devised originally in the 1950' and 1960's for the analysis of large and complicated mechanical structures particularly in the aerospace industry, in which the field variables are stress and strain, initially for static situations and then extended to dynamic ones, both in the frequency and time domains (for continuous or impulsive responses). Without the availability of high speed digital computers with large capacity memories the method would be totally impractical for any but the most trivial problem, however useful work can even be done on a desktop PC of minimum capacity. It was soon extended from discrete structures to continuous media and other energy fields such as fluid dynamics, electrodynamics, heat flow and of course acoustics, and so to situations where there is coupling between different fields; we are concerned here with the coupling between mechanical stress and strain and electric or magnetic fields through the piezoelectric or piezomagnetic effects. The piezoelectric effect is used much more than the piezomagnetic since the active materials are more readily available and are easier to use. In the analysis of sonar transducers, where both active and passive materials are used, several varieties of analysis exist together, for the basic mechanical structure, for the electromechanical coupling in the piezoelectric material, for the radiation into a continuous medium; full analysis of a powerful source may also include the flow of heat generated by internal losses. It is the dynamic response which is always investigated, but the static response to external pressure is important for transducers to be used at great depths. Most analyses assume linearity, but non-linear effects are sometimes important, for example in high power sources where piezoelectric nonlinearity can be present.

2. GENERAL PRINCIPLES

The basic principle can be stated very simply as the dividing up of a body into a number of simply defined components, the elements, the external behaviour of each of which can be described in discrete terms, so that a continuous problem is converted into a set of discrete problems. This is of particular value in most situations because the discontinuities within bodies and on their boundaries cannot be easily represented by analytical functions. A continuous situation would be defined by a system of differential equations, given that the structure is defined analytically, and these would be solved in a general sense by integration. In the Finite Element Method the behaviour within each element is defined analytically in a simplified way by assuming that the variables (mechanical displacement, stress, strain, electric field) are either constant through the element or vary in a simple way (linearly or quadratically at the most) with position; the overall behaviour of the element is then defined by what happens at certain points on the outside edges or surfaces, these are the nodes; adjacent elements are coupled by nodes in common, and because the same order of variation in the variables between the nodes must be used in the conjoined elements the common boundaries also coincide, thus satisfying the requirements for continuity between elements and equilibrium at the nodes. The responses referred to the nodes are derived essentially by applying the principles of virtual work and integrating through the element.

If the driving forces and responses are sinusoidal, they can be represented by complex functions, and so can the terms in the matrix of element properties. If energy is not dissipated in any way (no internal losses and no radiation), the complex factors cancel and the method of solution is essentially the same as for the static case except for the addition of inertial terms (dimensionally the product of mass and the square of the frequency). Internal losses are include by making the material properties complex, but radiation into an external medium is not so simple. The more common way of including it is by Boundary Elements, which are defined on the radiating surface, thereby reducing that part of the problem to a two-dimensional one instead of a three-dimensional one, but an alternative technique is through infinite Finite Elements. There are mathematical problems, such as singularities, which make the Boundary Element approach more complicated, and also matrices which are symmetrical in FE become unsymmetrical when BE components are included, thus substantially increasing the memory requirements for the same number of variables.

The analysis of the complete structure follows along the line of dividing it into elements, usually a large or even very large number, setting up the nodal response functions of each element, combining these into the global array covering the behaviour of the whole structure, adding in the external influences and constraints, and solving the resultant set of simultaneous equations which relate independent and dependent variables.

3 APPLICATIONS

The Underwater Acoustics Group of the Institute has been organising meetings on transducers for underwater applications for most of the life of the Institute, but in the first such meeting, the Transducer Workshop in 1976 (2), there was no mention of Finite Elements although the earliest, commonly cited, reference in the literature to FE Analysis applied to piezoelectric materials was in 1970 (1). The first reference to FE in any form in our Proceedings was in the meeting in December 1978, Transducer Arrays and Array Processing (3), where there was a brief mention of FEM being used to define the static mechanical stiffness of the mounting for a partial sphere transducer, but it was realised that this made very little contribution to the accuracy of the otherwise analytical methods used. Since then the FEM has been used increasingly in the designs described at our meetings, starting with one paper in the 1980 Conference, Transducers for Sonar Applications (4), on a ring-shell flex-tensional transducer; the software used was the DREA (Canada) in-house program MAVART, with the radiation load modelled by absorbing nodes terminating fluid elements surrounding the mechanical structure. The next Conference, New Techniques in Sonar Transducers (5), out of 15 papers there were three on FEM and these were of a general nature rather than on specific designs; the programs, where specified, were developed in house. At the next Conference, in 1987 (6), out of 17 papers there were three in which the FEM was a significant part; for one the program was a limited in-house development, but the other two made use of the major French development ATILA. By 1988 the Group realised that the FEM was becoming highly important in the design of transducers, and so a one-day Conference (7) specifically on this topic was arranged, with eight papers of a tutorial nature, more on fundamentals rather than on particular designs; most of the contributions referred to the commercial FE package PAFEC, which has been highly developed particularly for transducer design (not its only application of course), but one paper on quality assurance in structural analysis referred to the program SESAM69C which makes considerable use of substructuring to several levels. By 1990 development and use of FEM programs had increased markedly, and of 27 papers in that year's Conference (8) 8 were based on this subject, with a further paper on an allied technique originating in the electromagnetic field, Trans-mission Line Modelling (TLM), which has some limitations when applied to mechanical structures. The commercial code ABAQUS, which does not have piezoelectric elements but which is good for non-linear problems, was used in studying the static behaviour of a flextensional transducer at great depth. An FE/BE set of programs developed at Birmingham University (PHOEBE) was used in optimising the design of a simple flooded ring plus mounting for the best omnidirectional response. The codes ATILA and PAFEC, already mentioned, and various in-house codes were used. There was a gap of 5 years before the next conference in 1995 (9), at which 23 papers on transducers were presented, of these 13 made significant use of Finite Elements in a wide range of situations. A commercial package, ANSYS, was mentioned for the first time, this is a highly developed program for a wide range of situations including piezoelectrics and acoustic radiation; there was also a useful comparison between MAVART, which in its original form was for axisymmetry only, and PAFEC, which is fully three dimensional. The Conference on Numerical/Analytical Methods (10) was not primarily concerned with transducers, only four papers out of 17 being on them; the usual subjects of free-flooded rings and flextensionals came up, but one paper extended the ideas of substructuring to include piezoelectric coupling, and this can be useful in reducing the memory requirements in the analysis of arrays of identical elements. Finally in the current year, the Conference Transducers '99 (11) had 24 papers in all, 10 of which were based on the use of FE/BE methods. In conclusion one can say that these methods have contributed enormously to the design of sonar transducers, replacing a lot of practical trial and error in modifying designs which would otherwise have been developed intially by very approximate analytical methods. Conversely they have aided understanding of why apparently simple designs may not work in the way expected.

4 REFERENCES

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